is in the hole. Then, the assembly is completed after inserting the shaft to the desired depth.

V. Conclusion

With the ongoing development of micro machining technology, the need for micro assembly technology is growing. In this research, a micro parts assembly system with the following components is proposed.

1) *Micro gripper with two SMA coils as actuator:*
   This micro gripper has a loading capacity of more than 0.58 N, with mass of only 0.5 g.
2) *Micro RCC unit:*
   Links of this micro RCC unit are made of thin piano wires so that the unit has low translational/rotational stiffness.
3) *VCM-drive operating mechanism:*
   The mechanism creates linear motion with adjustable stiffness and measures the applied forces with the VCM current.
4) *Feed system of five precision motion stages*

In micro parts assembly, even positioning the shaft within the chamfer of the hole is difficult. An automatic assembly algorithm that searches the hole by sensing the force applied to the VCM-drive operating mechanism is proposed. The efficacy of the algorithm was proven by the experiment with a 97-μm diameter shaft with 100-μm diameter hole.

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Simplification of the Ray-Shooting Based Algorithm for 3-D Force-Closure Test

Yu Zheng and Wen-Han Qian

***Abstract*—This paper addresses a shortcut in the ray-shooting based algorithm proposed by Liu. His algorithm provides an efficient force-closure test of 3-D frictional grasps, which is formulated as a linear programming (LP) problem. We prove that the optimal objective value of the LP formulation indicates the force-closure property of grasps directly. Thus computing Q, d_1, and d_2 can be omitted.**

***Index Terms*—Duality, force closure, linear programming (LP), ray-shooting technique.**

Notations

n	Number of contacts.
r_i	Position vector of the ith contact $(i = 1, 2, \ldots, n)$.
m	Number of side facets linearizing a friction cone.
s_{ij}	The jth edge vector of the polyhedral cone at the ith contact $(i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, m)$.
w_{ij}	Primitive contact wrench $(i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, m)$.
N	Number of w_{ij}, $N = nm$.
W	Set of $w_{ij} (i = 1, 2, \ldots, n, j = 1, 2, \ldots, m)$.
$H(W)$	Convex hull of $w_{ij} (i = 1, 2, \ldots, n, j = 1, 2, \ldots, m)$.
$\partial H(W)$	Boundary of $H(W)$.
P	An interior point of $H(W)$.
O	Origin of the wrench space.
PO	Ray from P to O.
t	Direction vector of PO.
Q	Intersection point of $\partial H(W)$ with PO.
F	Facet of $H(W)$ intersected by PO.
$CT(W)$	Convex polytope dual to $H(W)$ after a coordinate translation of $-P$.
$\Vert \bullet \Vert$	2-norm of a vector.
d_1	Distance between P and Q.
d_2	Distance between P and O.
z	Objective function of Liu's LP formulation.
$z_{\max}$	Optimal objective value of Liu's LP formulation.

I. Introduction

During the past two decades, multifingered grasping was ardently studied. Many papers can be found on testing and planning force-closure grasps.

Early in 1983, Salisbury and Roth [1] proposed a necessary and sufficient condition for force closure; that is, the primitive contact wrenches $w_{ij} (i = 1, 2, \ldots, n, j = 1, 2, \ldots, m)$ of the grasp positively span the whole wrench space $\mathbb{R}^6$. This is equivalent to that the origin O of $\mathbb{R}^6$ lies strictly inside the convex hull $H(W)$ of $w_{ij} (i = 1, 2, \ldots, n, j = 1, 2, \ldots, m)$ [2], [3]. After that, however, no efficient algorithm was developed for 3-D frictional grasps to check whether or not O is an interior point of $H(W)$ until Liu [4] put forward a ray-shooting based

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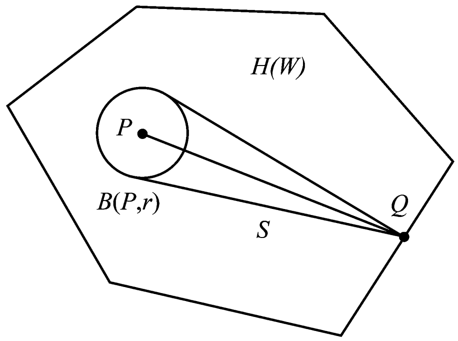

Fig. 1. Illustration of Lemma 1.

algorithm. The ray used in the algorithm starts from an interior point P of $H(W)$ to O and intersects the boundary $\partial H(W)$ of $H(W)$ at another point Q. P is taken at the centroid of $w_{ij} (i = 1, 2, \ldots, n, j = 1, 2, \ldots, m)$. Q is detected by linear programming (LP) based on the duality between convex polytopes. If the distance d_1 between P and Q is larger than that d_2 between P and O, then O is an interior point of $H(W)$, so the grasp is force closure; otherwise, the grasp is not force closure, since O is not inside $H(W)$. Later, the algorithm was applied to automatic generation of fixtures for polyhedral workpieces [5].

After repeatedly studying Liu's trailblazing work, we found a shortcut to make the algorithm simpler. Actually the optimal objective value of the LP formulation just equals to the ratio of d_2 to d_1. More straightforwardly, if the ratio is smaller than unity, the grasp is force closure; otherwise, the grasp is not force closure. In this way, computing Q, d_1, and d_2 is no longer required.

II. Key to the Ray-Shooting–Based Algorithm

Theorem 1: A grasp is force closure if and only if d_1 is larger than d_2 [4].

The following lemma supports the exactness of Theorem 1.

Lemma 1: Any point on segment PQ except Q is an interior point of $H(W)$.

Proof: Since P is an interior point of $H(W)$, there exists a ball $B(P, r)$ with center P and radius r lying strictly inside $H(W)$. Let S be the convex hull of $B(P, r)$ and Q. As shown in Fig. 1, PQ is the central axis of S and any point strictly between P and Q is an interior point of S. From the convexity of $H(W)$ it follows that $S \subset H(W)$. Thus any point on segment PQ except Q is an interior point of $H(W)$. □

Alternatively, the ray PO intersects $\partial H(W)$ at a unique point.

The crucial step of the algorithm is to detect the intersection point Q of $\partial H(W)$ with PO, which is a typical ray-shooting problem (see [4], Definition 2). Liu applied a coordinate translation of $-P$ on points in the wrench space $\mathbb{R}^6$ so that the facet F of $H(W)$ intersected by PO can be detected based on the duality between $H(W)$ and $CT(W)$, where

$$CT(W) = \Big\{ x \in \mathbb{R}^6 | (w_{ij} - P)^T x \leq 1 \quad \text{for } i = 1, 2, \ldots, n, \ j = 1, 2, \ldots, m \Big\}. \quad (1)$$

After the coordinate translation of $-P$, the ray-shooting problem is equivalent to an LP problem

$$\begin{cases} \text{Maximize } z = t^T x \\ \text{subject to } x \in CT(W). \end{cases} \quad (2)$$

Suppose that the optimal solution of the LP problem (2) is $E = [e_1, e_2, \ldots, e_6]^T$. Then, F lies in the hyperplane

$$E^T x = 1. \quad (3)$$

Thus, Q is the intersection of the hyperplane (3) with PO.

III. A Shortcut

Assume that the coordinate translation is applied so that P moves to the origin and the origin becomes an interior point of $H(W)$. Let $z_{\max}(t)$ denote the optimal objective value of the LP problem (2) with respect to t

$$z_{\max}(t) = t^T E. \quad (4)$$

Since $H(W)$ is a compact convex set that contains the origin as an interior point, $CT(W)$ is also a compact convex set containing the origin as an interior point [6, p. 142]. Thus $z_{\max}(t) > 0$ for any nonzero t.

In our opinion, to calculate Q, F is unnecessary. Indeed

$$Q = z_{\max}(t)^{-1} t. \quad (5)$$

Firstly, (5) represents a point on the ray PO. Secondly, after substituting (4) into (5), the latter satisfies (3). Hence, (5) is the intersection point of $\partial H(W)$ with PO.

Now, we give another proof of the preceding conclusion without introducing F and (3).

Proposition 1: $z_{\max}(t)^{-1} t$ is a point of $\partial H(W)$.

Proof: Since E is the optimal solution of the LP problem (2), $x^T z_{\max}(t)^{-1} t \leq z_{\max}(t)^{-1} E^T t = 1$ for $\forall x \in CT(W)$. According to the duality between $H(W)$ and $CT(W)$, $z_{\max}(t)^{-1} t \in H(W)$. Suppose that there is a closed ball $B(z_{\max}(t)^{-1} t, r)$ of radius $r > 0$ centered at $z_{\max}(t)^{-1} t$. Let $y = z_{\max}(t)^{-1} t + rt/\|t\|$. Obviously, $y \in B(z_{\max}(t)^{-1} t, r)$. Then, $E^T y = 1 + r z_{\max}(t)/\|t\| > 1$. Since $E \in CT(W)$, $y \notin H(W)$ for $\forall r > 0$, which implies $z_{\max}(t)^{-1} t \in \partial H(W)$. □

Lemma 2: Q is irrelevant to the magnitude of t.

Proof: Since E is irrelevant to the magnitude of t, from (4) and (5), $z_{\max}(\lambda t)^{-1} \lambda t = z_{\max}(t)^{-1} t$ for $\forall \lambda > 0$. □

As a consequence of Lemma 2 and Theorem 1, we have the following theorem.

Theorem 2: $z_{\max}(-P)$ is the ratio of d_2 to d_1. If $z_{\max}(-P) < 1$, the grasp is force closure; otherwise, it is not.

Proof: Since Q is irrelevant to the magnitude of t by Lemma 2, let $t = -P$. Substituting $t = -P$ into (5) yields

$$Q = -z_{\max}(-P)^{-1} P.$$

Then

$$d_1 = \left\| -z_{\max}(-P)^{-1} P \right\| = z_{\max}(-P)^{-1} \|P\|.$$

Since $d_2 = \|P\|$, we have

$$z_{\max}(-P) = \frac{d_2}{d_1}. \quad (6)$$

From (6), $z_{\max}(-P) < 1$ is equivalent to $d_1 > d_2$. Thus from Theorem 1 it follows that a grasp is force closure if and only if $z_{\max}(-P) < 1$. □

Theorem 2 reveals the geometric meaning of the optimal objective value of (2). By doing this, the ray-shooting based algorithm for force-closure test need not compute Q, d_1, and d_2, so that $O(1)$ time is saved (Steps 4 and 5 in the original algorithm [4] can be eliminated).

IV. Numerical Examples

We implement the algorithm using the optimization toolbox of MATLAB on Pentium-IV PC and verify the shortcut with two examples. Each friction cone is linearized by a 100-sided polyhedral cone ($m = 100$), of which the edge vectors are

$$d_j = \left[1 \ \mu \cos \frac{2j\pi}{m} \ \mu \sin \frac{2j\pi}{m}\right]^T, \qquad j = 1, 2, \ldots, m. \tag{7}$$

Then

$$s_{ij} = R_i d_j, \qquad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, m \tag{8}$$

where R_i is the relative orientation of the local coordinate frame at contact i with respect to the object frame.

Example 1: This is the first example in [4], which is a four-fingered grasp of a polyhedral object. The contact points r_i are given by

$$\begin{aligned} r_1 &= [2\ 0\ 0]^T \\ r_2 &= [0\ 1.5\ 0]^T \\ r_3 &= [0\ 0\ 2]^T \\ r_4 &= [1.2\ -2\ 0]^T. \end{aligned}$$

The relative orientations are expressed by

$$R_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$
$$R_2 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$R_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$
$$R_4 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

When $\mu = 0.3$, with the CPU time of 4.7 ms, we have

$$P = [-0.2138\ -0.0667\ -0.2076\ 0\ 0\ 0.1715]^T.$$

Let $t = -P$. The optimal objective value of (2) is

$$z_{\max}(-P) = 1.1926 > 1.$$

The CPU time is 156.5 ms. Hence, the grasp is not force closure. When $\mu = 0.5$, with the CPU time of 4.6 ms, we obtain

$$P = [-0.1797\ -0.0456\ -0.1667\ 0\ 0\ 0.1421]^T.$$

Then, with the CPU time of 132.5 ms, the LP problem (2) with respect to $t = -P$ turns out

$$z_{\max}(-P) = 0.7421 < 1.$$

Thus, the grasp is force closure.

The shortcut reaches the same result as [4].

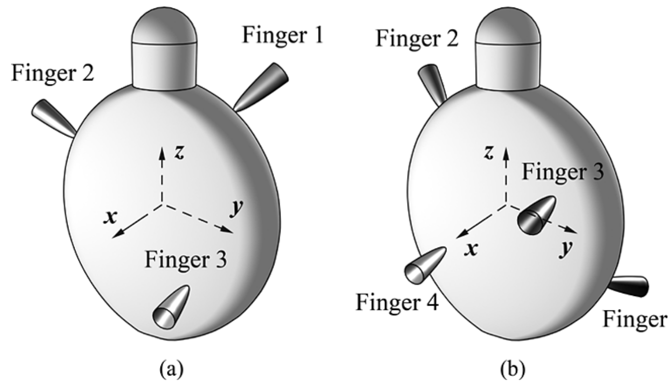

Fig. 2. Snuff bottle is gripped by (a) a three-fingered hand or (b) a four-fingered hand.

Example 2: As shown in Fig. 2, it is required to grasp a snuff bottle, whose body can be expressed by

$$\begin{cases} x = a \cos \beta \\ y = b_1 \cos \alpha \sin \beta \\ z = b_2 \sin \alpha \end{cases}$$

where $\alpha_0 - 4\pi/5 \le \alpha \le \alpha_0$, $0 \le \beta \le 2\pi$, $\alpha_0 = \arccos(a/b_1)$, $a = 7.5$, $b_1 = 30$, $b_2 = 35$.

Suppose that $\mu = 0.2$ at each contact. We first consider a three-fingered grasp [Fig. 2(a)], whose contacts are at

$$\begin{aligned} r_1 &= [-3.75\ 18.3712\ 24.7487]^T \ \left(\alpha_1 = \frac{\pi}{4},\ \beta_1 = \frac{2\pi}{3}\right) \\ r_2 &= [0\ -25.9808\ 17.5]^T \ \left(\alpha_2 = \frac{\pi}{6},\ \beta_2 = \frac{3\pi}{2}\right) \\ r_3 &= [6.9291\ 9.9424\ -17.5]^T \ \left(\alpha_3 = -\frac{\pi}{6},\ \beta_3 = \frac{\pi}{8}\right). \end{aligned}$$

The relative orientations are

$$R_1 = \begin{bmatrix} 0.7951 & 0.3082 & -0.5222 \\ -0.4869 & -0.1888 & -0.8528 \\ -0.3614 & 0.9324 & 0 \end{bmatrix}$$
$$R_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0.8963 & 0.4435 & 0 \\ -0.4435 & 0.8963 & 0 \end{bmatrix}$$
$$R_3 = \begin{bmatrix} -0.9927 & 0.0223 & -0.1187 \\ -0.1187 & 0.0027 & 0.9929 \\ 0.0225 & 0.9997 & 0 \end{bmatrix}.$$

With the CPU times of 3.1 ms and 164.9 ms, we have

$$P = [-0.0054\ 0.0422\ -0.0348\ -0.0757\ 0.5506\ -0.0295]^T$$
$$z_{\max}(-P) = 8.0629 > 1.$$

Thus, the grasp is not force closure.

Next we try a four-fingered grasp [Fig. 2(b)] having contacts

$$\begin{aligned} r_1 &= [-5.3033\ 18.3712\ -17.5]^T \ \left(\alpha_1 = -\frac{\pi}{6},\ \beta_1 = \frac{3\pi}{4}\right) \\ r_2 &= [-3.75\ -18.3712\ 24.7487]^T \ \left(\alpha_2 = \frac{\pi}{4},\ \beta_2 = \frac{4\pi}{3}\right) \\ r_3 &= [6.4952\ 13.8582\ 13.3939]^T \ \left(\alpha_3 = \frac{\pi}{8},\ \beta_3 = \frac{\pi}{6}\right) \\ r_4 &= [6.4952\ -13.8582\ -13.3939]^T \ \left(\alpha_4 = -\frac{\pi}{8},\ \beta_4 = \frac{11\pi}{6}\right). \end{aligned}$$

The relative orientations are

$$R_1 = \begin{bmatrix} 0.9563 & -0.0928 & -0.2774 \\ -0.2761 & 0.0268 & -0.9608 \\ 0.0966 & 0.9953 & 0 \end{bmatrix}$$
$$R_2 = \begin{bmatrix} 0.7951 & 0.3082 & 0.5222 \\ 0.4869 & 0.1888 & -0.8528 \\ -0.3614 & 0.9324 & 0 \end{bmatrix}$$
$$R_3 = \begin{bmatrix} -0.9876 & -0.0271 & -0.1544 \\ -0.1543 & -0.0042 & 0.9880 \\ -0.0274 & 0.9996 & 0 \end{bmatrix}$$
$$R_4 = \begin{bmatrix} -0.9876 & 0.0271 & 0.1544 \\ 0.1543 & -0.0042 & 0.9880 \\ 0.0274 & 0.9996 & 0 \end{bmatrix}.$$

With the CPU times of 4.7 ms and 175.8 ms, we have

$$P = [-0.0080\ 0.0023\ -0.0030\ -0.0449\ 0.0220\ -0.0359]^T$$
$$z_{\max}(-P) = 0.5159 < 1.$$

Hence, the grasp is force closure.

V. Conclusion and Future Work

A shortcut is found to simplify Liu's ray-shooting based algorithm for force-closure test [4]. The optimal objective value of the LP problem (2) with respect to $t = -P$ is the ratio of d_2 to d_1; that is, $z_{\max}(-P) = d_2/d_1$. If $z_{\max}(-P) < 1$, then the grasp is force closure; otherwise, it is not. Consequently, we can skip the steps of computing Q, d_1, and d_2. Having the geometric insight into the maximum $z_{\max}(-P)$, we can apply it to optimal grasp planning as a force-closure index. As this work goes beyond the topic of the paper, it is decent to leave it for the future.

Acknowledgment

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The Hierarchical Atlas

Brad Lisien, Deryck Morales, David Silver, George Kantor, Ioannis Rekleitis, and Howie Choset

***Abstract*—This paper presents a new map specifically designed for robots operating in large environments and possibly in higher dimensions. We call this map the *hierarchical atlas* because it is a multilevel and multiresolution representation. For this paper, the hierarchical atlas has two levels: at the highest level there is a topological map that organizes the free space into submaps at the lower level. The lower-level submaps are simply a collection of features. The hierarchical atlas allows us to perform calculations and run estimation techniques, such as Kalman filtering, in local areas without having to correlate and associate data for the entire map. This provides a means to explore and map large environments in the presence of uncertainty with a process named *hierarchical simultaneous localization and mapping*. As well as organizing information of the free space, the map also induces well-defined sensor-based control laws and a provably complete policy to explore unknown regions. The resulting map is also useful for other tasks such as navigation, obstacle avoidance, and global localization. Experimental results are presented showing successful map building and subsequent use of the map in large-scale spaces.**

***Index Terms*—Concurrent mapping and localization, generalized Voronoi diagram, Kalman filtering, mobile robots, simultaneous localization and mapping (SLAM), topological maps.**

I. Introduction

This paper presents a new map organization for mobile robots which embodies scalability in both storage and computation to address common robot tasks in large-scale environments. These tasks include simultaneous localization and mapping (SLAM), path planning, global localization, and obstacle avoidance in nonstatic environments. This paper addresses each of these tasks, and presents experimental results obtained with a mobile robot in a large environment containing cycles, to show how the new map is well-suited to address these tasks.

The successful implementation of these tasks depends on a reliable and usable map. With the choice of three basic types of maps, topological, grid-based, and feature-based, it seems that one must settle for drawbacks inherent in each type in order to take advantage of its particular benefits. Topological maps scale nicely to large planar environments and to environments of higher dimension by storing a minimal amount of information. Such a minimalistic representation lacks the necessary information to localize arbitrarily (can only localize to nodes in the topological graph) and to disambiguate similar topological regions.

Grid-based approaches offer discretized renditions of unstructured free spaces which can be used for many robot tasks. However, the high resolution required for accurate representations demands large amounts of memory to store and computation time to maintain. Feature-based methods extract distinct landmark features from the environment for use in robot localization, but do not explicitly address obstacles unless the obstacles have structured, observable characteristics. Feature- and

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