DRILL 3

$q_1 = \alpha \quad q_2 = \beta \quad q_3 = \gamma$

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>> RR = Rz(q1) * Ry(q2) *  Rz(q3)

RR =

[cos(q1)*cos(q2)*cos(q3) - sin(q1)*sin(q3), - cos(q3)*sin(q1) - cos(q1)*cos(q2)*sin(q3), cos(q1)*sin(q2), 0]
[cos(q1)*sin(q3) + cos(q2)*cos(q3)*sin(q1),   cos(q1)*cos(q3) - cos(q2)*sin(q1)*sin(q3), sin(q1)*sin(q2), 0]
[                         -cos(q3)*sin(q2),                             sin(q2)*sin(q3),         cos(q2), 0]
[                                        0,                                           0,               0, 1]

>>
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$\cos(q_2) = a_z$

$$\beta = q_2 = \mathrm{ATAN2}\left(\pm\sqrt{a_x^2 + a_y^2},\ a_z\right)$$

$$= 60^\circ \text{ and } -60^\circ$$

$\cos q_3 = -n_z / \sin(q_2) \qquad \sin q_3 = \frac{o_z}{\sin q_2}$

$$\gamma = q_3 = \mathrm{ATAN2}\left(\frac{o_z}{s_2},\ -n_z/s_2\right)$$

$\cos q_1 = \frac{a_x}{s_2} \qquad \sin q_1 = a_y/s_2$

$$\alpha = q_1 = \mathrm{ATAN2}\left(a_y/s_2,\ a_x/s_2\right)$$

Two solutions:

$(q_2 = 60^\circ,\ q_1 = 30^\circ,\ q_3 = 90^\circ)$

$(q_2 = -60^\circ,\ q_1 = -150^\circ,\ q_3 = -90^\circ)$

$$\omega = \begin{pmatrix}0\\0\\1\end{pmatrix}\dot{\alpha} + \underbrace{Rot(z,\alpha)\begin{pmatrix}0\\1\\0\end{pmatrix}}_{\text{2nd col (y-axis) of } Rot(z,\alpha)}\dot{\beta} + \underbrace{Rot(z,\alpha)Rot(y,\beta)}_{\text{z-axis: 3rd col'n of product } Rot(z,\alpha)Rot(y,\beta)}\begin{pmatrix}0\\0\\1\end{pmatrix}\dot{\gamma}$$

for $(\alpha,\beta,\gamma) = (30, 60, 90^\circ)$

$$\omega = \begin{pmatrix}0\\0\\1\end{pmatrix}1 + \begin{pmatrix}-0.5\\0.866\\0\end{pmatrix}1 + \begin{pmatrix}0.75\\0.433\\0.5\end{pmatrix}1$$

$$= \begin{pmatrix}0.25\\1.3\\1.5\end{pmatrix} {}^{\circ}/s$$

for $(\alpha,\beta,\gamma) = (-150^\circ, -60^\circ, -90^\circ)$

$$\omega = \begin{pmatrix}0\\0\\1\end{pmatrix}1 + \begin{pmatrix}0.5\\-0.866\\0\end{pmatrix}1 + \begin{pmatrix}0.75\\0.433\\0.5\end{pmatrix}1$$

$$= \begin{pmatrix}1.25\\-0.433\\1.5\end{pmatrix} {}^{\circ}/s$$

2. At that instant of time:

$$^{u}k = \begin{pmatrix} 0 \\ \cos 30^\circ \\ \sin 30^\circ \end{pmatrix} \qquad {}^{u}\omega = {}^{u}k\,(10) \text{ rad/sec}$$

$$^{u}T_H = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$^{u}T_H$ changing with time.

$$^{u}u_H = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\overline{{}^{H}\dot{P}_C} = {}^{H}T_u \; {}^{u}P_c$$

$^{H}T_u$: 4x4; ${}^{H}T_u\,{}^{u}P_c$ → changing with time

$^{H}P_C$ → constant since same rigid body (4x1)

$$^{u}\omega_H = {}^{u}\omega = {}^{u}k\,(10)$$

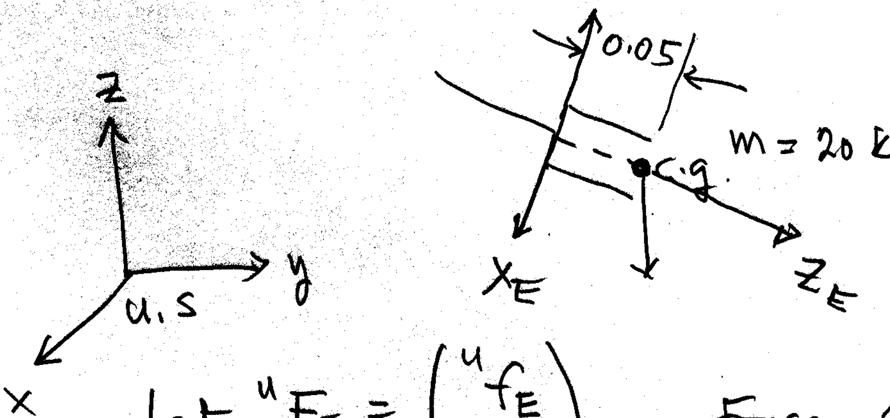
in 3x1, and 3x3

$$^{u}P_c = {}^{u}R_H \, {}^{H}P_c + {}^{u}P_H$$

$$^{u}u_c = {}^{u}\dot{R}_H \, {}^{H}P_c + {}^{u}u_H = \lfloor {}^{u}\omega_H \times \rfloor \, {}^{u}R_H \, {}^{H}P_c + {}^{u}u_H$$

$$= {}^{u}\omega_H \times ({}^{u}R_H \, {}^{H}P_c) + {}^{u}u_H$$

3.



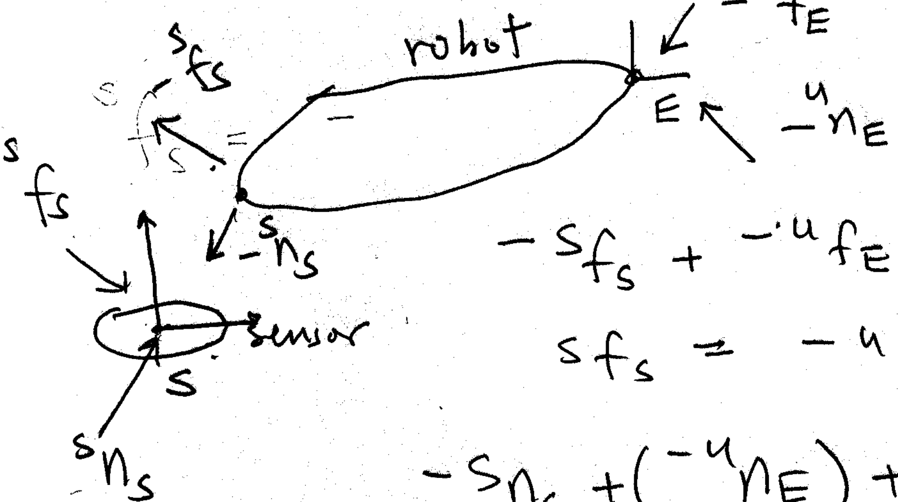
${}^uT_E$ = given at that instant of time

Let ${}^uF_E = \begin{pmatrix} {}^uf_E \\ {}^un_E \end{pmatrix}$ Force exerted on payload by E-E at frame E, + expressed in frame U.

$${}^uf_E = \begin{pmatrix} 0 \\ 0 \\ 20(9.8) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 196 \end{pmatrix} N \quad \text{along + z axis of frame U}$$

$${}^un_E = -\left(0.05\, Z_E \times \begin{pmatrix} 0 \\ 0 \\ -196 \end{pmatrix}\right)$$

where ${}^uZ_E$ = 3rd col (3x1) of ${}^uT_E$ (given)



S ∥ U

$$-{}^sf_s + -{}^uf_E = 0$$

$${}^sf_s = -{}^uf_E = \begin{pmatrix} 0 \\ 0 \\ -196 \end{pmatrix}$$

$$-{}^sn_s + (-{}^un_E) + {}^up_E \times (-{}^uf_E) = 0$$

$${}^sn_s = -{}^un_E + {}^up_E \times (-{}^uf_E)$$

${}^up_E$: Last col (3x1) of ${}^uT_E$ (given)

Sensor reading $\begin{pmatrix} {}^sf_s \\ {}^sn_s \end{pmatrix}$ N / N-m

4.

$${}^AT_C = {}^AT_B\,{}^BT_C$$

$${}^A\dot{T}_C = {}^A\dot{T}_B\,{}^BT_C + {}^AT_B\,{}^B\dot{T}_C$$

$$\begin{pmatrix} \lfloor {}^A\omega_C\times\rfloor\,{}^AR_C & {}^Au_C \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \lfloor {}^A\omega_B\times\rfloor\,{}^AR_B & {}^Au_B \\ 0 & 0 \end{pmatrix}\begin{pmatrix} {}^BR_C & {}^BP_C \\ 0 & 1 \end{pmatrix} +$$

$$\begin{pmatrix} {}^AR_B & {}^AP_B \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \lfloor {}^B\omega_C\times\rfloor\,{}^BR_C & {}^Bu_C \\ 0 & 0 \end{pmatrix}$$

$${}^Au_C = {}^A\omega_B \times {}^AR_B\,{}^BP_C + {}^Au_B + {}^AR_B\,{}^Bu_C \quad //$$

$$\lfloor {}^A\omega_C\times\rfloor\,{}^AR_C = \lfloor {}^A\omega_B\times\rfloor\,\underbrace{{}^AR_B\,{}^BR_C}_{{}^AR_C} + {}^AR_B\,\lfloor {}^B\omega_C\times\rfloor\,{}^BR_C$$

$${}^A\omega_C = {}^A\omega_B + {}^AR_B\,{}^B\omega_C \quad //$$

$${}^AV_C = \begin{pmatrix} {}^Au_C \\ {}^A\omega_C \end{pmatrix}$$