Drill Problem Set 1 ME4245/EE4304

1. Let ${}^A z_c = \overrightarrow{PQ} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \cdot \frac{1}{\sqrt{1^2+(-3)^2+2^2}}$ (normalized to 1)

Pick any ${}^A x_c$, ${}^A y_c$ such that ${}^A x_c \times {}^A y_c = {}^A z_c$

Let ${}^A y_c = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ $\perp$ ${}^A y_A \cdot ({}^A z_c) = 0$ since $\perp$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = 0 \rightarrow a - 3b + 2c = 0$$

Let $b=1$, $c=1$, then $a=1$

$$\therefore {}^A y_c = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \frac{1}{\sqrt{1^2+1^2+1^2}} \quad \text{(norm to 1)}$$

$$\left.\begin{aligned} {}^A x_c &= {}^A y_c \times {}^A z_c \\ {}^A P_c &= \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} \end{aligned}\right\} \rightarrow {}^A T_c = \begin{pmatrix} {}^A x_c & {}^A y_c & {}^A z_c & {}^A P_c \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^A T_{B_1} = {}^A T_c \; Rot(z, 30^\circ) \; {}^A T_c^{-1}$$

$${}^A T_c = \begin{pmatrix} 0.772 & 0.577 & 0.267 & 0 \\ -0.154 & 0.577 & -0.802 & 3 \\ -0.617 & 0.577 & 0.535 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^A T_{B_1} = \begin{pmatrix} 0.876 & -0.296 & -0.382 & 0.888 \\ 0.239 & 0.952 & -0.191 & 0.144 \\ 0.422 & 0.0762 & 0.904 & -0.229 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

1.

$${}^A x_c = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad {}^A y_c = \begin{pmatrix} -\cos 60^\circ \\ \sin 60^\circ \\ 0 \end{pmatrix}$$

$${}^A p_c = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$$

${}^A T_c$ = ✓ ${}^A T_c^{-1} \cdot {}^C T_A$ = ✓

2.

2 ~~${}^u T_{M_1} = \text{Rot}(z, 30^\circ)$~~

$${}^u x_c = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \quad {}^u y_c = \begin{pmatrix} 0 \\ -3 \\ -2 \end{pmatrix} \quad {}^u p_c = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

${}^u T_{C_1} = {}^u T_c \, \text{Rot}(z, 30^\circ)$ = ✓ ↓ ${}^u T_c = {}^M T_c$

${}^u T_{C_2} = {}^u T_{C_1} \, \text{Trans}(1,2,3)$ = ✓

${}^u T_{M_2} = {}^u T_{C_2} \, {}^{C_2} T_{M_2}$

~~${}^u T_{M_3} = {}^u T_{M_2} \, \text{Rot}(x, 45^\circ)$~~ ${}^u T_{M_3} = {}^u T_{M_2} \, \text{Rot}(x, 45^\circ)$

${}^u T_{C_3} = {}^u T_{M_3} \, {}^{M_3} T_{C_3}$

${}^u T_{C_3} = {}^u T_{C_2} \, {}^{C_2} T_{M_2} \, \text{Rot}(x, 45^\circ) \, {}^{M_3} T_{C_3}$

= ✓

${}^u T_{C_4} = \text{Rot}(y, 60^\circ) \, {}^u T_{C_3}$ = ✓

${}^u T_{M_4} = {}^u T_{C_4} \, {}^C T_M$ = ✓

3.

$${}^A x_C = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad {}^A y_C = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad {}^A z_C = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$${}^A p_C = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}$$

$${}^A x_B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad {}^A z_B = \begin{pmatrix} \sin 60^\circ \\ \cos 60^\circ \\ 0 \end{pmatrix}$$

$${}^A y_B = {}^A z_B \times {}^A x_B$$

$${}^A p_B = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$$

$${}^B T_C = {}^B T_A \, {}^A T_C$$

~~$T = {}^A T_B \, Rot(y, 45^\circ)$~~

$${}^B T_{C_1} = Rot(y, 45^\circ) \, {}^B T_C$$

$${}^B T_{C_2} = Rot(y, 45^\circ) \, {}^A T_C \, Rot(x, 30^\circ)$$

$${}^A T_{C_2} = {}^A T_B \, Rot(y, 45^\circ) \, {}^B T_C \, Rot(x, 30^\circ)$$

$= \checkmark$