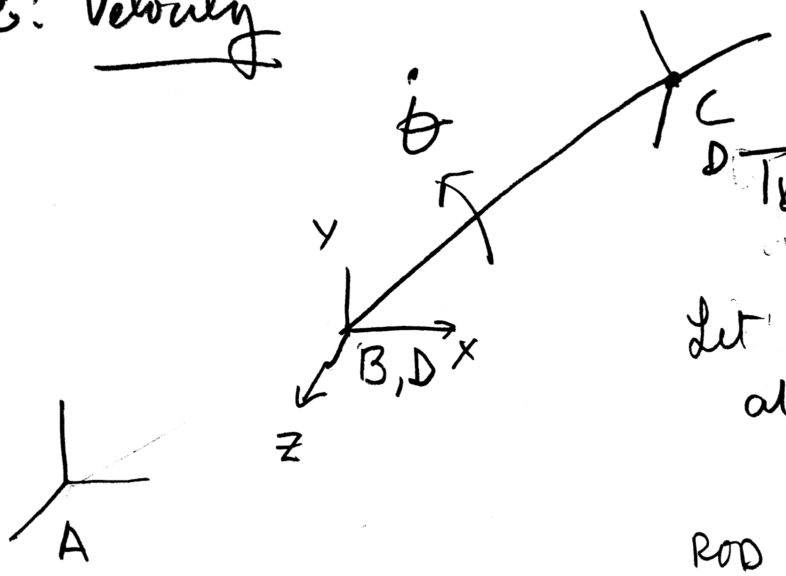
Example: Velocity



Let A, D be fixed

${}^{D}T_B$ = given - time varying

Let frame B + C be attached to Rod BC

ROD BC is rotating about Z_B, $\rightarrow$ ${}^{D}T_B$ is NOT fixed.

$${}^{D}U_C = {}^{D}W_C \times {}^{D}P_C \Rightarrow {}^{A}U_C = {}^{A}R_D \, {}^{D}U_C$$

$${}^{A}U_C = {}^{A}R_D \, {}^{D}W_C \times {}^{D}P_C$$

$$= {}^{A}W_C \times ({}^{A}P_C - {}^{A}P_D) \;//$$

Also

$${}^{A}U_C = {}^{A}W_B \times ({}^{A}P_C - {}^{A}P_B)$$

$$= {}^{A}W_B \times {}^{A}R_B \, {}^{B}P_C$$

$$= {}^{A}W_C \times {}^{A}R_B \, {}^{B}P_C$$

$$= {}^{A}W_C \times ({}^{A}P_C - {}^{A}P_B) \;//$$

since ${}^{A}W_B = {}^{A}W_C = {}^{A}Z_B \dot{\theta}$

$= {}^{A}R_D \, {}^{D}W_C$

What if Rod BC is also translating with $^A u_B$ and $^A \omega_B$ given

Two Methods to solve:

Method 1:

$$
\begin{aligned}
{}^A u_C &= {}^A u_B + {}^A R_D \left({}^D\omega_C \times ({}^D P_C - {}^D P_D) \right) + \\
&\quad {}^A\omega_B \times ({}^A P_C - {}^A P_D) \\
&= {}^A u_B + {}^A R_D\, {}^D\omega_C \times {}^A R_D ({}^D P_C - {}^D P_D) + {}^A\omega_B \times ({}^A P_C - {}^A P_D) \\
&= {}^A u_B + {}^A R_D\, {}^D\omega_C \times ({}^A P_C - {}^A P_D) + {}^A\omega_B \times ({}^A P_C - {}^A P_D) \\
&= {}^A u_B + \underbrace{\left({}^A\omega_B + {}^A R_D\, {}^D\omega_C \right)}_{{}^A\omega_C} \times ({}^A P_C - {}^A P_D) \\
&= {}^A u_B + {}^A\omega_C \times ({}^A P_C - {}^A P_D) \quad //
\end{aligned}
$$

Method 2 :

${}^{A}U_{C}$ = motion contribution from Frame B motion

+

motion contribution from Frame C motion with respect to B

$$= {}^{A}W_{B} \times {}^{A}P_{C} + {}^{A}R_{D}\,{}^{D}W_{C} \times ({}^{A}P_{C} - {}^{A}P_{D})$$

$${}^{A}P_{C} = {}^{A}P_{B} + {}^{A}R_{B}\,{}^{B}P_{C}$$

$${}^{A}U_{C} = {}^{A}W_{B} \times (\overbrace{{}^{A}P_{B} + {}^{A}R_{B}\,{}^{B}P_{C}}^{{}^{A}P_{C}}) + {}^{A}R_{D}\,{}^{D}W_{C} \times ({}^{A}P_{C} - {}^{A}P_{D})$$

$$= ({}^{A}W_{B} \times {}^{A}P_{B}) + {}^{A}W_{C} \times {}^{A}R_{B}\,{}^{B}P_{C} + {}^{A}R_{D}\,{}^{D}W_{C} \times ({}^{A}P_{C} - {}^{A}P_{D})$$

$$= {}^{A}U_{B} + \underbrace{({}^{A}W_{C} + {}^{A}R_{D}\,{}^{D}W_{C})}_{{}^{A}W_{C}} \times ({}^{A}P_{C} - {}^{A}P_{D}) \quad //$$

note ${}^{A}P_{D} = {}^{A}P_{B}$