Robotics - Quiz

75 mins

1.

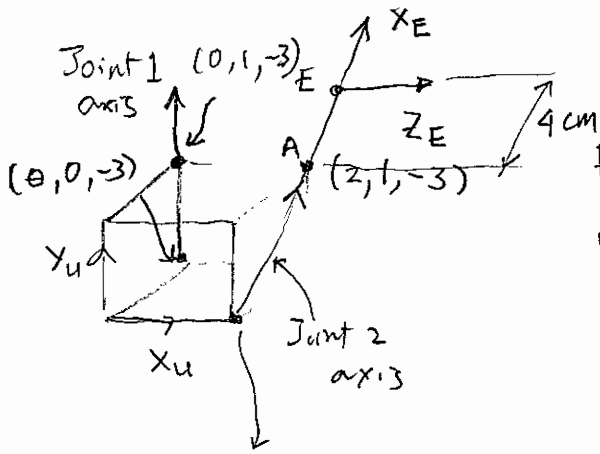


Fig. 1

Fig 1 shows 2 axes of a 2-joint robot. Both axes are rotational.

Frame U is fixed to the ground.

Frame E is fixed to the end-effector and located at 4 cm from A (corner)

All coords in Fig. 1 are expressed in Frame U, and are in units of cm.

Marks

9 (a) Attach frames to the robot according to the D-H convention.

8 (b) Determine all kinematic parameters that define the position & orientation of adjacent frames.

2 (c). Which of the kinematic parameters (in b) are the joint coordinates q_1, q_2 (for the 1st & 2nd joint respectively)?

8 (d). Determine an expression for ${}^{U}T_E$ as a function of the joint coords q_1, q_2. ${}^{U}T_E$ is the 4×4 homogeneous transformation matrix that describes the position and orientation of Frame E in Frame U.

2\.

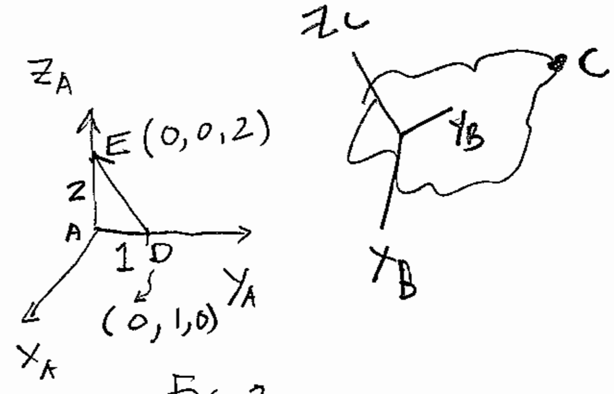


Fig 2.

Frame A is fixed to the ground. Frame B is attached to a moving body with point C at a different part of the body.

The Body is initially at ${}^{A}T_{B}$. It then undergoes the ffo sequence of motions.

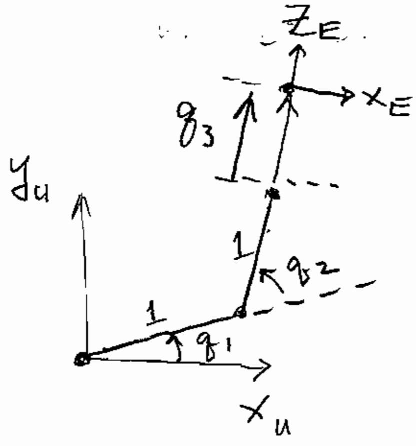
(1st) Rotation about the axis $\overrightarrow{DE}$ by 30°. Coordinates of D & E are expressed in Frame A. D & E are fixed to Frame A.

(2nd) Translation by (1 cm, 2 cm, 3 cm) along the x, y, z axes of Frame B.

Determine the final position of Frame C in Frame A. Assume that the coordinates of C in B is known to be (4cm, 5cm, 6 cm)

(20 marks)

3. Fig 3 shows a planar robot with 3 joints: 1st 2 are rotational and the 3rd joint is translational



The three joint coords. q_1, q_2, q_3 are shown in Fig 3 with their positive directions of motion + zero positions. The length of the 1st 2 moving links are 1 m each.

(a) Determine the expression for the full manipulator Jacobian matrix that relates the translational and angular velocities of frame E with the joint velocities, $\dot{q}_1, \dot{q}_2, \dot{q}_3$

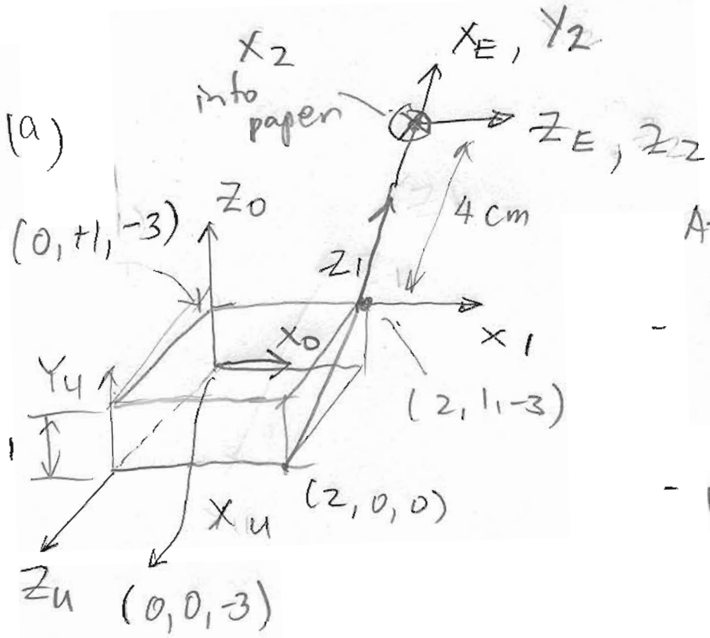
(15 marks)

(b) Does this robot have singularities. If so, what are the singularities of this robot. Explain your answer.

(15 marks)

1.

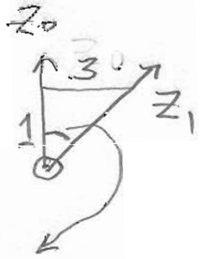
(a)



At configuration shown,

- X_u, X_1 + Z_E are parallel to each other.
- When robot moves, X_1 + Z_E moves.

(b).



i	θ	r	d	α
1	$q_{J1} = 0^\circ$	1 cm	2 cm	$-$ ATAN2(3, 1)
2	$q_{J2} = 90^\circ$	4 cm	0 cm	90°

(c)

$$q_{J1} = \theta_1$$

$$q_{J2} = \theta_2$$

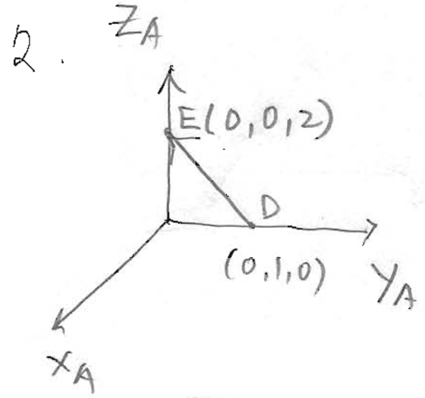
(d).

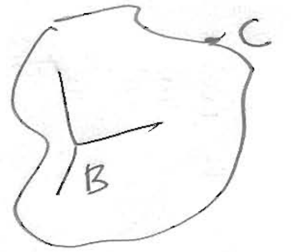
$$^{u}T_E = {}^{u}T_0 \; {}^{0}T_1 \; {}^{1}T_2 \; {}^{2}T_E$$

$$^{u}T_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \quad {}^{2}T_E = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^{i-1}T_i = \begin{pmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & d_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & d_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & r_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2.





$${}^{B}P_C = \begin{pmatrix} 4 \\ 5 \\ 6 \\ 1 \end{pmatrix}$$

Define a new Frame, D, whose z axis is $\overrightarrow{DE}$

$${}^{A}Z_D = \begin{pmatrix} 0-0 \\ 0-1 \\ 2-0 \\ 0 \end{pmatrix} \cdot \frac{1}{\sqrt{1^2+2^2}} = \begin{pmatrix} 0 \\ -1 \\ 2 \\ 0 \end{pmatrix} \frac{1}{\sqrt{5}}$$

${}^{A}X_D$ and ${}^{A}Y_D$ can be any 2 unit vectors such that they are $\perp$ to ${}^{A}Z_D$ and to each other

Let ${}^{A}X_D = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ $\quad {}^{A}X_D \cdot {}^{A}Z_D = 0$

$$a(0) + b(-1) + c(2) = 0$$

$$-b + 2c = 0$$

$c = 1$, $b = 2$ is a possibility. and a can be any number, e.g. 0

So ${}^{A}X_D = \begin{pmatrix} 0 \\ 2 \\ 1 \\ 0 \end{pmatrix} \cdot \frac{1}{\sqrt{2^2+1^2}} = \begin{pmatrix} 0 \\ 2 \\ 1 \\ 0 \end{pmatrix} \frac{1}{\sqrt{5}}$

$${}^{A}y_{D} = {}^{A}z_{D} \times {}^{A}x_{D}$$

$$= \begin{pmatrix} 0 \\ -1/\sqrt{5} \\ 2/\sqrt{5} \end{pmatrix} \times \begin{pmatrix} 0 \\ 2/\sqrt{5} \\ 1/\sqrt{5} \end{pmatrix} = \begin{pmatrix} -\frac{1}{5} - \frac{4}{5} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

$${}^{A}T_{D} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 2/\sqrt{5} & 0 & -1/\sqrt{5} & 1 \\ 1/\sqrt{5} & 0 & 2/\sqrt{5} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(1st) ${}^{A}T_{B_0} = {}^{A}T_{B}$

${}^{D}T_{B_0} = {}^{D}T_{A}\,{}^{A}T_{B_0}$ where ${}^{D}T_{A} = {}^{A}T_{D}^{-1}$

${}^{D}T_{B_1} = \mathrm{Rot}(z, 30^\circ)\,{}^{D}T_{B_0}$

where $\mathrm{Rot}(z, 30^\circ) = \begin{pmatrix} \cos 30^\circ & -\sin 30^\circ & 0 & 0 \\ \sin 30^\circ & \cos 30^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(2nd)

${}^{D}T_{B_2} = {}^{D}T_{B_1}\,\mathrm{Trans}(1, 2, 3)$

where $\mathrm{Trans}(4,5,6) = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$${}^{A}T_{B2} = {}^{A}T_{D}\,{}^{D}T_{B2}$$

$${}^{A}P_{C_2} = {}^{A}T_{B2}\,{}^{B2}P_{C2} \quad \text{where} \quad {}^{B2}P_{C_2} = {}^{B}P_{C}$$

$\therefore$

$${}^{A}P_{C_2} = {}^{A}T_{D}\,\underbrace{\overbrace{\mathrm{Rot}(z,30^{\circ})\,{}^{D}T_{A}\,{}^{A}T_{B_0}}^{{}^{D}T_{B_1}}\,\mathrm{Trans}(1,2,3)}_{{}^{D}T_{B_2}}\,{}^{B}P_{C}$$

3. $${}^{u}P_{E} = \begin{pmatrix} \overset{\cos(q_1)}{C_1} + \overset{\cos(q_1+q_2)}{C_{12}} + q_3(C_{12}) \\ S_1 + S_{12} + q_3(S_{12}) \\ 0 \end{pmatrix}$$

$${}^{u}u_{E} = \begin{pmatrix} -S_1\dot{q}_1 - S_{12}(\dot{q}_1+\dot{q}_2) - q_3 S_{12}(\dot{q}_1+\dot{q}_2) + \dot{q}_3 C_{12} \\ C_1\dot{q}_1 + C_{12}(\dot{q}_1+\dot{q}_2) + q_3 C_{12}(\dot{q}_1+\dot{q}_2) + \dot{q}_3 S_{12} \\ 0 \end{pmatrix}$$

$${}^{u}J_{E} = \begin{pmatrix} -S_1 - S_{12} - q_3 S_{12} & -S_{12} - q_3 S_{12} & C_{12} \\ C_1 + C_{12} + q_3 C_{12} & C_{12} + q_3 C_{12} & S_{12} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

Task : 3 DOF : $\begin{cases} u_x \\ u_y \\ \omega_z \end{cases}$

Task : Jacobian (reduced) =

$$\begin{pmatrix} -s_1 - s_{12} - q_3 s_{12} & -s_{12} - q_3 s_{12} & c_{12} \\ c_1 + c_{12} + q_3 c_{12} & c_{12} + q_3 c_{12} & s_{12} \\ 1 & 1 & 0 \end{pmatrix}$$

$$\det(\ ) = -s_{12}^2 - q_3 s_{12}^2 - c_{12}^2 - q_3 c_{12}^2 + \quad -1$$

$$\left[s_1 + s_{12}(1+q_3)\right] s_{12} +$$

$$\left[c_1 + c_{12}(1+q_3)\right] c_{12}$$

$$= -1 - q_3 + s_1 s_{12} + s_{12}^2(1+q_3) + c_1 c_{12} + c_{12}^2(1+q_3)$$

$$= -1 - q_3 + \cos q_2 + (1+q_3)$$

$$= \cos q_2 = 0$$

$q_2 = 90^\circ, -90^\circ$