Fig. 1 shows 2 axes of a 2-joint robot. Both axes are orthogonal.
Frame U is fixed to the ground.
Frame E is fixed to the end-effector and located at 4 cm from A (corner).
All co-ords in Fig. 1 are expressed in Frame U, and are in units of cm.

9. (a) Attach frames to the robot according to the 0-1-4 convention.

8. (b) Determine all kinematic parameters that define the position and orientation of adjacent frames.

2. (c) Which of the kinematic parameters are the joint coordinates $q_1, q_2$ (for the 1st & 2nd joint respectively)?

8. (d) Determine an expression for $T_{UE}$ as a function of the joint co-ords $q_1, q_2$. $T_{UE}$ is the 4x4 homogeneous transformation matrix that describes the position and orientation of Frame E in Frame U.
Frame A is fixed to the ground. Frame B is attached to a moving body with point C at a different part of the body.

The body is initially at \( T_B \). It then undergoes the following sequence of motions:

(1st) Rotation about the axis \( \overrightarrow{DE} \) by 30°. Coordinates of D and E are expressed in Frame A. D and E are fixed to Frame A.

(2nd) Translation by (1 cm, 2 cm, 3 cm) along the \( x, y, z \) axes of Frame B.

Determine the final position of Frame C in Frame A. Assume that the coordinates of C in B is known to be (4 cm, 5 cm, 6 cm). (20 marks)
3. Fig 3 shows a robot with 3 joints: 1st 2 are rotational and the 3rd joint is translational.

The three joint counts, \( q_1, q_2, q_3 \) are shown in Fig 3 with their positive directions, 3 motion & zero positions. The length of the 1st 2 moving links are 1 on each.

(a) Determine the expression for the full manipulator Jacobian matrix that relates the translational and angular velocities of link E with the joint velocities, \( q_1, q_2, q_3 \).

(15 marks)

(b) Does this robot have singularities? If so, what type of singularities does this robot have? Explain your answer.

(15 marks)
At configuration shown:
- \( X_0, X_1 + Z_E \) are parallel to each other.
- When robot moves, \( X_1 + Z_E \) moves.

\[
\begin{array}{c|cc|c|c}
\theta & r & d & 2 \\
1 & 0^\circ & 1 \text{ cm} & 2 \text{ cm} & 1 \text{ cm} \\
2 & 90^\circ & 4 \text{ cm} & 0 \text{ cm} & 90^\circ \\
\end{array}
\]

(c) \( q_1 = \Theta_1 \)
\( q_2 = \Theta_2 \)

(d) \( u_T^E = u_T^0 \cdot T_1 \cdot T_2 \cdot T_E \)

\[
\begin{align*}
T_0 &= \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\end{align*}
\]

\[
\begin{align*}
T_E &= \begin{pmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{pmatrix}
cos \Theta_1 & -sin \Theta_1 & 0 & 0 \\
sin \Theta_1 & cos \Theta_1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\end{align*}
\]
Define a new frame, \( D \), whose \( z \)-axis is \( \overrightarrow{DE} \).

\[
\mathbf{Z}_D = \begin{pmatrix}
0 & 0 \\
0 & 1 \\
2 & 0 \\
0 & 0
\end{pmatrix} = \begin{pmatrix}
0 \\
-1 \\
2 \\
0
\end{pmatrix} \frac{1}{\sqrt{1^2 + 2^2}} = \begin{pmatrix}
0 \\
-1 \\
2 \\
0
\end{pmatrix} \frac{1}{\sqrt{5}}
\]

\( \mathbf{X}_D \) and \( \mathbf{Y}_D \) can be any two unit vectors such that they are \( \perp \) to \( \mathbf{Z}_D \) and to each other.

Let \( \mathbf{X}_D = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \) and \( \mathbf{Y}_D = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = 0 \)

\( a(0) + b(-1) + c(2) = 0 \)

\( -b + 2c = 0 \)

\( c = 1, \ b = 2 \) is a possibility, as \( a \) can be any number, e.g. 0.

So \( \mathbf{X}_D = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \frac{1}{\sqrt{2^2 + 1^2}} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \frac{1}{\sqrt{5}} \)
\[ A_Y \cdot D = A_Z \cdot D \times A_X \cdot D \]

\[
\begin{pmatrix}
0 \\
-\frac{1}{\sqrt{5}} \\
\frac{2}{\sqrt{5}} \\
\frac{1}{\sqrt{5}}
\end{pmatrix}
\times
\begin{pmatrix}
0 \\
\frac{1}{\sqrt{5}} \\
\frac{2}{\sqrt{5}} \\
\frac{1}{\sqrt{5}}
\end{pmatrix}
= 
\begin{pmatrix}
\frac{1}{5} & \frac{4}{5} \\
0 & 0 \\
0 & 0
\end{pmatrix}
= 
\begin{pmatrix}
-1 \\
0 \\
0
\end{pmatrix}
\]

\[
A_{-T_0} = 
\begin{pmatrix}
0 & -1 & 0 & 0 & 0 \\
\frac{2}{\sqrt{5}} & 0 & -\frac{1}{\sqrt{5}} & 1 \\
\frac{1}{\sqrt{5}} & 0 & \frac{2}{\sqrt{5}} & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

((st)) \( A_{-T_{B_0}} = A_{-T_B} \)

\[ D_{T_{B_0}} = D_{T_A} \cdot A_{-T_{B_0}} \quad \text{where} \quad D_{T_A} = A_{-T_D}^{-1} \]

\[ D_{T_{B_1}} = \text{Rot}(Z, 30^\circ) \cdot D_{T_{B_0}} \]

\[ \text{where} \quad \text{Rot}(Z, 30^\circ) = 
\begin{pmatrix}
\cos 30^\circ & -\sin 30^\circ & 0 & 0 \\
\sin 30^\circ & \cos 30^\circ & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \]

(2nd) \( D_{T_{B_2}} = D_{T_{B_1}} \cdot \text{Trans}(4, 1, 2, 3) \)

\[ \text{where} \quad \text{Trans}(4, 1, 2, 3) = 
\begin{pmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 2 \\
0 & 0 & 1 & 3 \\
0 & 0 & 0 & 1
\end{pmatrix} \]
\[ \begin{align*}
\mathbf{A}_{TB} &= \mathbf{A} \mathbf{T}_D \mathbf{T}_{B_2} \\
\mathbf{A}_{PC_2} &= \mathbf{A} \mathbf{T}_{B_2} \mathbf{P}_{C_2} \quad \text{where} \quad \mathbf{B}_2 = \mathbf{B}_2 \\
\mathbf{A}_{P_{C_2}} &= \mathbf{A} \mathbf{T}_D \mathbf{Rot}(z, 30^\circ) \mathbf{T}_A \mathbf{A} \mathbf{T}_{B_0} \mathbf{Trans}(1, 2, 3) \mathbf{B} \mathbf{P}_C \end{align*} \]

3. \[ \begin{align*}
\mathbf{u}_{PE} &= \begin{pmatrix} C_1 + C_{12} + q_3 (C_{12}) \\
S_1 + S_{12} + q_3 (S_{12}) \\
0 \end{pmatrix} \\
\mathbf{u}_{PE} &= \begin{pmatrix} -S_1 \dot{q}_1 - S_{12}(\dot{q}_1 + \dot{q}_2) - q_3 S_{12}(\dot{q}_1 + \dot{q}_2) + \dot{q}_3 C_{12} \\
C_1 \dot{q}_1 + C_{12}(\dot{q}_1 + \dot{q}_2) + q_3 C_{12}(\dot{q}_1 + \dot{q}_2) + \dot{q}_3 S_{12} \\
0 \end{pmatrix} \\
\mathbf{u}_{PE} &= \begin{pmatrix} -S_1 - S_{12} - q_3 S_{12} \\
C_1 + C_{12} + q_3 C_{12} \\
0 \end{pmatrix} \begin{pmatrix} -S_1 - S_{12} - q_3 S_{12} \\
C_1 + C_{12} + q_3 C_{12} \\
0 \end{pmatrix} \begin{pmatrix} C_{12} \\
S_{12} \\
0 \end{pmatrix}
\end{align*} \]
Task: 3 DOF: \{ u_x, u_y, w_z \}

Task: Jacobian (reduced) =

\[
\begin{pmatrix}
-s_1 - s_{12} q_3 s_{12} & -s_{12} q_3 s_{12} & c_{12} \\
 c_1 + c_{12} q_3 s_{12} & c_{12} + q_3 c_{12} & s_{12} \\
 1 & 1 & 0
\end{pmatrix}
\]

\[
\det(C) = -s_{12}^2 - q_3 s_{12}^2 - c_{12}^2 - q_3 c_{12}^2 + 1
\]

\[
= -1 - q_3 + s_1 s_{12} + s_{12}^2 (1 + q_3) + c_1 c_{12} + c_{12}^2 (1 + q_3)
\]

\[
= -1 - q_3 + c_0 + q_2 + 2 (1 + q_3)
\]

\[
\Rightarrow q_2 = 90^\circ, -90^\circ
\]