National University of Singapore
Faculty of Engineering

EE5106/MCH5209/ME5402: Advanced Robotics

Quiz 1 **Open Book Examination** 17 Feb 2003 **(90 mins)**

1. Refer to Figure 1. The origin of Frame B is in (3,0,0). The X and Y axis of Frame {B} passes through coordinates (3,0,-4) and (2,5,0), respectively. All coordinates are expressed in Frame {A}. Determine ${}^{A}T_{B}$ that describes the position and orientation of Frame {B} in Frame {A}.

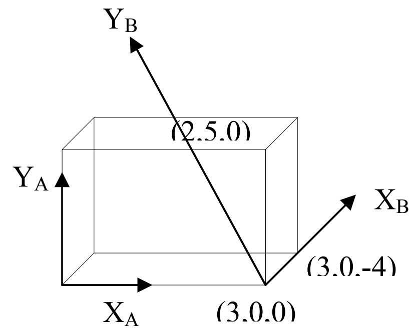

Figure 1

2. There are 2 fixed frames {A} and {B} in space whose position and orientation relative to each other are known as ${}^{A}T_{B}$. Frame {C} is attached to a rigid moving body. Initially C is at a known position and orientation described by ${}^{A}T_{C}$. The body then undergoes the following sequence of motions:
   a) Rotation about the x-axis of Frame {B} by 30 degrees, followed by
   b) Translation along itself (Frame {C}) by (1, 2, 3)

   Determine the new position and orientation of Frame {C} in Frame {A}.

3. Figure 2 shows a robot with 1 rotational joint followed by a translational joint. Frame {U} is attached to the base of the robot and is fixed to the ground. Frame {E} is attached to the end-effector as shown in Figure 2.

   a) Assign all necessary frames to completely describe the kinematics of the robot.
   b) Fill in the table of kinematic parameters and indicate which parameters change as the robot moves (i.e., which are the joint coordinates?).
   c) Given

$$ {}^{U}T_{E} = \begin{pmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

   Which element(s) in ${}^{U}T_{E}$ are variables (or change values as the robot moves)?

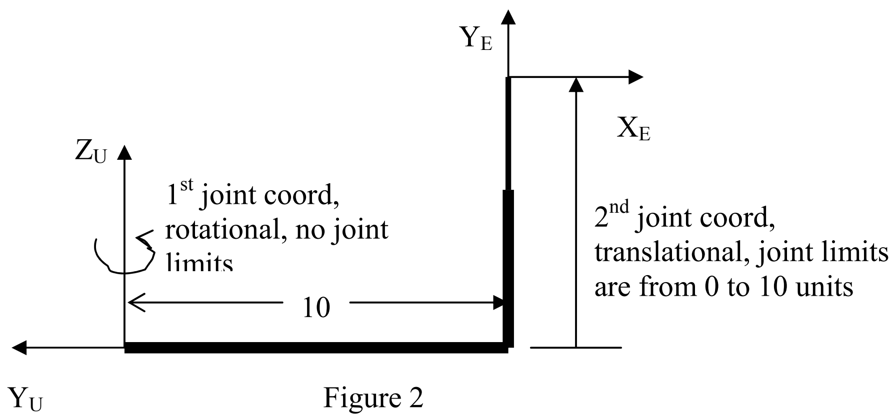

Figure 2

4. Figure 3 shows two robots ($Robot_1$ and $Robot_2$) with base frames $\{0_1\}$ and $\{0_2\}$, and end-effector frames $\{E_1\}$ and $\{E_2\}$. $Robot_2$ is rigidly attached to the end-effector of $Robot_1$ ($Robot_1$ carries $Robot_2$), with ${}^{E_1}T_{0_2}$ known and constant. The kinematic models of two robots are completely known. The manipulator Jacobians for each robot are known to be:

$$ {}^{0_1}J_{E_1} = \begin{pmatrix} J_{v1} \\ J_{\omega 1} \end{pmatrix} \qquad {}^{0_2}J_{E_2} = \begin{pmatrix} J_{v2} \\ J_{\omega 2} \end{pmatrix} $$

where J_v and J_ω represent the translational and angular velocity partitions of the Jacobian. Determine an expression for the manipulator Jacobian of the combined robot system, ${}^{0_1}J_{E_2}$, as a function of J_{vi}, $J_{\omega i}$, ${}^{E_1}T_{0_2}$, and other parameters derived from the kinematics of each robot.

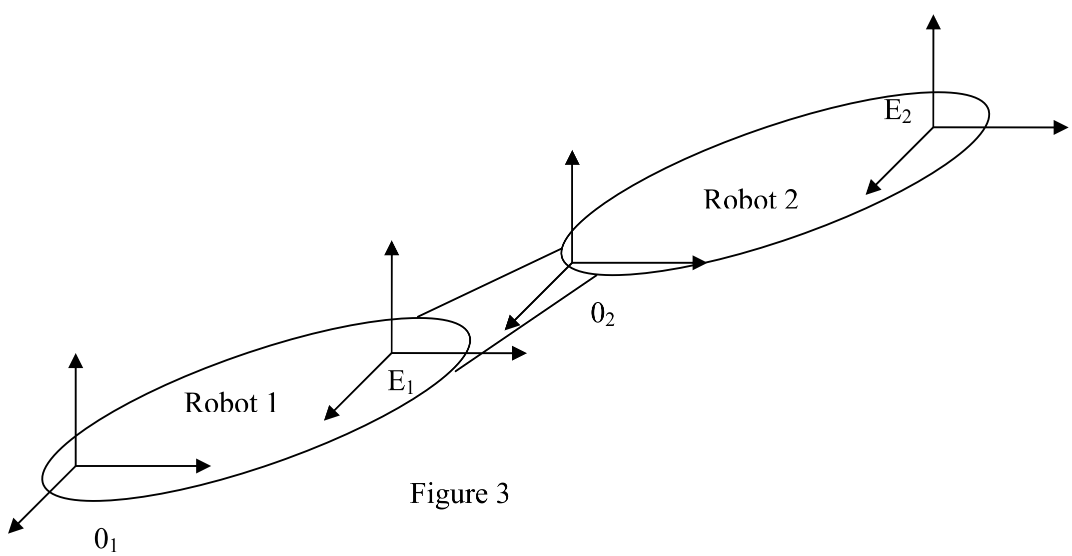

Figure 3