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Improving grasp quality evaluation

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ABSTRACT

The capability to equilibrate external wrenches is crucial in optimal grasp planning. This paper presents a new method for evaluating this capability when the external wrench is unknown. Two criteria are reformulated using the L_2 distance function, and further transformed into two nonlinear optimization problems. The differentiability of the objective functions and choice of initial conditions for global optimization are discussed. Keeping all the merits, that the criteria are applicable to grasps of 3-D objects with any contact types, and that the friction cones are not linearized, this work endows them with several new virtues: (a) Their formulation and computation are unified for both force-closure and non-force-closure grasps; (b) They are independent of the choice of coordinate frame and unit; (c) The object geometry is taken into account; (d) The computational efficiency is even higher than some methods by linearizing the friction cones.

1. Introduction

The capability of a grasp to equilibrate external wrenches on the grasped object is the key index for choosing a good grasp. During the past two decades, various evaluation methods were proposed [1–15]. Until now, the criteria can be generally described as the scale factor of a required wrench set such that the scaled set just fits within a grasp wrench set.

The grasp wrench set consists of the resultant wrenches that can be generated by the grasp with limited contact forces. Each contact force is bounded by its friction cone. The overall contact forces are limited by limiting the sum or the maximum of their normal components. Accordingly, by linearizing friction cones, the grasp wrench set is given by the convex hull of the primitive contact wrenches, or of their Minkowski sums [1,2]. Therefore, for any required wrench set, there are two scale factors with respect to the two grasp wrench sets.

Selections of required wrench sets are various. In the absence of any task information, Kirkpatrick et al. [1], and Ferrari and Canny [2] selected the 6-D unit ball centered at the origin. In this case, Miller and Allen [3] used the "Qhull" program to compute the former scale factor. Borst et al. [4] put forward an incremental algorithm for computing both. Xiong et al. [5] calculated the latter one using nonlinear programming technique. Zhu and Wang [6] substituted a polytope for the ball and computed the former factor by solving linear programming problems. Liu et al. [7] expressed two scale factors as min–max and max–min problems. Zheng and Qian [8] clarified their difference. As the force and moment components of a wrench have different units and the latter depends on the chosen coordinate frame and length unit, changing either frame or unit will alter the grasp wrench sets but have no effect on the unit ball. As a result, the scale factors vary with the change. One popular remedy is replacing the ball by a set with the same variances as the grasp wrench sets [9–13]. Li and Sastry [9] suggested a task-oriented ellipsoid. Pollard [10] offered an object wrench set comprising the external wrenches that are yielded by acting pure forces on the object surface. Combining the ideas of [9,10], Borst et al. [11] adopted an ellipsoid enclosing an object wrench set. Strandberg and Wahlberg [12] generalized the idea of [10] to 3-D objects with frictional point contacts and added an offset wrench to denote some other kinds of external wrenches. Watanabe and Yoshikawa [13] utilized a convex polyhedral required wrench set for facilitating the computation. Other remedies include dividing the moment component by a length, to eliminate the unit dependence [2,10], treating the force and moment components separately to avoid the ambiguity between them [14], and considering the change of coordinate frame and length unit in establishing the wrench set, so as to compute the largest scale factors over all possible changes [15].

The above work remarkably enhanced the grasp quality evaluation. Each method has its virtues and weakness, as listed in Table 1. Some methods cannot apply to non-force-closure grasps, or treat them differently from force-closure grasps [1–7,9–15]. The method [8] formulates the two cases as a single optimization problem, but it has other weaknesses. Users often find difficulty in selecting among these methods. In view of the situation, this paper seeks a method with all the virtues for computing the

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Table 1
Comparing different methods of grasp quality evaluation.

Reference	Knowledge of external wrenches	Non-force-closure case	Dependent on frame or unit	Contact types	Linearizing the friction cone	Considering the object geometry
[1]	Not required	N.A.	Both	FPC	/	No
[2]	Not required	N.A.	Frame	FPC, PCwF	Yes	No
[3]	Not required	Separate treatment	Frame	FPC, PCwF	Yes	No
[4]	Not required	N.A.	Both	FPC, PCwF	Yes	No
[5]	Not required	N.A.	Both	FPC, PCwF	No	No
[6]	Not required	Separate treatment	Both	FPC, PCwF	Yes	No
[7]	Not required	N.A.	Both	FPC, PCwF, SFC	No	No
[8]	Not required	Unified treatment	Both	FPC, PCwF	Yes	No
[9]	Required	N.A.	No	FPC, PCwF, SFC	No	No
[10]	Required	N.A.	No	FPC	/	Yes
[11]	Required	N.A.	No	FPC, PCwF	No	Yes
[12]	Required	N.A.	No	FPC, PCwF	Yes	Yes
[13]	Required	N.A.	No	FPC, PCwF	No	No
[14]	Not required	N.A.	Frame	FPC, PCwF	No	No
[15]	Not required	N.A.	No	FPC, PCwF	Yes	No
This work	Not required	Unified treatment	No	FPC, PCwF, SFC	No	Yes

FPC, PCwF, and SFC denote frictionless point contact, point contact with friction, and soft finger contact, respectively.

two scale factors. Different from [9–13], we consider the case that the external wrench is entirely unknown, and still take the required wrench set to be the unit ball to equally consider all forms of external wrench in all directions. An improvement over our previous work [8], the grasp wrench sets are formulated for all the three contact types without linearizing friction cones. The moment origin is set at the centroid of contact positions so that the grasp wrench sets are frame independent. Rather than dividing the moment components [2,10], the force components are multiplied by the average distance from contact positions to their centroid so that the grasp wrench sets have the same scales in all wrench directions. Furthermore, by doing this, the scale factors of the unit ball in the grasp wrench sets are directly proportional to the average distance, so that a good grasp should have wide-spread contact positions. Finally, either scale factor is cast into a nonlinear optimization problem. The objective function is differentiable almost everywhere and its derivative is calculated in closed form. As the optimization problem may have local optima, the choice of initial conditions for attaining the global optimum is addressed. Unlike those linearization-based methods for computing the two scale factors [3,6,8,15], this method needs neither to calculate the primitive contact wrenches and their Minkowski sums, nor to determine every facet of a grasp wrench set. Thus it is more efficient, especially when the grasp wrench set is taken to be the Minkowski sum of the primitive contact wrenches and has many vertices [15].

2. Preliminaries

In this section, we introduce the statics involved in grasp quality evaluation and a distance function, which will be used later to formulate the grasp quality criteria.

2.1. Basic knowledge about multi-fingered grasping

Consider an object grasped by an m-fingered robot hand, which makes m_0 frictionless point contacts (FPCs), m_f point contacts with friction (PCwFs), and m_s soft finger contacts (SFCs) with the object surface. Thus $m = m_0 + m_f + m_s$. Let $\boldsymbol{r}_i$ be the position vector of contact i ($i = 1, 2, \ldots, m$), $\boldsymbol{n}_i$ the unit inward normal, and $\boldsymbol{o}_i$ and $\boldsymbol{t}_i$ two unit tangent vectors satisfying $\boldsymbol{n}_i = \boldsymbol{o}_i \times \boldsymbol{t}_i$, all of which are described in the coordinate frame attached to the object.

The contact force $\boldsymbol{f}_i$ can be expressed in the local coordinate frame $\{\boldsymbol{n}_i, \boldsymbol{o}_i, \boldsymbol{t}_i\}$ by

FPC: $\boldsymbol{f}_i = [f_{i1}]$

PCwF: $\boldsymbol{f}_i = [f_{i1} \quad f_{i2} \quad f_{i3}]^{\mathrm{T}}$

SFC: $\boldsymbol{f}_i = [f_{i1} \quad f_{i2} \quad f_{i3} \quad f_{i4}]^{\mathrm{T}}$,

where f_{i1} is the normal force; f_{i2} and f_{i3} are two tangential force components along $\boldsymbol{o}_i$ and $\boldsymbol{t}_i$, respectively; f_{i4} is the spin moment about the contact normal. To avoid separation and slip at contact, $\boldsymbol{f}_i$ must satisfy one of the following contact constraints:

$$\text{FPC: } F_i = \{\boldsymbol{f}_i \in \mathbb{R} \mid f_{i1} \geq 0\} \tag{1}$$

$$\text{PCwF: } F_i = \left\{\boldsymbol{f}_i \in \mathbb{R}^3 \mid f_{i1} \geq 0, \sqrt{f_{i2}^2 + f_{i3}^2} \leq \mu_i f_{i1}\right\} \tag{2}$$

$$\text{SFCl: } F_i = \left\{\boldsymbol{f}_i \in \mathbb{R}^4 \mid f_{i1} \geq 0, \frac{\sqrt{f_{i2}^2 + f_{i3}^2}}{\mu_i} + \frac{|f_{i4}|}{\mu_{si}} \leq f_{i1}\right\} \tag{3}$$

$$\text{SFCe: } F_i = \left\{\boldsymbol{f}_i \in \mathbb{R}^4 \mid f_{i1} \geq 0, \sqrt{\frac{f_{i2}^2 + f_{i3}^2}{\mu_i^2} + \frac{f_{i4}^2}{\mu_{si}'^{\,2}}} \leq f_{i1}\right\}, \tag{4}$$

where μ_i is the coefficient of tangential friction at contact i, and μ_{si} and μ'_{si} are the coefficients of torsional friction for SFC with linear (SFCl) and elliptic (SFCe) models [16], respectively. For PCwF, F_i is a convex cone of $\mathbb{R}^3$ known as the Coulomb friction cone. For FPC and SFC, F_i are convex cones of $\mathbb{R}^1$ and $\mathbb{R}^4$, which we call the friction cones likewise.

The image $\boldsymbol{G}_i(F_i)$ of F_i under the mapping $\boldsymbol{G}_i$ into the wrench space $\mathbb{R}^6$ is a convex cone, which comprises all the feasible wrenches from contact i, where

$$\text{FPC: } \boldsymbol{G}_i = \begin{bmatrix} \boldsymbol{n}_i \\ \boldsymbol{r}_i \times \boldsymbol{n}_i \end{bmatrix} \tag{5}$$

$$\text{PCwF: } \boldsymbol{G}_i = \begin{bmatrix} \boldsymbol{n}_i & \boldsymbol{o}_i & \boldsymbol{t}_i \\ \boldsymbol{r}_i \times \boldsymbol{n}_i & \boldsymbol{r}_i \times \boldsymbol{o}_i & \boldsymbol{r}_i \times \boldsymbol{t}_i \end{bmatrix} \tag{6}$$

$$\text{SFC: } \boldsymbol{G}_i = \begin{bmatrix} \boldsymbol{n}_i & \boldsymbol{o}_i & \boldsymbol{t}_i & \boldsymbol{0} \\ \boldsymbol{r}_i \times \boldsymbol{n}_i & \boldsymbol{r}_i \times \boldsymbol{o}_i & \boldsymbol{r}_i \times \boldsymbol{t}_i & \boldsymbol{n}_i \end{bmatrix}. \tag{7}$$

Then the Minkowski sum $\sum_{i=1}^{m} \boldsymbol{G}_i(F_i) = \boldsymbol{G}(F)$ is a convex cone in $\mathbb{R}^6$ consisting of all the resultant wrenches that the robot hand can exert on the grasped object, where $\boldsymbol{G} = [\boldsymbol{G}_1 \quad \boldsymbol{G}_2 \quad \cdots \quad \boldsymbol{G}_m]$ and $F = \prod_{i=1}^{m} F_i$. A grasp is said to be force-closure if $\boldsymbol{G}(F) = \mathbb{R}^6$ [17].

2.2. Previous results concerning the distance function

Let S be a nonempty compact convex subset of $\mathbb{R}^n$ and $B_0 = \{\boldsymbol{u} \in \mathbb{R}^n \mid \boldsymbol{u}^{\mathrm{T}}\boldsymbol{u} = 1\}$ the unit sphere centered at the origin $\mathbf{0}$. The unit vector $\boldsymbol{u}$ specifies a direction in $\mathbb{R}^n$. The distance between $\mathbf{0}$ and S is defined by

$$d(\mathbf{0}, S) = \begin{cases} \min\limits_{\lambda B_0 \cap S \neq \emptyset, \lambda \geq 0} \lambda, & \text{if } \mathbf{0} \notin \text{int } S \\ \min\limits_{\lambda B_0 \subset S, \lambda \leq 0} \lambda, & \text{if } \mathbf{0} \in \text{int } S, \end{cases} \tag{8}$$

where int(·) denotes the interior of a set. This distance definition contains the cases of $\mathbf{0}$ being separated from or contained in the interior of S. In other words, $d(\mathbf{0}, S)$ means the radius of the largest ball centered at $\mathbf{0}$ separated from S or contained in S. Its value is positive or negative, respectively.

Define a real-valued function q_S on S as

$$q_S(\boldsymbol{u}) = \min_{\boldsymbol{x} \in S} \boldsymbol{u}^{\mathrm{T}}\boldsymbol{x}, \tag{9}$$

where $\boldsymbol{u} \in \mathbb{R}^n$. Our previous work [8] has shown

$$d(\mathbf{0}, S) = \max\nolimits_{\boldsymbol{u}^{\mathrm{T}}\boldsymbol{u}=1} q_S(\boldsymbol{u}). \tag{10}$$

Let S_1 and S_2 be nonempty compact subsets of $\mathbb{R}^n$. Then we have the following properties, which are useful for computing $q_S(\boldsymbol{u})$:

1. $q_{\text{conv}S_1}(\boldsymbol{u}) = q_{S_1}(\boldsymbol{u})$, where conv(·) denotes the convex hull of a set.
2. $q_{S_1 \cup S_2}(\boldsymbol{u}) = \min\{q_{S_1}(\boldsymbol{u}), q_{S_2}(\boldsymbol{u})\}$.
3. $q_{\alpha_1 S_1 \pm \alpha_2 S_2}(\boldsymbol{u}) = \alpha_1 q_{S_1}(\boldsymbol{u}) + \alpha_2 q_{S_2}(\pm\boldsymbol{u})$ for $\alpha_1 \geq 0$ and $\alpha_2 \geq 0$.
4. $q_{\boldsymbol{R}(S_1)}(\boldsymbol{u}) = q_{S_1}(\boldsymbol{R}^{\mathrm{T}}\boldsymbol{u})$, where $\boldsymbol{R} \in \mathbb{R}^{n' \times n}$ and n' denotes a positive integer.
5. $q_{S_1 \times S_2}(\boldsymbol{u}) = q_{S_1}(\boldsymbol{u}_1) + q_{S_2}(\boldsymbol{u}_2)$, where S_1 and S_2 are subsets of $\mathbb{R}^{n_1}$ and $\mathbb{R}^{n_2}$, $\boldsymbol{u}_1 \in \mathbb{R}^{n_1}$, $\boldsymbol{u}_2 \in \mathbb{R}^{n_2}$, and $\boldsymbol{u} = [\boldsymbol{u}_1^{\mathrm{T}} \quad \boldsymbol{u}_2^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n_1+n_2}$.

3. Reformulation of the grasp quality criteria

In all previous work [2–4,6,8,12,15,18], the primitive contact wrenches are derived from finite edges for approximating the nonlinear friction cones. In this section, we first provide a precise expression of the primitive contact wrench set based on the original friction cones (1)–(4). Then the grasp wrench sets and the grasp quality criteria are formulated accurately. Their physical meanings and differences are elucidated. Finally, a new remedy is given to keep the grasp quality criteria invariant under a change of coordinate frame and dimension unit.

3.1. Primitive contact wrench sets

From (1)–(4), the friction cone F_i can be rewritten as

$$F_i = \text{co}\, U_i, \tag{11}$$

where co (·) denotes the set of all nonnegative linear combinations of the elements in a set, also known as the convex cone with apex at the origin $\mathbf{0}$ generated by the set, and the set U_i has one of the following forms:

$$\text{FPC: } U_i = \{\boldsymbol{f}_i \in \mathbb{R} \mid f_{i1} = 1\} \tag{12}$$

$$\text{PCwF: } U_i = \left\{\boldsymbol{f}_i \in \mathbb{R}^3 \mid f_{i1} = 1, \sqrt{f_{i2}^2 + f_{i3}^2} = \mu_i\right\} \tag{13}$$

$$\text{SFCl: } U_i = \left\{\boldsymbol{f}_i \in \mathbb{R}^4 \mid f_{i1} = 1, \frac{\sqrt{f_{i2}^2 + f_{i3}^2}}{\mu_i} + \frac{|f_{i4}|}{\mu_{si}} = 1\right\} \tag{14}$$

$$\text{SFCe: } U_i = \left\{\boldsymbol{f}_i \in \mathbb{R}^4 \mid f_{i1} = 1, \sqrt{\frac{f_{i2}^2 + f_{i3}^2}{\mu_i^2} + \frac{f_{i4}^2}{\mu_{si}'^2}} = 1\right\}. \tag{15}$$

This means that F_i can be generated by a basic set U_i and U_i is called the *primitive contact force set*. From (12)–(15) it can be seen that U_i is a singleton for FPC but an infinite set given by a nonlinear equation for PCwF or SFC. In [2–4,6,8,12,15,18], a subset of U_i with finite elements is used instead, so that from (11) the friction cone F_i consists of only the nonnegative linear combinations of these elements, and the nonlinearity is eliminated. Hereinafter, however, we still use the original nonlinear models. Their geometric meanings can be realized by decomposing (13)–(15) as

$$U_i = N_i \times T_i, \tag{16}$$

where

$$N_i = \{f_{i1} \in \mathbb{R} \mid f_{i1} = 1\} \tag{17}$$

and T_i takes one of the following forms:

$$\text{PCwF: } T_i = \left\{[f_{i2}\, f_{i3}]^{\mathrm{T}} \in \mathbb{R}^2 \mid \sqrt{f_{i2}^2 + f_{i3}^2} = \mu_i\right\} \tag{18}$$

$$\text{SFCl: } T_i = \left\{[f_{i2}\, f_{i3}\, f_{i4}]^{\mathrm{T}} \in \mathbb{R}^3 \mid \frac{\sqrt{f_{i2}^2 + f_{i3}^2}}{\mu_i} + \frac{|f_{i4}|}{\mu_{si}} = 1\right\} \tag{19}$$

$$\text{SFCe: } T_i = \left\{[f_{i2}\, f_{i3}\, f_{i4}]^{\mathrm{T}} \in \mathbb{R}^3 \mid \sqrt{\frac{f_{i2}^2 + f_{i3}^2}{\mu_i^2} + \frac{f_{i4}^2}{\mu_{si}'^2}} = 1\right\}. \tag{20}$$

The set T_i depicts a circle of $\mathbb{R}^2$ for PCwF, a bicone of $\mathbb{R}^3$ for SFCl, and an ellipsoid of $\mathbb{R}^3$ for SFCe, as shown in Fig. 1.

From (11) it follows that

$$\boldsymbol{G}_i(F_i) = \boldsymbol{G}_i(\text{co}\, U_i) = \text{co}\, (\boldsymbol{G}_i(U_i)) = \text{co}\, W_i,$$

where W_i is just the *primitive contact wrench set*:

$$W_i = \boldsymbol{G}_i(U_i). \tag{21}$$

The set W_i for FPC is a singleton, while W_i for PCwF or SFC is an infinite set, which consists of all primitive contact wrenches at contact i.

Combining (1)–(4) and (12)–(15) we see that the convex hull of U_i, denoted by convU_i, is equal to $F_i \cap \{\boldsymbol{f}_i | f_{i1} = 1\}$. Then from (21), the convex hull conv W_i of W_i has the following meaning:

$$\text{conv}\, W_i = \boldsymbol{G}_i(\text{conv}U_i) = \{\boldsymbol{G}_i\boldsymbol{f}_i | \boldsymbol{f}_i \in F_i \text{ and } f_{i1} = 1\}. \tag{22}$$

3.2. Grasp wrench sets and grasp quality criteria

To establish the grasp wrench sets, we first define W^k as the union of Minkowski sums of different k ($k = 1, 2, \ldots, m$) of W_i, $i = 1, 2, \ldots, m$, given by

$$W^k = \bigcup_{i_1=1}^{m-k+1} \bigcup_{i_2=i_1+1}^{m-k+2} \cdots \bigcup_{i_k=i_{k-1}+1}^{m} (W_{i_1} + W_{i_2} + \cdots + W_{i_k}), \tag{23}$$

where $i_1, i_2, \ldots, i_k$ designate k of W_i, $i = 1, 2, \ldots, m$ to add up. In particular, $W^1 = \bigcup_{i=1}^m W_i$ and $W^m = \sum_{i=1}^m W_i$. There are $n = \binom{k}{m}$ different selections of k of W_i, $i = 1, 2, \ldots, m$, as depicted in Fig. 2. Let W_c^k be the convex hull of W^k. From (23) we have

$$\begin{aligned} W_c^k &= \text{conv}\left(\bigcup_{i_1=1}^{m-k+1} \bigcup_{i_2=i_1+1}^{m-k+2} \cdots \bigcup_{i_k=i_{k-1}+1}^{m} (\text{conv}\, W_{i_1} \right. \\ &\quad \left. + \text{conv}\, W_{i_2} + \cdots + \text{conv}\, W_{i_k})\right) \\ &= \left\{\sum_{j=1}^{n} \lambda_j \text{conv}\, W_j | \sum_{j=1}^{n} \lambda_j = 1 \text{ and } 0 \leq \lambda_j \leq 1\right\}, \end{aligned} \tag{24}$$

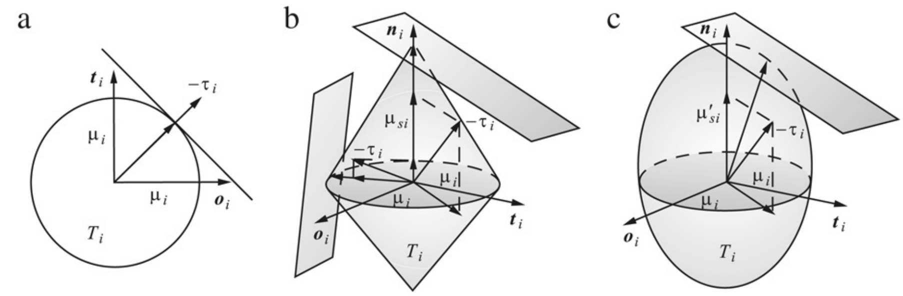

Fig. 1. The set T_i for (a) PCwF, (b) SFCl, and (c) SFCe.

$$\begin{array}{c|cccccccccc} & \multicolumn{10}{c}{\overbrace{\hspace{20em}}^{m}} \\ \lambda_1 & W_1 & W_2 & \cdots & W_k & \times & \times & \cdots & \cdots & \cdots & \times \\ \lambda_2 & W_1 & W_2 & \cdots & \times & W_{k+1} & \times & \cdots & \cdots & \cdots & \times \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \lambda_n & \times & \times & \cdots & \cdots & \cdots & \times & W_{m-k+1} & W_{m-k+2} & \cdots & W_m \end{array} \left.\vphantom{\begin{array}{c}1\\1\\1\\1\end{array}}\right\} n$$

Fig. 2. Different selections of k of W_i, $i = 1, 2, \ldots, m$. Totally there are $n = \binom{k}{m}$ selections. Each W_i appears in $n_i = \binom{k-1}{m-1}$ selections. The nonnegative scalars $\lambda_j, j = 1, 2, \ldots, n$ with $\sum_{j=1}^{n} \lambda_j = 1$ give a convex combination of the n selections.

where $\operatorname{conv} W_j = \operatorname{conv} W_{i_1} + \operatorname{conv} W_{i_2} + \cdots + \operatorname{conv} W_{i_k}$. From (22) it follows that $\operatorname{conv} W_j$ consists of the wrenches generated by $\boldsymbol{f}_i \in F_i$, $i = 1, 2, \ldots, m$ with $f_{i1} = 1$ for $i = i_1, i_2, \ldots, i_k$ and $f_{i1} = 0$ otherwise and $\sum_{i=1}^{m} f_{i1} = k$, which means that only k contacts are really working and the rest $m - k$ contacts are idle (they do not take part in generating the resultant wrench). Then the wrenches in W_c^k are generated by $\boldsymbol{f}_i \in F_i$, $i = 1, 2, \ldots, m$ with $\sum_{i=1}^{m} f_{i1} = \sum_{j=1}^{n} \lambda_j k = k$. Moreover, from Fig. 2 we see that only $n_i = \binom{k-1}{m-1} < n$ selections contain W_i. Then f_{i1} of $\boldsymbol{f}_i \in F_i$ in generating W_c^k is equal to the sum of n_i of λ_j, $j = 1, 2, \ldots, n$, which is not more than $\sum_{j=1}^{n} \lambda_j = 1$. Hence, W_c^k has the physical meaning:

$$\begin{aligned} W_c^k &= \left\{ \boldsymbol{G}\boldsymbol{f} | \boldsymbol{f} \in F, \sum_{i=1}^{m} f_{i1} = k, \text{ and } \max_{1 \le i \le m} f_{i1} \le 1 \right\} \\ &= \boldsymbol{G}(F \cap \Omega^k), \end{aligned} \tag{25}$$

where $\Omega^k = \left\{ \boldsymbol{f} \mid \sum_{i=1}^{m} f_{i1} = k \text{ and } \max_{1 \le i \le m} f_{i1} \le 1 \right\}$. Particularly, $\Omega^1 = \left\{ \boldsymbol{f} \mid \sum_{i=1}^{m} f_{i1} = 1 \right\}$ and $\Omega^m = \{ \boldsymbol{f} \mid f_{i1} = 1 \text{ for } i = 1, 2, \ldots, m \}$. Let $W^M = \bigcup_{k=1}^{m} W^k$ and W_c^M be the convex hull of W^M. From (25) we have

$$W_c^M = \operatorname{conv}\left(\bigcup_{k=1}^{m} W^k \right) = \operatorname{conv}\left(\bigcup_{k=1}^{m} W_c^k \right) = \boldsymbol{G}(F \cap \Omega^M), \tag{26}$$

where $\Omega^M = \left\{ \boldsymbol{f} \mid \sum_{i=1}^{m} f_{i1} \ge 1 \text{ and } \max_{1 \le i \le m} f_{i1} \le 1 \right\}$. The sets W_c^1 and W_c^M are so-called grasp wrench sets. According to (25) and (26), they consist of the resultant wrenches that can be produced by the contact forces in $F \cap \Omega^1$ and $F \cap \Omega^M$, respectively. A grasp is force-closure if and only if $\boldsymbol{0} \in \operatorname{int} W_c^1$ or $\boldsymbol{0} \in \operatorname{int} W_c^M$.

Remark 1. Using the distance function (8), the grasp quality criteria in [1,2] can be directly reformulated as $d(\boldsymbol{0}, W_c^1)$ and $d(\boldsymbol{0}, W_c^M)$. A grasp is force-closure if and only if $d(\boldsymbol{0}, W_c^1)$ or $d(\boldsymbol{0}, W_c^M)$ is negative. If this is so, their absolute values indicate the largest resultant wrenches in the worst directions yielded by the contact forces in $F \cap \Omega^1$ and $F \cap \Omega^M$, respectively; otherwise they imply how far the grasp is from achieving force-closure.

Remark 2. In the early stages of seeking the optimal grasp, the tentative grasps are often tested to be non-force-closure (see Example 2). Namely, $d(\boldsymbol{0}, W_c^1)$ or $d(\boldsymbol{0}, W_c^M)$ is positive. This value serves as a guide for further optimization. The foregoing unified formulas for both force-closure and non-force-closure greatly facilitate the work.

Remark 3. From (24) and (26), W_c^1 results only from the primitive contact wrench sets W_i, $i = 1, 2, \ldots, m$, while W_c^M comes from not only the primitive contact wrench sets themselves but also their Minkowski sums. This leads to W_c^1 being just a proper subset of W_c^M and the constraint Ω^1 on the force magnitude for W_c^1 is much stronger than the constraint Ω^M for W_c^M. Therefore, the absolute value of $d(\boldsymbol{0}, W_c^1)$ is not greater than that of $d(\boldsymbol{0}, W_c^M)$. The maximum normal contact force is determined by the capability of the fingers and also limited by the material strength of the gripped object and the gripper. Hence the maximum normal contact force is a compulsory upper bound and $d(\boldsymbol{0}, W_c^M)$ is a reasonable criterion. However, computing it previously was more difficult, and therefore the sum of the normal contact forces or correspondingly $d(\boldsymbol{0}, W_c^1)$ is used as a substitute in many classical papers.

3.3. Modification for frame and unit invariances

A good grasp quality criterion should be independent of the choice or invariant under a change of object coordinate frame. Let $\boldsymbol{Q} \in SO(3)$ and $\boldsymbol{p} \in \mathbb{R}^3$ denote the changes of the orientation and position of object coordinate frame, respectively. Then the contact position vector, unit inward normal, and unit tangent vectors are changed by

$$\boldsymbol{r}_i' = \boldsymbol{Q}\boldsymbol{r}_i + \boldsymbol{p}, \quad \boldsymbol{n}_i' = \boldsymbol{Q}\boldsymbol{n}_i, \quad \boldsymbol{o}_i' = \boldsymbol{Q}\boldsymbol{o}_i, \quad \text{and} \quad \boldsymbol{t}_i' = \boldsymbol{Q}\boldsymbol{t}_i. \tag{27}$$

Then from (5)–(7), the matrix $\boldsymbol{G}_i$ is changed by

$$\boldsymbol{G}_i' = \operatorname{diag}(\boldsymbol{Q}, \boldsymbol{Q})\boldsymbol{G}_i + \boldsymbol{P}_i,$$

where $\operatorname{diag}(\boldsymbol{Q}, \boldsymbol{Q}) \in \mathbb{R}^{6\times 6}$ and $\boldsymbol{P}_i$ has one of the following forms:

$$\text{FPC: } \boldsymbol{P}_i = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{p} \times \boldsymbol{Q}\boldsymbol{n}_i \end{bmatrix}$$

$$\text{PCwF: } \boldsymbol{P}_i = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{p} \times \boldsymbol{Q}\boldsymbol{n}_i & \boldsymbol{p} \times \boldsymbol{Q}\boldsymbol{o}_i & \boldsymbol{p} \times \boldsymbol{Q}\boldsymbol{t}_i \end{bmatrix}$$

$$\text{SFC: } \boldsymbol{P}_i = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{p} \times \boldsymbol{Q}\boldsymbol{n}_i & \boldsymbol{p} \times \boldsymbol{Q}\boldsymbol{o}_i & \boldsymbol{p} \times \boldsymbol{Q}\boldsymbol{t}_i & \boldsymbol{0} \end{bmatrix}.$$

If $\boldsymbol{p} = \boldsymbol{0}$, then $\boldsymbol{P}_i$ is a zero matrix, and from (21) the set W_i is changed only by the matrix $\operatorname{diag}(\boldsymbol{Q}, \boldsymbol{Q})$. Since $\operatorname{diag}(\boldsymbol{Q}, \boldsymbol{Q})$ is an orthogonal matrix, the distances $d(\boldsymbol{0}, W_c^1)$ and $d(\boldsymbol{0}, W_c^M)$ remain unchanged. However, if $\boldsymbol{p} \ne \boldsymbol{0}$, then $\boldsymbol{P}_i$ causes additional change in only three moment components of a wrench vector, which in turn might alter the values of $d(\boldsymbol{0}, W_c^1)$ and $d(\boldsymbol{0}, W_c^M)$. Similarly, changing the unit of the object dimension also alters the quantities

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of the moment components of a wrench but has no effect on the force components, which may change the values of $d(\mathbf{0}, W_c^1)$ and $d(\mathbf{0}, W_c^M)$ as well. So far, therefore, the grasp quality criteria are not invariant under a change of frame position and unit.

To remedy this drawback, we modify $\boldsymbol{G}_i$ as follows:

$$\text{FPC: } \boldsymbol{G}_i = \begin{bmatrix} R\boldsymbol{n}_i \\ (\boldsymbol{r}_i - \boldsymbol{r}_0) \times \boldsymbol{n}_i \end{bmatrix} \tag{28}$$

$$\text{PCwF: } \boldsymbol{G}_i = \begin{bmatrix} R\boldsymbol{n}_i & R\boldsymbol{o}_i & R\boldsymbol{t}_i \\ (\boldsymbol{r}_i - \boldsymbol{r}_0) \times \boldsymbol{n}_i & (\boldsymbol{r}_i - \boldsymbol{r}_0) \times \boldsymbol{o}_i & (\boldsymbol{r}_i - \boldsymbol{r}_0) \times \boldsymbol{t}_i \end{bmatrix} \tag{29}$$

$$\text{SFC: } \boldsymbol{G}_i = \begin{bmatrix} R\boldsymbol{n}_i & R\boldsymbol{o}_i & R\boldsymbol{t}_i & \mathbf{0} \\ (\boldsymbol{r}_i - \boldsymbol{r}_0) \times \boldsymbol{n}_i & (\boldsymbol{r}_i - \boldsymbol{r}_0) \times \boldsymbol{o}_i & (\boldsymbol{r}_i - \boldsymbol{r}_0) \times \boldsymbol{t}_i & \boldsymbol{n}_i \end{bmatrix}, \tag{30}$$

where

$$\boldsymbol{r}_0 = \frac{1}{m} \sum_{i=1}^{m} \boldsymbol{r}_i \tag{31}$$

$$R = \frac{1}{m} \sum_{i=1}^{m} \|\boldsymbol{r}_i - \boldsymbol{r}_0\| . \tag{32}$$

This modification means that the moment origin is selected at the centroid of the contact positions, and the force components of a wrench are multiplied by the average distance from the contact positions to the centroid. Then the moment origin may probably not coincide with the origin of the chosen object coordinate frame.

Theorem 1. *By the modification given by (28)–(32), the values of $d(\mathbf{0}, W_c^1)$ and $d(\mathbf{0}, W_c^M)$ are invariant under a change of object coordinate frame and similarly invariant under a change of dimension unit.*

Proof. First, we prove the frame invariance. From (27), (31), and (32) it follows that

$$\boldsymbol{r}'_0 = \frac{1}{m} \sum_{i=1}^{m} \boldsymbol{r}'_i = \frac{1}{m} \sum_{i=1}^{m} (\boldsymbol{Q}\boldsymbol{r}_i + \boldsymbol{p}) = \boldsymbol{Q}\boldsymbol{r}_0 + \boldsymbol{p} \tag{33}$$

$$R' = \frac{1}{m} \sum_{i=1}^{m} \left\|\boldsymbol{r}'_i - \boldsymbol{r}'_0\right\| = \frac{1}{m} \sum_{i=1}^{m} \|\boldsymbol{Q}(\boldsymbol{r}_i - \boldsymbol{r}_0)\| = R. \tag{34}$$

From (31) and (33) we have

$$\boldsymbol{r}'_i - \boldsymbol{r}'_0 = \boldsymbol{Q}(\boldsymbol{r}_i - \boldsymbol{r}_0). \tag{35}$$

Substituting (27), (34) and (35) into (28)–(30), we obtain

$$\boldsymbol{G}'_i = \mathrm{diag}(\boldsymbol{Q}, \boldsymbol{Q})\boldsymbol{G}_i.$$

The matrix $\mathrm{diag}(\boldsymbol{Q}, \boldsymbol{Q}) \in SO(6)$ preserves the distances $d(\mathbf{0}, W_c^1)$ and $d(\mathbf{0}, W_c^M)$.

The change of dimension unit only affects the contact position vector, which can be described by $\boldsymbol{r}'_i = \lambda \boldsymbol{r}_i$, where λ is a positive scalar. Then from (31) and (32) we see that $\boldsymbol{r}'_i - \boldsymbol{r}'_0 = \lambda(\boldsymbol{r}_i - \boldsymbol{r}_0)$ and $R' = \lambda R$. Substituting them into (28)–(30) yields

$$\boldsymbol{G}'_i = \lambda \boldsymbol{G}_i.$$

This implies that the values of $d(\mathbf{0}, W_c^1)$ and $d(\mathbf{0}, W_c^M)$ will also be scaled by λ. □

Now the performance quality of any grasp on an object can be evaluated by $d(\mathbf{0}, W_c^1)$ or $d(\mathbf{0}, W_c^M)$ in any selected, fixed, object coordinate frame and dimension unit. Furthermore, from (28)–(30) we see that the values of $d(\mathbf{0}, W_c^1)$ and $d(\mathbf{0}, W_c^M)$ are related to $\boldsymbol{r}_i - \boldsymbol{r}_0$, $i = 1, 2, \ldots, m$. To minimize either of them (should be negative) in optimizing the contact positions $\boldsymbol{r}_i$, $i = 1, 2, \ldots, m$, increasing $\|\boldsymbol{r}_i - \boldsymbol{r}_0\|$, $i = 1, 2, \ldots, m$ is helpful. From (31) and (32) we have

$$\boldsymbol{r}_i - \boldsymbol{r}_0 = \frac{1}{m} \sum_{i'=1}^{m} (\boldsymbol{r}_i - \boldsymbol{r}'_i).$$

Then

$$\sum_{i=1}^{m} \|\boldsymbol{r}_i - \boldsymbol{r}_0\| = \frac{1}{m} \sum_{i=1}^{m} \left\| \sum_{i'=1}^{m} (\boldsymbol{r}_i - \boldsymbol{r}'_i) \right\| \le \frac{1}{m} \sum_{i=1}^{m} \sum_{i'=1}^{m} \left\| \boldsymbol{r}_i - \boldsymbol{r}'_i \right\| .$$

Note that $\left\|\boldsymbol{r}_i - \boldsymbol{r}'_i\right\|$ is the distance between two contact positions. Then the above inequality implies that the increase in $\|\boldsymbol{r}_i - \boldsymbol{r}_0\|$, $i = 1, 2, \ldots, m$ helps to increase the distance between any two contact positions, so that the optimal contact positions will spread more widely on the object surface. This enables the optimal grasp to produce larger moments on the object under the same contact force limit. In this sense, the effect of the object geometry is counted in the grasp quality criteria.

4. Computation of the grasp quality criteria

In this section, the two grasp quality criteria are transformed into two optimization problems. The analytical formulas for computing the objective functions and their derivatives are derived. The strategy for finding the globally maximum values of the objective functions is given. It will be shown that this computational method, without linearizing the friction cones, is even simpler than that in [8], which uses the linearized friction model.

4.1. Computing formulas

It follows directly from (10) that, no matter whether the grasp is force-closure or not, $d(\mathbf{0}, W_c^1)$ and $d(\mathbf{0}, W_c^M)$ can be computed by solving the following optimization problems, respectively:

$$d(\mathbf{0}, W_c^1) = \max_{\boldsymbol{u}^{\mathrm{T}}\boldsymbol{u}=1} q_{W_c^1}(\boldsymbol{u}) \tag{36}$$

$$d(\mathbf{0}, W_c^M) = \max_{\boldsymbol{u}^{\mathrm{T}}\boldsymbol{u}=1} q_{W_c^M}(\boldsymbol{u}). \tag{37}$$

Either of the above is maximized w.r.t. the direction $\boldsymbol{u}$. Here max means the worst (see Remark 1 in Section 3.2). From (24), (26) and Points 1 and 2 given in Section 2.2, the objective functions of (36) and (37) can be calculated respectively by

$$q_{W_c^1}(\boldsymbol{u}) = q_{W^1}(\boldsymbol{u}) = \min_{1 \le i \le m} q_{W_i}(\boldsymbol{u}) \tag{38}$$

$$q_{W_c^M}(\boldsymbol{u}) = q_{W^M}(\boldsymbol{u}) = \min_{1 \le k \le m} q_{W^k}(\boldsymbol{u}). \tag{39}$$

From (23) and Points 2 and 3 we have

$$q_{W^k}(\boldsymbol{u}) = \min_{1 \le i_1 < i_2 < \cdots < i_k \le m} q_{W_{i_1} + W_{i_2} + \cdots + W_{i_k}}(\boldsymbol{u}) = \min_{1 \le i_1 < i_2 < \cdots < i_k \le m} \left(q_{W_{i_1}}(\boldsymbol{u}) + q_{W_{i_2}}(\boldsymbol{u}) + \cdots + q_{W_{i_k}}(\boldsymbol{u}) \right). \tag{40}$$

Eq. (40) shows that the minimum value of the function q over the Minkowski sums of W_i, $i = 1, 2, \ldots, m$ is just equal to the minimum value over the scalar sums of the function q of W_i, $i = 1, 2, \ldots, m$. By this nice property, we do not need to figure out the Minkowski sums of W_i, $i = 1, 2, \ldots, m$, and thus the complexity of computing $d(\mathbf{0}, W_c^M)$ is almost the same as $d(\mathbf{0}, W_c^1)$. From (21) and Point 4 we further obtain

$$q_{W_i}(\boldsymbol{u}) = q_{U_i}(\boldsymbol{G}_i^{\mathrm{T}}\boldsymbol{u}) = q_{U_i}(\boldsymbol{d}_i),$$

where

$$\boldsymbol{d}_i = \boldsymbol{G}_i^{\mathrm{T}}\boldsymbol{u} \tag{41}$$

and $\boldsymbol{d}_i$ has one of the following forms:

FPC: $\boldsymbol{d}_i = [d_{i1}]$
PCwF: $\boldsymbol{d}_i = [d_{i1}\ d_{i2}\ d_{i3}]^{\mathrm{T}}$
SFC: $\boldsymbol{d}_i = [d_{i1}\ d_{i2}\ d_{i3}\ d_{i4}]^{\mathrm{T}}$.

For FPC, from (12) it is evident that $q_{U_i}(\boldsymbol{d}_i) = d_{i1}$. For PCwF and SFC, from (16), (17), and Point 5 we may rewrite $q_{U_i}(\boldsymbol{d}_i)$ as

$$q_{U_i}(\boldsymbol{d}_i) = q_{N_i}(d_{i1}) + q_{T_i}(\boldsymbol{\tau}_i) = d_{i1} + q_{T_i}(\boldsymbol{\tau}_i),$$

where $\boldsymbol{\tau}_i = [d_{i2}\ d_{i3}]^{\mathrm{T}}$ for PCwF and $\boldsymbol{\tau}_i = [d_{i2}\ d_{i3}\ d_{i4}]^{\mathrm{T}}$ for SFC. Taking $\boldsymbol{\tau}_i$ as an inward normal or $-\boldsymbol{\tau}_i$ as an outward normal to a supporting hyperplane of T_i, we see that the function $q_{T_i}(\boldsymbol{\tau}_i)$ on T_i attains its value at the point where the hyperplane supports T_i, as depicted in Fig. 1. Thus from (18)–(20) we derive

PCwF: $q_{T_i}(\boldsymbol{\tau}_i) = -\mu_i\sqrt{d_{i2}^2 + d_{i3}^2}$

SFCl: $q_{T_i}(\boldsymbol{\tau}_i) = -\max\left\{\mu_i\sqrt{d_{i2}^2 + d_{i3}^2},\ \mu_{si}\,|d_{i4}|\right\}$

SFCe: $q_{T_i}(\boldsymbol{\tau}_i) = -\sqrt{\mu_i^2(d_{i2}^2 + d_{i3}^2) + \mu_{si}'^2 d_{i4}^2}$.

Finally, we attain

$$\text{FPC: } q_{W_i}(\boldsymbol{u}) = d_{i1} \tag{42}$$

$$\text{PCwF: } q_{W_i}(\boldsymbol{u}) = d_{i1} - \mu_i\sqrt{d_{i2}^2 + d_{i3}^2} \tag{43}$$

$$\text{SFCl: } q_{W_i}(\boldsymbol{u}) = d_{i1} - \max\left\{\mu_i\sqrt{d_{i2}^2 + d_{i3}^2},\ \mu_{si}\,|d_{i4}|\right\} \tag{44}$$

$$\text{SFCe: } q_{W_i}(\boldsymbol{u}) = d_{i1} - \sqrt{\mu_i^2(d_{i2}^2 + d_{i3}^2) + \mu_{si}'^2 d_{i4}^2}. \tag{45}$$

Formulas (36) and (37) for computing $d(\mathbf{0}, W_c^1)$ and $d(\mathbf{0}, W_c^M)$ are nonlinear optimization problems with only one constraint $\boldsymbol{u}^{\mathrm{T}}\boldsymbol{u} = 1$. They can be interpreted in terms of the infinitesimal motion and the virtual work [19]. Let $\boldsymbol{u} = [\boldsymbol{\varepsilon}^{\mathrm{T}}/R \quad \boldsymbol{\varphi}^{\mathrm{T}}]^{\mathrm{T}}$, where $\boldsymbol{\varepsilon} \in \mathbb{R}^3$ is an infinitesimal translation of the grasped object and $\boldsymbol{\varphi} \in \mathbb{R}^3$ is an infinitesimal rotation. Then $q_{W_i}(\boldsymbol{u})$ calculates the minimum work generated by $\boldsymbol{f}_i \in F_i \cap \{\boldsymbol{f}_i | f_{i1} = 1\}$ w.r.t. the total infinitesimal motion $\boldsymbol{\varepsilon}$ and $\boldsymbol{\varphi}$, and $q_{W_c^1}(\boldsymbol{u})$ and $q_{W_c^M}(\boldsymbol{u})$ are the minimum total work that can be generated by the contact forces in $F \cap \Omega^1$ and $F \cap \Omega^M$, respectively. Either of them indicates whether the grasp can restrain $\boldsymbol{\varepsilon}$ and $\boldsymbol{\varphi}$. In (36) and (37), $\boldsymbol{u}$ is taken to be a unit vector, which defines a direction of infinitesimal motion. In this sense, the criteria evaluate the capability of the grasp to restrain motions over all directions. If their values are negative, then all the infinitesimal motions of the object can be restrained. Suppose that $\boldsymbol{u}^*$ is an optimal solution of (36) or (37). Then $\boldsymbol{u}^*$ gives the worst direction of infinitesimal motion for the grasp to restrain, since the minimum total work w.r.t. $\boldsymbol{u}^*$ is maximal. If the value of a criterion is nonnegative, then the infinitesimal motions along $\boldsymbol{u}^*$ cannot be restrained.

4.2. Differentiability of the objective functions

Herein we figure out the differentiability of the objective functions $q_{W_c^1}(\boldsymbol{u})$ and $q_{W_c^M}(\boldsymbol{u})$ of (36) and (37) and deduce their derivatives so that gradient-based search methods can be used to compute (36) and (37).

Theorem 2. *The function q_S satisfies the Lipschitz continuity, i.e., $|q_S(\boldsymbol{u} + \Delta\boldsymbol{u}) - q_S(\boldsymbol{u})| \le \xi\,\|\Delta\boldsymbol{u}\|$, where $\xi > 0$ is independent of $\boldsymbol{u}$ and $\Delta\boldsymbol{u}$.*

Proof. From (9) we have $q_S(\boldsymbol{u} + \Delta\boldsymbol{u}) \ge q_S(\boldsymbol{u}) + q_S(\Delta\boldsymbol{u})$ and $q_S(\boldsymbol{u}) \ge q_S(\boldsymbol{u} + \Delta\boldsymbol{u}) + q_S(-\Delta\boldsymbol{u})$. Then $q_S(\Delta\boldsymbol{u}) \le q_S(\boldsymbol{u} + \Delta\boldsymbol{u}) - q_S(\boldsymbol{u}) \le -q_S(-\Delta\boldsymbol{u})$. Noticing that $q_S(\Delta\boldsymbol{u}) = \min_{\boldsymbol{x}\in S} \Delta\boldsymbol{u}^{\mathrm{T}}\boldsymbol{x} \ge -\xi\,\|\Delta\boldsymbol{u}\|$ and $-q_S(-\Delta\boldsymbol{u}) = \max_{\boldsymbol{x}\in S} \Delta\boldsymbol{u}^{\mathrm{T}}\boldsymbol{x} \le \xi\,\|\Delta\boldsymbol{u}\|$, where $\xi = \max_{\boldsymbol{x}\in S} \|\boldsymbol{x}\|$ is positive and independent of $\boldsymbol{u}$ and $\Delta\boldsymbol{u}$, we then obtain $|q_S(\boldsymbol{u} + \Delta\boldsymbol{u}) - q_S(\boldsymbol{u})| \le \xi q_S(\Delta\boldsymbol{u})$. □

The Lipschitz continuity implies that $q_{W_c^1}(\boldsymbol{u})$ and $q_{W_c^M}(\boldsymbol{u})$ are differentiable almost everywhere w.r.t. $\boldsymbol{u}$. The partial derivative of $q_{W_c^1}(\boldsymbol{u})$ w.r.t. the element u_l $(l = 1, 2, \ldots, 6)$ of $\boldsymbol{u}$ is determined by

$$\frac{\partial q_{W_c^1}(\boldsymbol{u})}{\partial u_l} = \frac{\partial q_{W_{i^*}}(\boldsymbol{u})}{\partial u_l} = \sum_{h=1}^{q_{i^*}} \frac{\partial q_{W_{i^*}}(\boldsymbol{u})}{\partial d_{i^*h}} \cdot \frac{\partial d_{i^*h}}{\partial u_l}, \tag{46}$$

where i^* is the index for which $q_{W_c^1}(\boldsymbol{u}) = q_{W_{i^*}}(\boldsymbol{u})$ from (38), $\partial d_{i^*h}/\partial u_l$ is equal to the (l, h) entry of $\boldsymbol{G}_{i^*}$ from (41), and $\partial q_{W_{i^*}}(\boldsymbol{u})/\partial d_{i^*h}$ can be easily computed from (42)–(45). It should be indicated that i^* depends on $\boldsymbol{u}$ and may not be unique for some $\boldsymbol{u}$. At such $\boldsymbol{u}$, $\partial q_{W_{i^*}}(\boldsymbol{u})/\partial d_{i^*h}$ may not be continuous; thus $q_{W_c^1}(\boldsymbol{u})$ may not be differentiable.

Similarly, the partial derivative of $q_{W_c^M}(\boldsymbol{u})$ w.r.t. u_l can be calculated by

$$\frac{\partial q_{W_c^M}(\boldsymbol{u})}{\partial u_l} = \frac{\partial q_{W_{i_1^*}}(\boldsymbol{u})}{\partial u_l} + \frac{\partial q_{W_{i_2^*}}(\boldsymbol{u})}{\partial u_l} + \cdots + \frac{\partial q_{W_{i_k^*}}(\boldsymbol{u})}{\partial u_l}, \tag{47}$$

where $i_1^*, i_2^*, \ldots, i_k^*$ satisfy $q_{W_c^M}(\boldsymbol{u}) = q_{W_{i_1^*} + W_{i_2^*} + \cdots + W_{i_k^*}}(\boldsymbol{u})$ and $\partial q_{W_{i_k^*}}(\boldsymbol{u})/\partial u_l$ can be computed by (46). Also, $q_{W_c^M}(\boldsymbol{u})$ may not be differentiable at $\boldsymbol{u}$ for which $q_{W_c^M}(\boldsymbol{u})$ can be attained with multiple choices of $i_1^*, i_2^*, \ldots, i_k^*$.

In (36) and (37), $\boldsymbol{u}$ is taken to be a point on the 6-D unit sphere $B_0 = \left\{\boldsymbol{u} \in \mathbb{R}^6 \mid \boldsymbol{u}^{\mathrm{T}}\boldsymbol{u} = 1\right\}$. First, there are only few irregular points in B_0 where $q_{W_c^1}(\boldsymbol{u})$ or $q_{W_c^M}(\boldsymbol{u})$ is nondifferentiable. Then irregular $\boldsymbol{u}$ is rarely encountered in the numerical computation. Besides, in any small neighborhood of an irregular $\boldsymbol{u}$ in B_0 there exist many regular points where $q_{W_c^1}(\boldsymbol{u})$ or $q_{W_c^M}(\boldsymbol{u})$ is still differentiable. Thus, in case of $q_{W_c^1}(\boldsymbol{u})$ or $q_{W_c^M}(\boldsymbol{u})$ being nondifferentiable, it is easy to find a small perturbation imposed on $\boldsymbol{u}$ to force it away from these irregular points.

4.3. Global optimum and computational complexity

Usually the nonlinear optimization problems (36) and (37) have local maxima. In the present problem, however, as the dimensions are not high and the single constraint is very simple, the global maxima could be found just by properly selecting different points on the sphere B_0 for the initial values of $\boldsymbol{u}$. A good choice is the set of vertices of the 6-D regular simplex circumscribed by B_0, namely

$$\boldsymbol{w}_v = \left(\boldsymbol{w}_v^0 - \frac{1}{7}\sum_{v=1}^{7}\boldsymbol{w}_v^0\right) \Bigg/ \left\|\boldsymbol{w}_v^0 - \frac{1}{7}\sum_{v=1}^{7}\boldsymbol{w}_v^0\right\|$$
$$\text{for } v = 1, 2, \ldots, 7,$$

where

$$\boldsymbol{w}_1^0 = \begin{bmatrix}1 & 1 & 1 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}$$
$$\boldsymbol{w}_2^0 = \begin{bmatrix}-1 & -1 & 1 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}$$
$$\boldsymbol{w}_3^0 = \begin{bmatrix}-1 & 1 & -1 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}$$
$$\boldsymbol{w}_4^0 = \begin{bmatrix}1 & -1 & -1 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}$$
$$\boldsymbol{w}_5^0 = \begin{bmatrix}0 & 0 & 0 & \sqrt{5} & 0 & 0\end{bmatrix}^{\mathrm{T}}$$
$$\boldsymbol{w}_6^0 = \begin{bmatrix}0 & 0 & 0 & \sqrt{5}/5 & 2\sqrt{30}/5 & 0\end{bmatrix}^{\mathrm{T}}$$
$$\boldsymbol{w}_7^0 = \begin{bmatrix}0 & 0 & 0 & \sqrt{5}/5 & 2/\sqrt{30} & \sqrt{42}/3\end{bmatrix}^{\mathrm{T}}.$$

Let $d_v(\mathbf{0}, W_c^1)$ be the value of $d(\mathbf{0}, W_c^1)$ computed by solving (36) w.r.t. $\boldsymbol{w}_v$. Then $d(\mathbf{0}, W_c^1)$ is taken to be the maximum of $d_v(\mathbf{0}, W_c^1)$ for all v. $d(\mathbf{0}, W_c^M)$ can be computed in the same way.

The total time complexity of computing the criteria is dependent on the method used to solve (36) or (37) as well as the number of initial values for $\boldsymbol{u}$ to be tried. The computation cost is mainly spent on evaluating the objective functions $q_{W_c^1}(\boldsymbol{u})$ and $q_{W_c^M}(\boldsymbol{u})$ of (36) and (37). From (38)–(45), their evaluations take merely algebraic operations. If SFCe is adopted, then computing $q_{W_c^1}(\boldsymbol{u})$ requires $5m_0+17m_f+23m_s$ additions, $6m_0+21m_f+29m_s$ multiplications, and $m_f + m_s$ square roots from (41)–(43) and (45), and $m-1$ comparisons from (38); computing $q_{W_c^M}(\boldsymbol{u})$ needs 2^m-m-1 additions and 2^m-2 comparisons from (39) and (40) in addition to those from (41)–(43) and (45). Moreover, since (36) and (37) contain only 6 variables and a simple equality constraint, it is not difficult for modern nonlinear optimization methods to solve them.

Assume that the friction cone F_i is linearized; that is, U_i is replaced by a finite subset, say $\{\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_N\}$. The selection of the subset can be found in [2–4,6,8,12,15,18], from which we know that $N = 1$ for FPC, $N \geq 3$ for PCwF, and $N \geq 5$ for SFC. For better linearization quality, N is taken to be a value much greater than the minimum, especially for SFC. From (9), (21), and (41) we have

$$q_{W_i}(\boldsymbol{u}) = \min\left\{\boldsymbol{d}_i^{\mathrm{T}}\boldsymbol{s}_1, \boldsymbol{d}_i^{\mathrm{T}}\boldsymbol{s}_2, \ldots, \boldsymbol{d}_i^{\mathrm{T}}\boldsymbol{s}_N\right\}. \tag{48}$$

For PCwF, computing $q_{W_i}(\boldsymbol{u})$ by (48) requires $2N$ additions, $3N$ multiplications, and $N-1$ comparisons, while by (43) it takes 2 additions, 3 multiplications, and 1 square root. For SFC, (48) undergoes $3N$ additions, $4N$ multiplications, and $N-1$ comparisons, while (45) takes 3 additions, 5 multiplications, and 1 square root. It is clearly shown that the method proposed here for computing the criteria not only avoids the loss of accuracy caused by linearization but also increases the computational efficiency.

5. Numerical examples

We implement the proposed grasp quality evaluation method using Matlab on a Pentium-M 1.8 GHz notebook and verify its performance with two examples. Formulas (36) and (37) are computed by the function **fmincon** of Matlab with the initial values for $\boldsymbol{u}$ given in Section 4.3. Assume $\mu = 0.2$ and $\mu_s = 0.4$ mm.

Example 1. This example is to verify the proposed method of grasp quality evaluation. The object is an 80 mm × 30 mm × 30 mm cuboid, grasped with two couples of antipodal PCwFs on the midlines of four facets (see Fig. 3). Such a grasp configuration is easier to implement in practice. Let the contacts move apart from the centers of the cuboid in opposite directions by distances s_1 and s_2, respectively. With only two variables, the computed results can be visualized: the values of $d(\mathbf{0}, W_c^1)$ and $d(\mathbf{0}, W_c^M)$ versus (s_1, s_2) are plotted in Fig. 4, which constitute two smooth and continuous

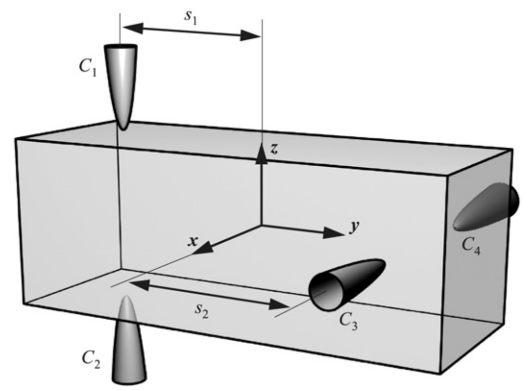

Fig. 3. A cuboid grasped with four PCwFs.

surfaces without local minima and jumps. Apparently, the minima of $d(\mathbf{0}, W_c^1)$ and $d(\mathbf{0}, W_c^M)$ w.r.t. (s_1, s_2) are both attained at $s_1 = 40$ mm and $s_2 = 40$ mm, and the contacts are by the sides of the facets and far from each other, which implies that the two criteria consider the object geometry. The average CPU times for computing the two criteria at each point are 1.98 s and 1.82 s, respectively.

To demonstrate the advantage of this method over the linearization-based method [8], we replace the friction cone F_i by a 4-sided polyhedral cone and compute the function $q_{W_i}(\boldsymbol{u})$ by (48). Then computing $d(\mathbf{0}, W_c^1)$ and $d(\mathbf{0}, W_c^M)$ by (36) and (37) takes 1.79 s and 2.06 s, respectively, which are slightly shorter or even longer than the required CPU times without linearizing the friction cones. Moreover, the values of $d(\mathbf{0}, W_c^1)$ and $d(\mathbf{0}, W_c^M)$ computed by the two methods have evident differences, as displayed in Fig. 4. The average differences are 0.25 N mm and 1.01 N mm, respectively. In addition, Fig. 4 also reveals that the linearization distorts the gradient flows of $d(\mathbf{0}, W_c^1)$ and $d(\mathbf{0}, W_c^M)$, which may lead to the optimal grasp planning falling into other undesirable locally optimal grasps.

In Fig. 4 it can be seen that the absolute value of $d(\mathbf{0}, W_c^1)$ is much less than that of $d(\mathbf{0}, W_c^M)$. This is due to the constraint Ω^1 on the magnitude of the contact forces for W_c^1 being much stronger than the one Ω^M for W_c^M, as discussed in Section 3.2.

Example 2. This example tries to use the criteria in optimal grasp planning. The object is a hammer involving two spheres S_1 and S_2, an ellipsoid S_3, and a cylinder S_4 (Fig. 5):

$$S_1 : \begin{cases} x = 100\cos\alpha\cos\beta \\ y = 100\cos\alpha\sin\beta \\ z = 100\sin\alpha - 68, \end{cases} \qquad S_2 : \begin{cases} x = 100\cos\alpha\cos\beta \\ y = 100\cos\alpha\sin\beta \\ z = 100\sin\alpha + 68, \end{cases}$$

$$S_3 : \begin{cases} x = 20\cos\alpha\cos\beta \\ y = 20\cos\alpha\sin\beta \\ z = 50\sin\alpha, \end{cases} \qquad S_4 : \begin{cases} x = 8\cos\beta \\ y = \alpha \\ z = 10\sin\beta. \end{cases}$$

The contacts are so arranged: C_1, C_2, and C_3 are FPCs on S_1, S_2, S_3, respectively, and C_4 and C_5 are SFCs on S_4. Each contact position is specified by (α_i, β_i). Fix (α_1, β_1), (α_2, β_2), (α_3, β_3) to $(23\pi/50, 0)$, $(-23\pi/50, \pi)$, $(0, 3\pi/2)$, and change (α_4, β_4) and (α_5, β_5) in $[40, 120] \times [0, \pi]$ and $[40, 120] \times [\pi, 2\pi]$, respectively. Then the computation of optimal grasps can be formulated as

$$\begin{cases} \text{minimize } d(\mathbf{0}, W_c^1) \text{ or } d(\mathbf{0}, W_c^M) \\ \text{s.t. } (\alpha_4, \beta_4) \in [40, 120] \times [0, \pi],\ (\alpha_5, \beta_5) \in [40, 120] \times [\pi, 2\pi]. \end{cases}$$

This problem is solved by the function **fmincon** of Matlab and the maximum iteration number is taken to be 10. Fig. 6 exhibits many trials with the initial values of (α_4, β_4) and (α_5, β_5) randomly set by the computer. Saving the best grasp so far, such

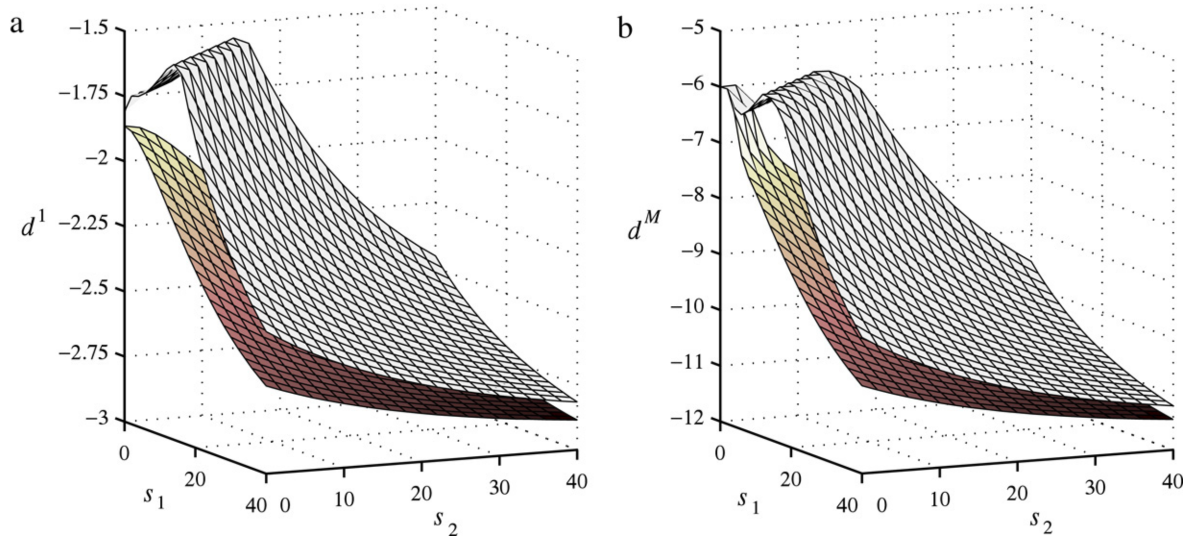

Fig. 4. $d(\mathbf{0}, W_c^1)$ and $d(\mathbf{0}, W_c^M)$ versus (s_1, s_2). The upper patches express the results computed using the linearized friction cones, while the lower ones are computed without linearizing the friction cones. Each patch contains 21×21 points. (a) $d(\mathbf{0}, W_c^1)$ versus (s_1, s_2). (b) $d(\mathbf{0}, W_c^M)$ versus (s_1, s_2).

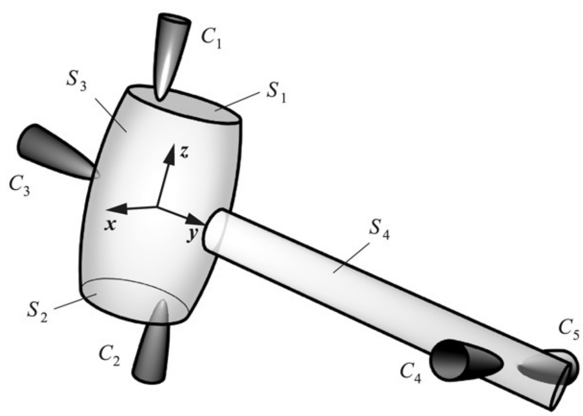

Fig. 5. A hammer grasped with FPCs C_1, C_2, C_3 and SFCs C_4, C_5.

trials can go on automatically until a satisfying one turns out. Now the minima of $d(\mathbf{0}, W_c^1)$ and $d(\mathbf{0}, W_c^M)$ are -3.0704 and -8.6342, respectively, and the corresponding grasps are the same, as depicted in Fig. 5. The average CPU times for a single trial based on the two criteria are 155.75 s and 159.53 s, respectively. Normally, optimal grasp planning can proceed offline. One may take as many trials as possible within acceptable time to find a relatively better grasp. Developing an efficient method for the trial will definitely increase the chance to attain a good grasp.

Fig. 6 shows that, with the same initial values, the optimized grasps according to the two criteria might be different. This is because the criteria have different gradients w.r.t. the parameters specifying the contact positions, and optimizations often converge to different local minima. Finding the globally optimal grasp is a challenging task, since any criterion usually has many local minima w.r.t the contact positions that make the global minimum hard to find and verify. Fortunately, the best of many scattered locally optimal grasp configurations often has a pleasing $d(\mathbf{0}, W_c^1)$ or $d(\mathbf{0}, W_c^M)$, as shown in Fig. 6. Whether it is the global optimum is not so important, since our goal is a reliable force-closure grasp.

6. Conclusions

Based on a systematical study of the previous work, this paper aims to improve the grasp quality evaluation, which is the kernel of grasping problems. Making strides towards perfection in all aspects (Table 1), two grasp quality criteria are formulated as the distances between the wrench origin and two grasp wrench sets. Their computations are cast into nonlinear optimization problems, essentially better than the approaches using the linearized friction cones. To optimize by some gradient-based search methods, the derivatives of the objective functions are calculated in closed form

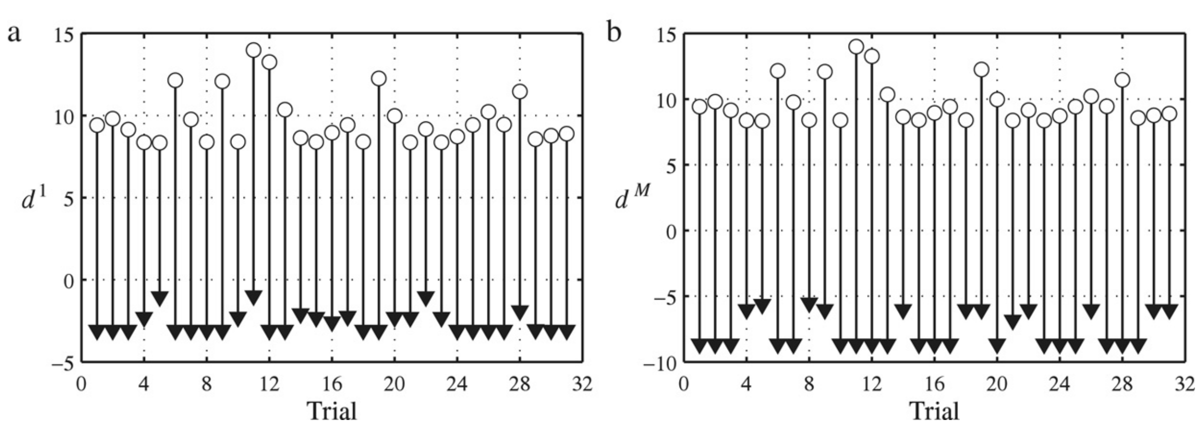

Fig. 6. Multiple trials of grasp optimization according to $d(\mathbf{0}, W_c^1)$ or $d(\mathbf{0}, W_c^M)$ with the same initial configurations. The circles denote the initial values, while the downward triangles denote the minimized values. (a) Adopting $d(\mathbf{0}, W_c^1)$ as the objective function. (b) Adopting $d(\mathbf{0}, W_c^M)$ as the objective function.

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and the choice of initial conditions is addressed. It is shown that the computational method proposed in this paper is as fast as the linearization-based ones. Moreover, the criterion $d(\mathbf{0}, W_c^M)$, whose computation was considered to be much more complex previously, can be calculated as efficiently as the criterion $d(\mathbf{0}, W_c^1)$ now.

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