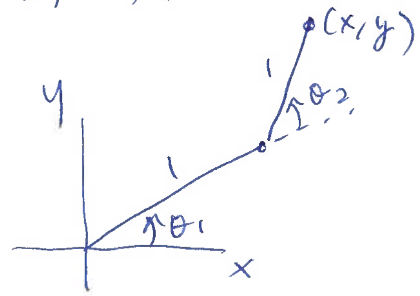
Robot Inverse Kinematics of Position



$x = C_1 + C_{12}$

$y = S_1 + S_{12}$

$S_i = \sin\theta_i$

$C_i = \cos\theta_i$

$C_{12} = \cos(\theta_1+\theta_2)$

$S_{12} = \sin(\theta_1+\theta_2)$

Three possibilities to manipulate Eqns.

① square both sides

$$\left.\begin{array}{l} x^2 = C_1^2 + C_{12}^2 + 2C_{12}C_1 \\ y^2 = S_1^2 + S_{12}^2 + 2S_{12}S_1 \end{array}\right\} \oplus \rightarrow x^2+y^2 = C_1^2 + S_1^2 + C_{12}^2 + S_{12}^2 + 2C_2$$

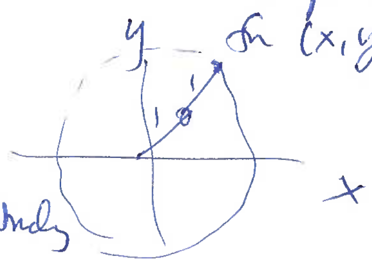
$$x^2+y^2 = 2 + 2C_2 \rightsquigarrow C_2 = \frac{x^2+y^2-2}{2}$$ Case 2

Note that $|C_2| < 1 \therefore x^2+y^2-2 \le 2$

$x^2+y^2 \le 4$ workspace for (x, y)

$S_2 = \pm\sqrt{1-C_2^2}$

2 solns for θ_2



② multiply both sides by C_1 and S_1, respectively

$$\left.\begin{array}{l} C_1 x = C_1^2 + C_{12}C_1 \\ S_1 y = S_1^2 + S_{12}S_1 \end{array}\right\} \oplus \rightarrow xC_1 + yS_1 = 1 + C_2 \quad (2)$$

this looks like Case 4 which gives 2 solns.

So we may think there are 4 solns. But we have to be careful of extraneous roots. Let's use another manipulation of Eqns to get rid of extraneous roots.

③ multiply both sides by S_1 and C_1 respectively and subtract:

$$\left.\begin{array}{l} S_1 x = S_1C_1 + C_{12}S_1 \\ C_1 y = S_1C_1 + S_{12}C_1 \end{array}\right\} \ominus \quad xS_1 - yC_1 = -S_2 \quad (3)$$

also looks like Case 4

But . . .

② and ③ : via case 6

$$a\cos\theta - b\sin\theta = c$$
$$a\sin\theta + b\cos\theta = d$$

$a = x \qquad b = -y \qquad c = 1 + C_2$

$$x\cos\theta_1 + y\sin\theta_1 = 1 + C_2$$
$$x\sin\theta_1 - y\cos\theta_1 = -S_2$$

$d = -S_2$

$$\theta = ATAN2\left(\frac{ad - bc}{ac + bd}\right)$$ 1 soln.

$$\therefore \quad \theta_1 = ATAN2\left(\frac{-xS_2 + y(1+C_2)}{x(1+C_2) + yS_2}\right)$$

1 soln.

Note

$a^2 + b^2 = c^2 + d^2$

Note

$$x^2 + y^2 = (1 + C_2)^2 + S_2^2 = 1 + 2C_2 + C_2^2 + S_2^2$$
$$x^2 + y^2 = 2 + 2C_2 = 2(1 + C_2)$$
$$\text{or } C_2 = \frac{x^2 + y^2}{2} - 1$$

same eqn as before

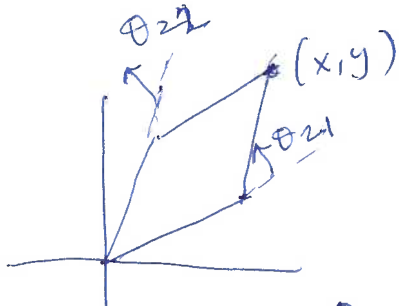
This eqn can also be used to compute θ_2.

2 solutions.

1st : $$\begin{cases} \theta_2 = ATAN2\left(\dfrac{\sqrt{1 - C_2^2}}{C_2}\right) = \theta_{21} \\ \theta_1 = ATAN2\left(\dfrac{-xS_2 + y(1+C_2)}{x(1+C_2) + yS_2}\right) \end{cases}$$

when $C_2 = \dfrac{x^2 + y^2}{2} - 1$

2nd : $$\begin{cases} \theta_2 = ATAN2\left(-\sqrt{1 - C_2^2} / C_2\right) = \theta_{22} \\ \theta_1 = ATAN2\left(\dfrac{-xS_2 + y(1+C_2)}{x(1+C_2) + yS_2}\right) \end{cases}$$



$\theta_{21} = -\theta_{22}$