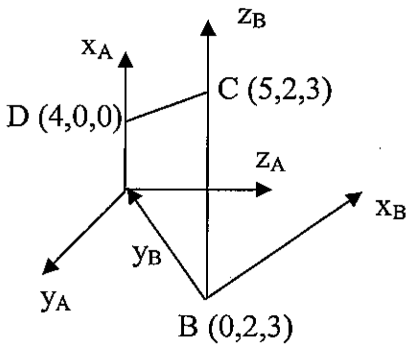
ME4245E Robot Kinematics, Dynamics and Control
Quiz 1 (Aug 2009) 1 hr

Name: ______________________ Matric number: ______________

The figure below shows Frames A and B and rigid rod CD. The coordinates indicated are all expressed in Frame A (in meters). You don't need to evaluate the expressions, but make sure all expressions are complete with known quantities. Provide all answers in this sheet. Maximum score is 140 pts.



1. Determine the position of Frame B in A. (10 pts)
Ans:

$${}^A P_B = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$$

2. Determine the orientation of Frame B in A. (10 pts)
Ans:

$${}^A z_B = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad {}^A y_B = \begin{pmatrix} 0 \\ -2 \\ -3 \end{pmatrix} \quad {}^A x_B = {}^A y_B \times {}^A z_B$$

$${}^A R_B = \begin{pmatrix} {}^A x_B & {}^A y_B & {}^A z_B \end{pmatrix}$$

3. Express the position and orientation of Frame B with respect to A as a homogeneous transformation matrix. (10 pts)
Ans:

$${}^A T_B = \begin{pmatrix} {}^A x_B & {}^A y_B & {}^A z_B & {}^A P_B \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

4. An orientation vector $\vec{v}_{CD}$ is directed from C to D. Determine this vector, expressed in frame A (10 pts)
Ans:

$$\vec{v}_{CD} = \begin{pmatrix} 4-5 \\ 0-2 \\ 0-3 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix}$$

normalized to 1:

$$\vec{v}_{CD} = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix} \times \frac{1}{\sqrt{1+4+9}} = \begin{pmatrix} -1/\sqrt{14} \\ -2/\sqrt{14} \\ -3/\sqrt{14} \end{pmatrix}$$

5. The rod (CD) undergoes the following sequence of motions:
   1) Rotation around X_B by 60 degrees, followed by
   2) Translation along itself (from C to D) by 2 m. (60 pts)

   a) Determine the new position of D in A

Ans:

$$ {}^{A}P_{D_0} = {}^{A}P_{D} = \begin{pmatrix} 4 \\ 0 \\ 0 \\ 1 \end{pmatrix} \qquad {}^{B}P_{D_0} = {}^{B}T_{A}\,{}^{A}P_{D_0} = {}^{A}T_{B}^{-1}\,{}^{A}P_{D_0} $$

$$ {}^{B}P_{D_1} = Rot(x, 60^\circ)\,{}^{B}P_{D_0} \quad \text{where } Rot(x,60^\circ) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 60^\circ & -\sin 60^\circ & 0 \\ 0 & \sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{A}P_{D_1} = {}^{A}T_{B}\,{}^{B}P_{D_1} = {}^{A}T_{B}\,Rot(x,60^\circ)\,{}^{A}T_{B}^{-1}\,{}^{A}P_{D_0} $$

$$ {}^{A}P_{D_2} = {}^{A}P_{D_1} + 2\,{}^{A}v_{CD_1} \quad \text{where } {}^{A}V_{CD_1} \text{ is given below.} $$

   b) Determine the new orientation of CD.

Ans:

$$ {}^{A}V_{CD_0} = {}^{A}V_{CD} \text{ from Question 5} $$

$$ {}^{A}V_{CD_1} = {}^{A}T_{B}\,Rot(x,60^\circ)\,{}^{A}T_{B}^{-1} \begin{pmatrix} {}^{A}V_{CD_0} \\ 0 \end{pmatrix} $$

$$ {}^{A}V_{CD_2} = {}^{A}V_{CD_1} \text{ since only translation.} $$

Another solution: Define a new frame, Frame C where Z axis is $\vec{V}_{CD}$

${}^{A}T_{C}$ can then be obtained, ${}^{B}T_{C} = {}^{B}T_{A}\,{}^{A}T_{C} = {}^{A}T_{B}^{-1}\,{}^{A}T_{C}$

$$ {}^{B}T_{C_1} = Rot(x,60^\circ)\,{}^{B}T_{C} \qquad Trans(0,0,2) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{B}T_{C_2} = {}^{B}T_{C_1}\,Trans(0,0,2) $$

$$ {}^{A}T_{C_2} = {}^{A}T_{B}\,{}^{B}T_{C_2} $$

$$ {}^{A}V_{CD_2} = {}^{A}T_{C_2} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \text{ (3rd column)} $$

also ${}^{A}P_{D_2} = {}^{A}T_{C_2}\,{}^{C_2}P_{D_2}$ $|\vec{CD}|$

${}^{C_2}P_{D_2} = (0, 0, \sqrt{14}, 1)^T$

6. The orientation of a body B in Frame A is described by the three angles, θ_1, θ_2, θ_3 and the following equation: (40 pts)

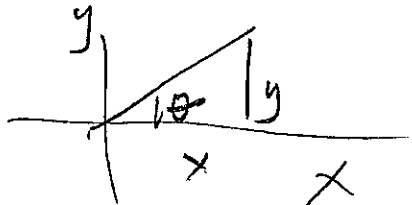
$$
{}^{A}R_{B} = Rot(y,\theta_1)Rot(z,\theta_2)Rot(y,\theta_3)
$$

$$
= \begin{pmatrix} \cos\theta_1\cos\theta_2\cos\theta_3 - \sin\theta_1\sin\theta_3 & -\cos\theta_1\sin\theta_2 & \cos\theta_1\cos\theta_2\sin\theta_3 + \sin\theta_1\cos\theta_3 \\ \cos\theta_3\sin\theta_2 & \cos\theta_2 & \sin\theta_3\sin\theta_2 \\ -\sin\theta_1\cos\theta_2\cos\theta_3 - \cos\theta_1\sin\theta_3 & \sin\theta_1\sin\theta_2 & -\sin\theta_1\cos\theta_2\sin\theta_3 + \cos\theta_1\cos\theta_3 \end{pmatrix}
$$

Given ${}^{A}R_{B}$, it is desired to solve for the three angles θ_1, θ_2, θ_3. For this quiz, please just provide the **complete** solution for **any one** angle. You can choose any angle you want to solve.

Ans:

$$
\begin{pmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{pmatrix}
$$



$\theta = ATAN2(y, x)$

$n_y = \cos\theta_3 \sin\theta_2$, $\quad \cos\theta_3 = n_y / \sin\theta_2$

$a_y = \sin\theta_3 \sin\theta_2$, $\quad \sin\theta_3 = a_y / \sin\theta_2$

$\therefore$

$\theta_3 = ATAN2(a_y, n_y)$ if $\sin\theta_2 > 0$

$= ATAN2(-a_y, -n_y)$ if $\sin\theta_2 < 0$

Similarly

$\theta_1 = ATAN2(o_z, -o_x)$ if $\sin\theta_2 > 0$

$= ATAN2(-o_z, +o_x)$ if $\sin\theta_2 < 0$

If $\sin\theta_2 = 0$, $(\theta_2 = 0° \sim 180°)$, $\rightarrow o_y = \pm 1$

If $\theta_2 = 0°$, $o_y = 1$

$$
R = \begin{pmatrix} \cos(\theta_1+\theta_3) & 0 & \sin(\theta_1+\theta_3) \\ 0 & 1 & 0 \\ -\sin(\theta_1+\theta_3) & 0 & \cos(\theta_1+\theta_3) \end{pmatrix}
$$

$(\theta_1+\theta_3) = ATAN2(a_x, a_z)$

infinite sol'n;

if $\theta_2 = 180°$, $o_y = -1$

$$R = \begin{pmatrix} -\cos(\theta_1 - \theta_3) & 0 & \sin(\theta_1 - \theta_3) \\ 0 & -1 & 0 \\ \sin(\theta_1 - \theta_3) & 0 & \cos(\theta_1 - \theta_3) \end{pmatrix}$$

$\theta_1 - \theta_3 = \text{ATAN2}\,(a_x, a_z)$ infinite sol'n,

$\theta_2 = \text{ATAN2}(\pm\sqrt{1 - o_y^2}, o_y)$

Summary:

If $|o_y| \neq 1$, 2 sol'ns:

$$\boxed{\begin{aligned} \theta_2 &= \text{ATAN2}(\sqrt{1-o_y^2}, o_y) \\ \theta_1 &= \text{ATAN2}(o_z, -o_x) \\ \theta_3 &= \text{ATAN2}(o_y, n_y) \end{aligned}} \qquad \boxed{\begin{aligned} \theta_2 &= \text{ATAN2}(-\sqrt{1-o_y^2}, o_y) \\ \theta_1 &= \text{ATAN2}(-o_z, o_x) \\ \theta_3 &= \text{ATAN2}(-o_y, n_y) \end{aligned}}$$

If $o_y = +1$, $\theta_2 = 0°$, $\theta_1 + \theta_3 = \text{ATAN2}(a_x, a_z)$ — inf sol'n

If $o_y = -1$, $\theta_2 = 180°$ $\theta_1 - \theta_3 = \text{ATAN2}(+a_x, a_z)$ - inf sol'ns.