ME4245 Robot Kinematics, Dynamics and Control
Quiz 2 (September 2009), 1 hr

Name: ______________________________ Matric number: ______________

1. A robot manipulator has two joints whose axes are shown in Fig. 1. The 1st joint is rotational while the 2nd joint is translational. Assign frames to the robot according to the Denavit Hartenberg (DH) convention discussed in class. Provide the DH parameters of the robot and indicate which parameter(s) is (are) the joint variable (s). (All coordinates are expressed in Frame U).

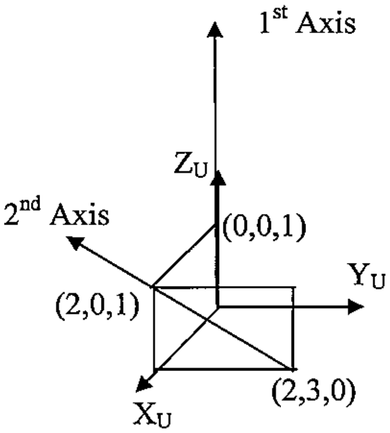

Fig. 1

2. There are 2 moving frames B and C and a fixed frame A. At an instant of time the following are known:

$^{A}P_{B}$ $\quad ^{A}R_{B}$ $\quad ^{A}P_{C}$ $\quad ^{A}R_{C}$ $\quad ^{A}u_{B}$ $\quad ^{A}\omega_{B}$ $\quad ^{A}u_{C}$ $\quad ^{A}\omega_{C}$

Determine the relative velocities of C with respect to B, i.e., find $^{B}u_{C}$ $\quad ^{B}\omega_{C}$

3. Fig. 2 shows a planar robot with three joints: the 1st and 3rd joints are rotational, while the 2nd joint is translational. (The third link has a length of 1 m.)

   a. Determine the expression for the full manipulator Jacobian.
   b. For the task of positioning the end point of the last link in Frame U at (x,y), are there singular configurations?

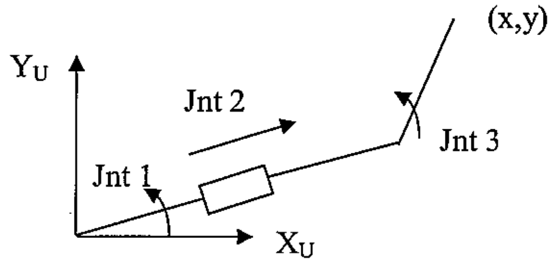

Fig 2

4. The robot in Fig. 2 carries a payload of mass 10 kg at the joint configuration shown in Fig. 3 (q_1 = 30 degrees, q2 = 1 m (length of UA), and q_3 = 60 degrees). Frame E is attached to the last link of the robot. The last link (AE) has a length of 1 m. The payload has its centre of gravity located at coordinates (2, 3) m in Frame E. Determine the joint efforts (torques or forces at each of the three joints). Assume that the links have negligible weights. The acceleration due to gravity is along the negative Y_U direction ($g = 9.8\ m/s^2$).

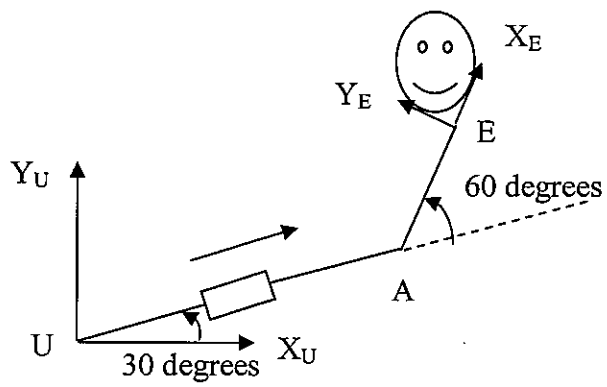

Fig. 3

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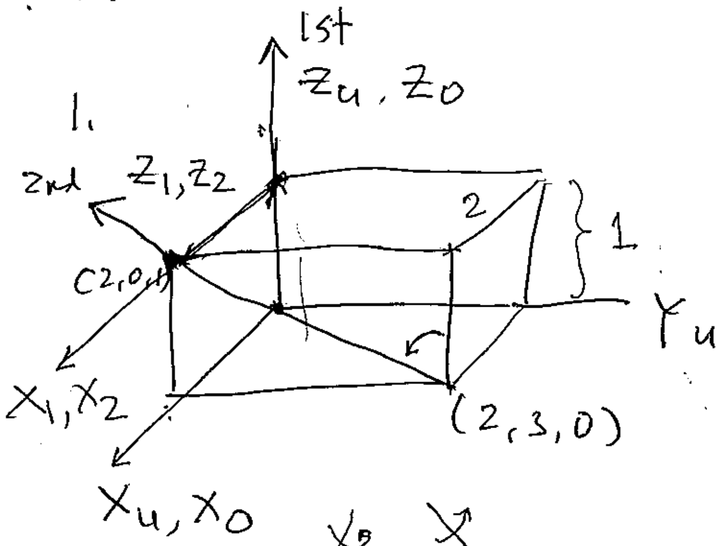
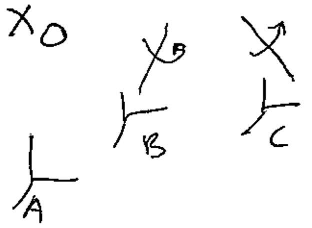
1.



	θ	r	d	α
1	$q_1 = 0^\circ$	1	2	ATAN2(3/1
2	0°	$q_2 = 0$	0	0°

2.



$${}^A T_B\,{}^B T_C = {}^A T_C$$

$${}^A\dot{T}_C = {}^A T_B\,{}^B\dot{T}_C + {}^A\dot{T}_B\,{}^B T_C$$

$$\begin{pmatrix} {}^A\hat{\omega}_C\,{}^A R_C & {}^A u_C \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} {}^A R_B & {}^A p_B \\ 0 & 1 \end{pmatrix}\begin{pmatrix} {}^B\hat{\omega}_C\,{}^B R_C & {}^B u_C \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} {}^A\hat{\omega}_B\,{}^A R_B & {}^A u_B \\ 0 & 0 \end{pmatrix}\begin{pmatrix} {}^B R_C & {}^B p_C \\ 0 & 1 \end{pmatrix}$$

$${}^A\hat{\omega}_C\,\cancel{{}^A R_C} = {}^A R_B\,{}^B\hat{\omega}_C\,\underbrace{{}^B R_C}_{{}^B R_A\,\cancel{{}^A R_C}} + {}^A\hat{\omega}_B\,\underbrace{{}^A R_B\,{}^B R_C}_{\cancel{{}^A R_C}}$$

$${}^A\hat{\omega}_C = {}^A R_B\,{}^B\hat{\omega}_C\,{}^B R_A + {}^A\hat{\omega}_B \qquad 3\times 3$$

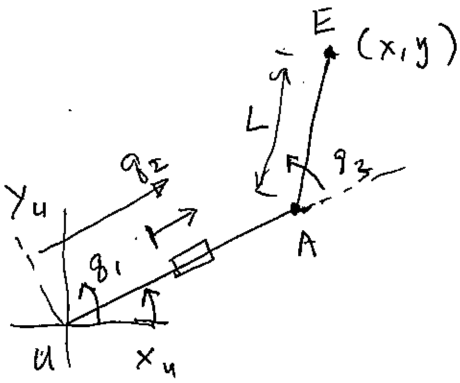
$${}^A\omega_C = {}^A R_B\,{}^B\omega_C + {}^A\omega_B \quad ✓$$

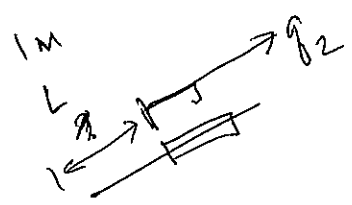
$$\boxed{{}^B\omega_C = {}^A R_B^T\left({}^A\omega_C - {}^A\omega_B\right)}$$

$${}^A u_C = {}^A R_B\,{}^B u_C + {}^A\hat{\omega}_B\,{}^A R_B\,{}^B p_C + {}^A u_B \quad ✓$$

$$\boxed{{}^B u_C = {}^A R_B^T\left({}^A u_C - {}^A u_B - {}^A\hat{\omega}_B\,{}^A R_B\,{}^B p_C\right)}$$

3.





uA = variable length = q_2

$$x = q_2 C_1 + L C_{13}$$
$$y = q_2 S_1 + L S_{13}$$
$$\dot{x} = -q_2 S_1 \dot{q}_1 + \dot{q}_2 C_1 - L S_{13}(\dot{q}_1 + \dot{q}_3)$$
$$\dot{y} = q_2 C_1 \dot{q}_1 + \dot{q}_2 S_1 + L C_{13}(\dot{q}_1 + \dot{q}_3)$$
$$^u\omega_z = \dot{q}_1 + \dot{q}_3$$

$$J = \begin{pmatrix} -q_2 S_1 - L S_{13} & C_1 & -L S_{13} \\ q_2 C_1 + L C_{13} & S_1 & L C_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Another Method.

$$J = [\, J_1 \;\; J_2 \;\; J_3 \,] =$$

$$J_1 = \begin{pmatrix} Z_0 \times (P_3 - P_0) \\ Z_0 \end{pmatrix} \qquad J_2 = \begin{pmatrix} Z_1 \\ 0 \end{pmatrix} \qquad J_3 = \begin{pmatrix} Z_2 \times P_3 - P_2 \\ Z_2 \end{pmatrix}$$

$$Z_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad P_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad P_3 = \begin{pmatrix} q_2 C_1 + L C_{13} \\ q_2 S_1 + L S_{13} \\ 0 \end{pmatrix}$$

$$P_2 = \begin{pmatrix} q_2 C_1 \\ q_2 S_1 \\ 0 \end{pmatrix} \qquad Z_1 = \begin{pmatrix} \cos q_1 \\ \sin q_1 \\ 0 \end{pmatrix} \qquad Z_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Task (x, y)

$$J = \begin{pmatrix} -q_2 S_1 - L S_{13} & C_1 & -L S_{13} \\ q_2 C_1 + L C_{13} & S_1 & L C_{13} \end{pmatrix}$$

all possible 2×2

$$\det \begin{pmatrix} -q_2 S_1 - L S_{13} & C_1 \\ q_2 C_1 + L C_{13} & S_1 \end{pmatrix} = -q_2 S_1^2 - L S_{13} S_1 - q_2 C_1^2 - L C_{13} C_1$$

$$= -q_2 - L(C_{13} C_1 + S_{13} S_1)$$

$$= -q_2 - L C_3 = 0.$$

$$\det \begin{pmatrix} C_1 & -L S_{13} \\ S_1 & L C_{13} \end{pmatrix} = L C_{13} C_1 + L S_{13} S_1$$

$$= L(C_3) = 0$$

$$\det \begin{pmatrix} -q_2 S_1 - L S_{13} & -L S_{13} \\ q_2 C_1 + L C_{13} & L C_{13} \end{pmatrix} = -q_2 S_1 L C_{13} - L^2 S_{13} C_{13} + q_2 C_1 L S_{13} + L^2 S_{13} C_{13}$$

$$= +q_2 L (S_{13} C_1 - C_{13} S_1)$$

$$= +q_2 L S_3 = 0$$

__when:__ $q_2 = 0$, $C_3 = 0$, $\theta_3 = \pm 90°$ all determinants $= \emptyset$

at $\theta_3 = \pm 90°$, q_2 and q_3 has same effect on e-e motion and if $q_2 = 0$ also, then q_1 & q_3 has same effect.

Another method:

$$\det(J J^T) = 0$$

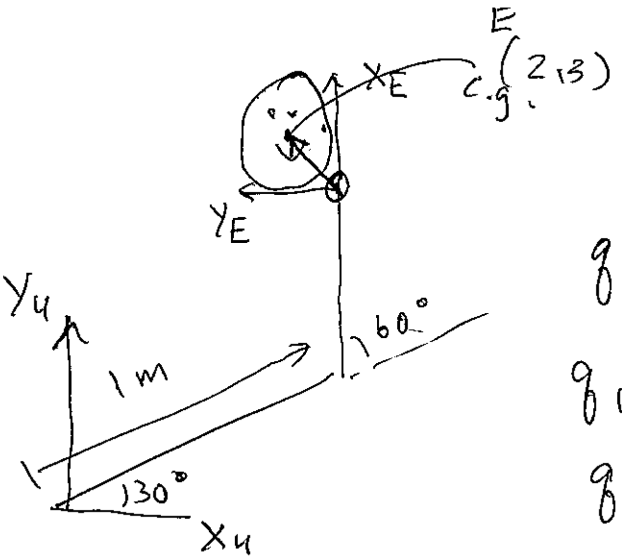
find q_1, q_2, q_3 such that

$$\det(J J^T) = L^2 + q_2^2 + \frac{L^2 q_2^2}{2} + 2 L q_2 C_3 - \frac{1}{2}(-2 + q_2^2)\cos(2 q_3)$$

If $q_2 = 0$, $\det(J J^T) = L^2 + L^2 \cos(2 q_3)$

If $q_3 = \pm 90°$ also $\det(J J^T) = 0$

4.



$${}^{E}P_{cg} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

$$\left.\begin{array}{l} q_2 = 1 \\ q_1 = 30^\circ \\ q_3 = 60^\circ \end{array}\right\} \phi$$

$${}^{4}J_E = \begin{pmatrix} -q_2 S_1 - L S_{13} & C_1 & -L S_{13} \\ q_2 C_1 + L C_{13} & S_1 & L C_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

joint efforts

$$3\times 1 \rightarrow \underline{\tau} = {}^{4}J_E^{T}\ {}^{4}F_E \qquad {}^{4}F_E = \begin{pmatrix} {}^{4}f_E \\ {}^{4}n_E \end{pmatrix}$$

$${}^{4}f_E = \begin{pmatrix} 0 \\ 10(9.8) \\ 0 \end{pmatrix} \qquad {}^{4}n_E = \underline{{}^{4}R_E}\ {}^{E}P_{cg} \times {}^{4}f_E$$

$${}^{4}R_E = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & +1 \end{pmatrix}$$

$\tau =$ torque (joint 1)
force (joint 2)
torque (joint 3)