CHAPTER 3

Robot Trajectory Planning & Kinematic Robot Control

After this chapter, the students are expected to learn the following:

1. Plan point to point trajectories in joint space and task space
2. Plan trajectories with via points
3. Plan trajectories with velocity and acceleration constraints
4. Kinematic control algorithm using resolved motion rate control

Trajectory Generation

- Polynomial Trajectory
  - subject to constraints
  - boundary conditions

- Initial & Final Positions $\theta(0) = \theta_0$, $\theta(t_f) = \theta_f$

  Initial & Final Velocities $\dot{\theta}(0) = 0$, $\dot{\theta}(t_f) = 0$

  } 4 constraints

Trajectory Generation

Cubic Polynomial

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

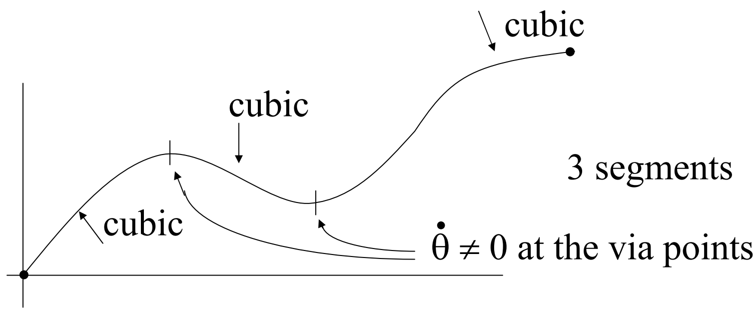
Four Equations (constraints) in Four Unknowns ($a_0, \ldots, a_3$)

$$a_0 = \theta_0 \qquad a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0)$$

$$a_1 = 0 \qquad a_3 = \frac{-2}{t_f^3}(\theta_f - \theta_0)$$

Trajectory Generation

- Cubic Polynomials with Via Points



– Choose velocities at via points
  - kinetics
  - continuity in acceleration
  - others

Trajectory Generation

- Initial & final positions
  " " " velocities
  " " " acceleration
  } 6 constants

Quintic Polynomial

$\theta(t) = a_0 + a_1 t + a_2 t^2 + a3t3 + a_4 t^4 + a_5 t^5$

6 Equations in 6 unknowns:

$a_0 = \theta_0$

$a_1 = \theta_0$

$a_2 = \frac{\theta_0}{2}$

Trajectory Generation

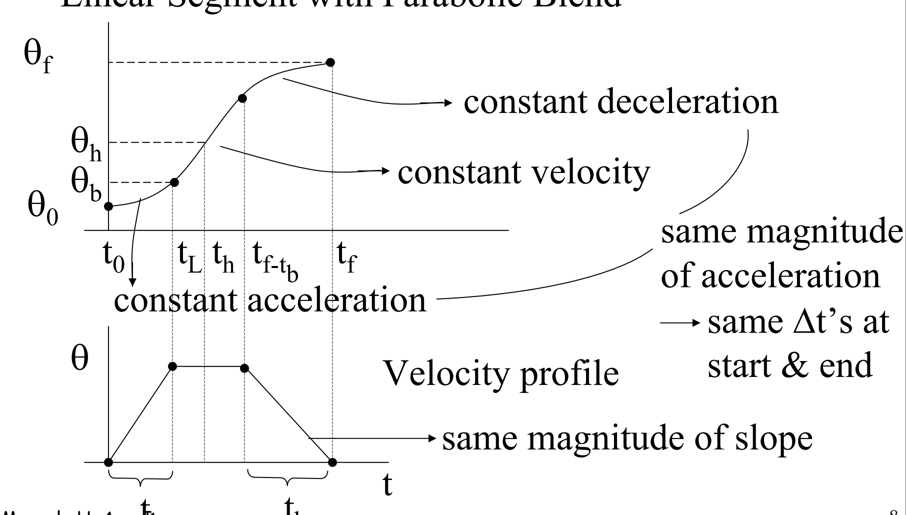
$$a_3 = \frac{20\theta_f - 20\theta_0 - (\delta\,\dot{\theta}_f + 12\dot{\theta}_0)t_f - (3\ddot{\theta}_0 - \ddot{\theta}_f)t_f^2}{2t_f^3}$$

$$a_4 = \frac{30\theta_0 - 30\theta_f + (14\,\dot{\theta}_f + 16\dot{\theta}_0)t_f + (3\ddot{\theta}_0 - 2\ddot{\theta}_f)t_f^2}{2t_f^4}$$

$$a_5 = \frac{12\theta_f - 12\theta_0 - (6\,\dot{\theta}_f + 6\dot{\theta}_0)t_f - (\ddot{\theta}_0 - \ddot{\theta}_f)t_f^2}{2t_f^5}$$

Trajectory Generation

- Linear Segment with Parabolic Blend



Trajectory Generation

– There are many possible solutions, given

$\theta(0) = \theta_0 \qquad \dot{\theta}(0) = 0$

$\theta(t_f) = \theta_f \qquad \dot{\theta}(t_f) = 0$

$$\theta_b = \theta_0 + \tfrac{1}{2}\ddot{\theta}\, t_b^2 \qquad \text{—— (1)}$$

$$\dot{\theta}_b = \overset{0}{\cancel{\dot{\theta}_0}} + \ddot{\theta} t_b = \frac{\theta_h - \theta_b}{t_h - t_b} \qquad \text{—— (2)}$$

$$\text{where } \theta_h = \frac{1}{2}\left(\frac{\theta_f + \theta_0}{2}\right)$$

Trajectory Generation

Given θ_0, θ_f and t_f = (move time)

Can choose $\ddot{\theta}$ and t_b to satisfy (1) & (2)

Combining (1) & (2) and since $t_f = 2t_h$, we get

$$\ddot{\theta} t_b^2 - \ddot{\theta} t_f t_b + (\theta_f - \theta_0) = 0$$

$$t_b = \frac{t_f}{2} - \frac{\sqrt{\ddot{\theta}^2 t_f^{\,2} - 4\ddot{\theta}(\theta_f - \theta_0)}}{2\ddot{\theta}}$$

Trajectory Generation

For t_b to exists,

$$\ddot{\theta} \geq \frac{4(\theta_f - \theta_0)}{t_f^{\,2}}$$ (acceleration must be sufficiently high)

When equality holds:

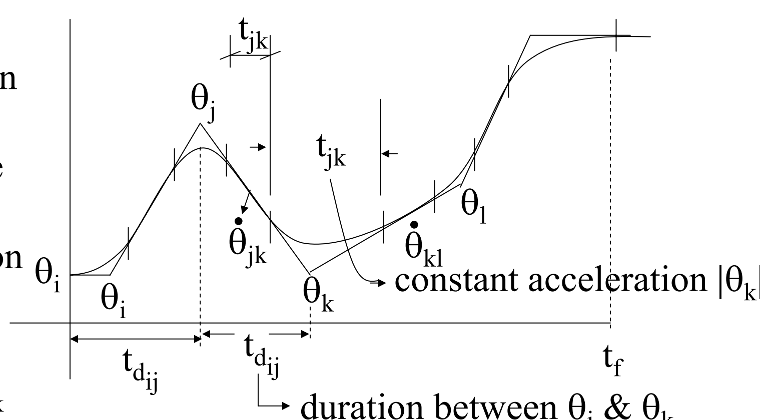
$$t_b = \frac{t_f}{2} = t_h$$ (no constant velocity)

Trajectory Generation

- Linear Segments with Parabolic Blods & with Via Points

given: - all points θ_k
- all duration $t_{d_{jk}}$
- magnitude of acceleration $|\theta_k|$

compute: bold times t_k



Trajectory Generation

Interior Paths:

$$\dot{\theta}_{jk} = \frac{\theta_k - \theta_j}{t_{d_{jk}}} \qquad \dot{\theta}_{kl} = \frac{\theta_l - \theta_k}{t_{d_{kl}}}$$

$$\ddot{\theta}_k = \mathrm{SGN}(\dot{\theta}_{kl} - \dot{\theta}_{jk})\,|\ddot{\theta}_k|$$

$$t_k = \frac{\dot{\theta}_{kl} - \dot{\theta}_{jk}}{\ddot{\theta}_k}$$

$$t_{jk} = t_{d_{jk}} - ½\, t_j - ½\, t_k$$

Trajectory Generation

First Segment (difficult because entire blod region crowded in duration)

$$\frac{\theta_2 - \theta_1}{t_{d_{12}} - \frac{1}{2}t_1} = \ddot{\theta}_1 t_1 = \text{velocity at } 1^{st} \text{ line segment} = \dot{\theta}_{12}$$

where $\ddot{\theta}_1 = \mathrm{SGN}(\theta_2 - \theta_1)\,|\ddot{\theta}_1|$

$$\therefore t_1 = t_{d_{12}} - \sqrt{{t_{d_{12}}}^2 - \frac{2(\theta_2 - \theta_1)}{\theta_1}}$$

$$\dot{\theta}_{12} = \frac{\theta_2 - \theta_1}{t_{d12} - \frac{1}{2}t_1}$$

$$t_{12} = t_{d_{12}} - t_1 - ½\, t_2$$

Trajectory Generation

Last segment connecting θ_{n-1} & θ_n

$$\frac{\theta_n - \theta_{n-1}}{t_{d_{(n-1)n}} - \frac{1}{2} t_n} = -\ddot{\theta}_n t_n \qquad \text{where } \ddot{\theta}_n = \mathrm{SGN}(\theta_n - \theta_{n-1}) \, |\ddot{\theta}_n|$$

$$t_n = t_{d_{(n-1)n}} - \sqrt{t_{d_{(n-1)n}}^{\,2} + \frac{2(\theta_n - \theta_{n-1})}{\theta_n}}$$

$$\dot{\theta}_{(n-1)n} = \frac{\theta_n - \theta_{n-1}}{t_{d_{(n-1)n}} - \frac{1}{2} t_n}$$

$$t_{(n-1)n} = t_{d_{(n-1)n}} - t_n - \tfrac{1}{2} t_{n-1}$$

Resolved Motion Rate Control

– Kinematic Control without the need for solving the inverse Kinematics of Position

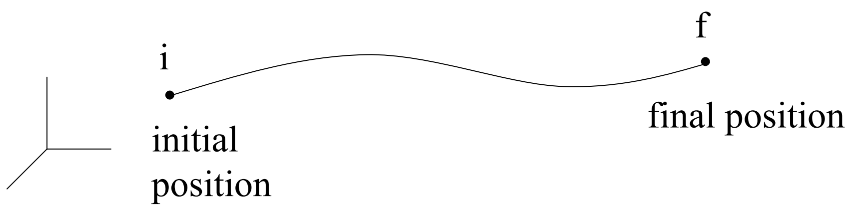
– Need Joint level control which is available in robot controllers

δq δx

– Command joint motion such that desired e-e motion is achieved

Resolved Motion Rate Control

1) Given a Trajectory $x(t) \in R^m$ in task space



2) Divide Trajectory into small segments according to sample time on reference Trajectory update rate

3) At x_k, compute $E(x_k)$, compute $J_0(q_k)$

Resolved Motion Rate Control

4) Compute $\Delta x_k = x_{k+1} - x_k$

5) Compute $\delta x_{0,\,k} = E(x_k)^+ \, \Delta \, x_k$

6) Compute $\delta q = J_0^{\#}(q_k)\, \delta x_{0,\,k} + [In - J_0^{\#}(q^k)\, J_0(q_k)]\, \delta q_0$

7) Command δq to robot controller
(Robot moves from q_k to q_{k+1}) $(\delta q = q_{k+1} - q_k)$

8) Go to step 3 until x_k reaches x_f