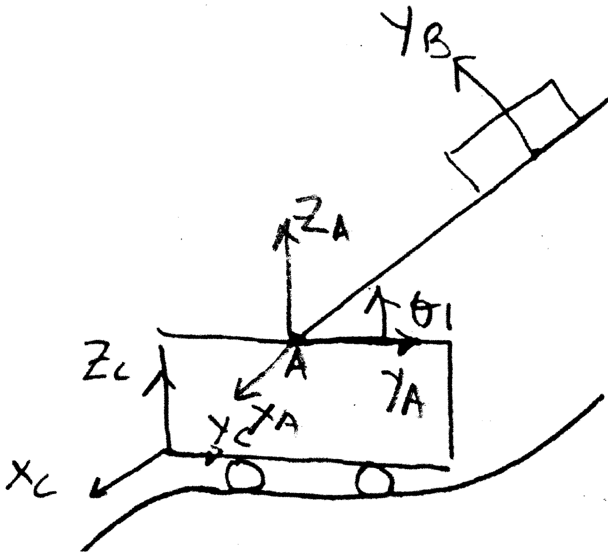
ME4245 Quiz 1.2 – 13 October 2014, 16:00-17:00

1. Figure 1 shows a cart supporting an inclined plane via joint driven by a motor. The motor rotates the plane about joint A (planar rotation). A block slides down the inclined plane. Frame C is attached to the cart, and Frame B is attached to the block. Frame U is fixed to the ground. At a certain instant of time, shown in Figure 1, the following are known:

   - Position and orientation of Frame C in U, ${}^{U}T_{C}$
   - Position and orientation of Frame B in C, ${}^{C}T_{B}$
   - Translational and angular velocity of C in U, ${}^{U}u_{C}$ and ${}^{U}w_{C}$, respectively
   - Block is moving down the inclined plane at 2 m/s
   - Inclined plane rotating at 30 degrees/s counterclockwise when it is an angle of 45 degrees with respect to the horizontal (AB makes an able of 45 degrees with respect to the Yc axis.)



Let Frame A be also attached to cart with Frame A parallel to Frame C

${}^{C}P_{A}$ must be given!

Figure 1

(a) Determine the expression for the translational velocity of the origin of Frame B with respect to Frame U as a function of the known quantities.

(b) Determine the expression for the angular velocity of block B with respect to Frame U as function of the known quantities.

2. Figure 2 shows a planar robot with 2 joints: the first is rotational, moving the 1st link about the z axis in a counter-clockwise fashion; the 2nd joint is translational and moves the 2nd link along the longitudinal axis of the 1st link. Frame U is attached to the ground and its origin coincides with the location of the 1st joint.

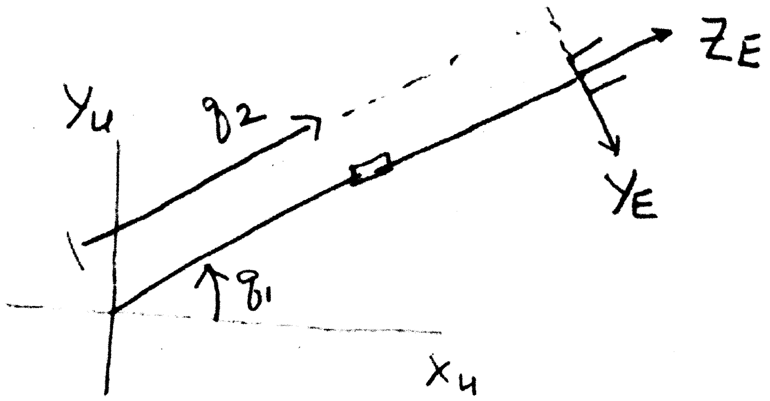


Figure 2

(a) Determine the expression for the full manipulator Jacobian relating the translational and angular velocity of the end-effector (Frame E) with the joint velocities.

(b) The end-effector is carrying a tool with Frame B attached to the tool tip. Frame B is attached to the end-effector with ${}^{E}T_{B}$ known and fixed (constant). Determine the expression of the full manipulator Jacobian relating the translational and angular velocity of Frame B in Frame U,as a function of the manipulator Jacobian in (a)

(c) If the task of the robot is for its end-effector to translate along the XY plane of Frame U, determine the joint coordinates in which the robot becomes singular, if any.

(d) If the task of the robot is for its end-effector to translate along an axis parallel to Xu and rotate about Zu, determine the joint coordinates in which the robot becomes singular, if any.

(e) If the task of the robot is for its end-effector to only translate along an axis parallel to Xu, determine the joint coordinates in which the robot becomes singular, if any.

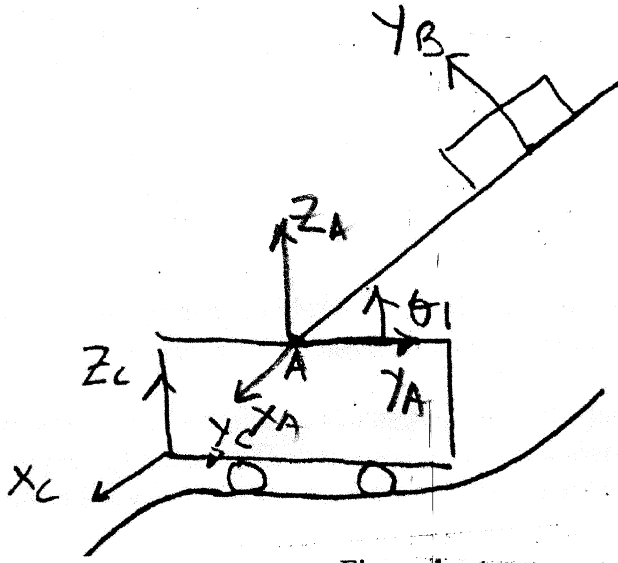


Figure 1

Let Frame A be also attached to cart with Frame A parallel to Frame C. ${}^{C}P_A$ must be given!

(a)

$$ {}^{A}V_B = {}^{u}V_C + \underbrace{{}^{u}\omega_C \times {}^{u}R_C\,{}^{C}P_B}_{\text{motion contribution of } {}^{u}\omega_C} + \underbrace{{}^{u}R_A\,{}^{A}\omega_B \times \left({}^{u}P_B - {}^{u}P_A\right)}_{\text{motion contribution of rotating inclined plane}} $$

$$ + \underbrace{{}^{u}R_C\,{}^{C}V_B}_{\text{Block sliding down inclined plane}} $$

$$ {}^{C}V_B = 2\begin{pmatrix} 0 \\ -\cos 45^\circ \\ -\sin 45^\circ \end{pmatrix} $$

Note: Need to know ${}^{u}P_A$

$$ {}^{A}R_C = I_{3\times3} $$

$$ {}^{A}\omega_B = \begin{pmatrix} 30 \times \pi/180^\circ \\ 0 \\ 0 \end{pmatrix} $$

$$ {}^{u}P_A = {}^{u}P_C + {}^{C}P_A $$ //

(b) $$ {}^{u}\omega_B = {}^{u}\omega_C + {}^{u}R_C\,{}^{C}\omega_B $$ //

(a) Another method: using Transformation matrices.

$$^{u}T_B = {}^{u}T_C \; {}^{C}T_A \; {}^{A}T_B \qquad \text{Note. } {}^{C}\dot{T}_A = 0$$

$$^{u}\dot{T}_B = {}^{u}\dot{T}_C \; {}^{C}T_B + {}^{u}T_C \; {}^{C}T_A \; {}^{A}\dot{T}_B$$

$$\begin{pmatrix} {}^{u}\hat{\omega}_B \, {}^{u}R_B & {}^{u}u_B \\ 0 & 0 \end{pmatrix} = \left(\begin{array}{c|c} {}^{u}\hat{\omega}_C \, {}^{u}R_C & {}^{u}u_C \\ \hline 0 & 0 \end{array}\right) \left(\begin{array}{c|c} {}^{C}R_B & {}^{C}P_B \\ \hline 0 & 1 \end{array}\right) +$$

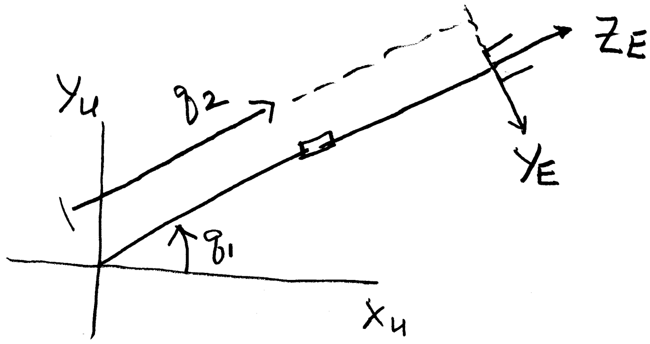
$$\begin{pmatrix} {}^{u}R_A & {}^{u}P_A \\ 0 & 1 \end{pmatrix} \left(\begin{array}{c|c} {}^{A}\hat{\omega}_B \, {}^{A}R_B & {}^{A}u_B \\ \hline 0 & 0 \end{array}\right)$$

$$^{u}u_B = \underbrace{{}^{u}\hat{\omega}_C \, {}^{u}R_C \, {}^{C}P_B} + \underbrace{{}^{u}u_C} + {}^{u}R_A \, {}^{A}u_B$$

$$^{A}u_B = {}^{A}\omega_B \times {}^{A}P_B + \begin{pmatrix} 0 \\ -2\cos 45 \\ -2\sin 45 \end{pmatrix}$$

$$^{u}u_B = \underbrace{{}^{u}\omega_C \times {}^{u}R_C \, {}^{C}P_B}_{{}^{u}P_B - {}^{u}P_C} + {}^{u}u_C + \underbrace{{}^{u}R_A \, {}^{A}\omega_B \times {}^{A}P_B}_{\checkmark} + \underbrace{{}^{u}R_A}_{= {}^{u}R_C} \begin{pmatrix} 0 \\ -2\cos 45 \\ -2\sin 45 \end{pmatrix}$$

2.



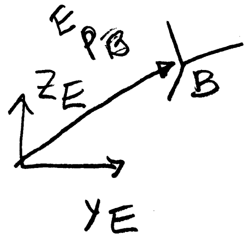
(a) $x = q_2 \cos q_1 \quad \dot{x} = \dot{q}_2 \cos q_1 - q_2 \sin q_1 \dot{q}_1$

$y = q_2 \sin q_1 \quad \dot{y} = \dot{q}_2 \sin q_1 + q_2 \cos q_1 \dot{q}_1$

$\omega_z = \dot{q}_1$

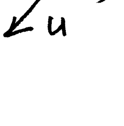
$$
{}^u J_E = \begin{pmatrix} -q_2 \sin q_1 & \cos q_1 \\ q_2 \cos q_1 & \sin q_1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}
$$

(b)



$$
{}^u u_B = {}^u u_E + {}^u R_E {}^E p_B \times {}^u w_E
$$

$$
= {}^u u_E + \widehat{{}^u R_E {}^E p_B}\, {}^u w_E
$$



${}^u w_E = {}^u w_B$ (same rigid body)

$$
\begin{bmatrix} {}^u u_B \\ {}^E w_B \end{bmatrix} = \left[\begin{array}{c|c} I & {}^u R_E {}^E p_B \\ \hline 0 & I \end{array}\right] \begin{bmatrix} {}^u u_E \\ {}^u w_E \end{bmatrix}
$$

$$
{}^u J_B \cancel{\dot{q}} = \quad " \quad {}^u J_E \cancel{\dot{q}} \quad //
$$

(c) 1st 2 rows of Jacobian u_x, u_y Task

$$\det(J) = -q_2 \sin^2 q_1 - q_2 \cos^2 q_1 = 0$$

$$= -q_2 = 0 \qquad q_2 = 0$$ //

(d). 1st and last row, u_x, w_z Task

$$\det(J) = -\cos q_1 = 0$$

$$q_1 = 90^\circ \text{ or } 270^\circ$$ //

(e). Task is only u_x !!!

1st row.

$$q_2 \sin q_1 = 0 \qquad q_2 = 0, \text{ and } q_1 = 90^\circ$$ //

$$\cos q_1 = 0$$