ME4245E Robot Kinematics, Dynamics and Control
Quiz 1 21 Sept 2011, 18:30-19:30

Name: ______________________________ Matric number:______________

Please make sure all your solutions are complete. For problems 1(b), 2, and 3(c), you need not evaluate nor compute the final numeric value of your answers.

1. Frames A and B are attached rigidly to a rectangular block as shown in Figure 1.

   (a) (10 marks) Determine the homogeneous transformation matrix ${}^{O}T_{A}$ that describes the position and orientation of Frame A in Frame O.

   (b) (30 marks) The block undergoes the following rigid motion in sequence:
      a. Rotation about Z axis of Frame 0 by 30 degrees,
      b. Translation along Frame A by (2, 3, 4) m,
      c. Rotation about X axis of Frame B by 60 degrees

      Determine the new position and orientation of Frame B (express this as a homogeneous transformation matrix). in Frame O

2. (20 marks) The block in Fig 1 is translating at the rate of (1, 2, 3) m/s and rotating at (4, 5, 6) /s with respect to Frame O. Determine the translational and angular velocities of Frame A. in Frame O

3. Fig. 2 shows a robot with two rotational joints. The first joint axis is along AB (vertical) and the 2nd joint axis is along CD (on a horizontal plane).
   (a) (15 marks) Assign frames to the robot according to the Denavit Hartenberg convention given in class.
   (b) (15 marks) Fill in the table of kinematic parameters and indicate which parameter is the joint variable
   (c) (10 marks) Derive the expression for the full Manipulator Jacobian.

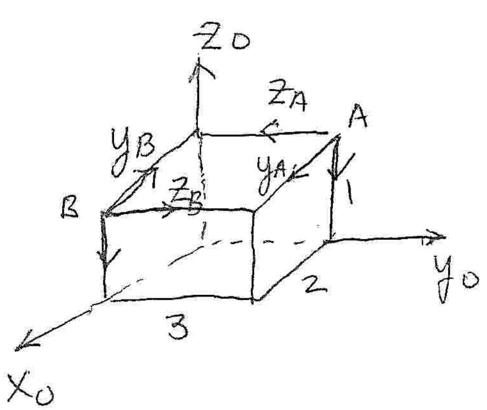
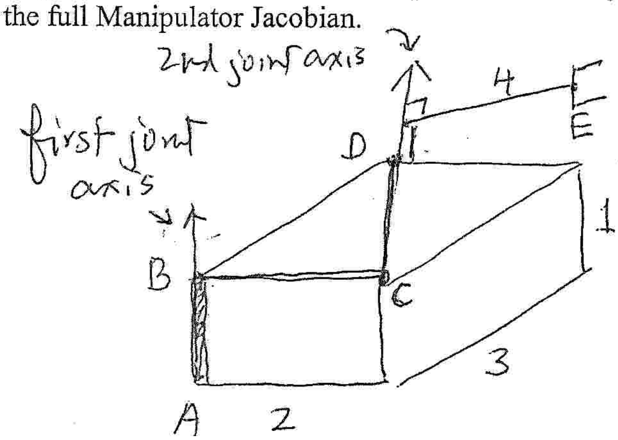


Fig. 1



AB = fixed to ground

BCD = 1st moving link

DE = 2nd moving link

Fig. 2

1. (a) ${}^{O}P_A = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$ ${}^{O}Z_A = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$

${}^{O}X_A = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$ ${}^{O}Y_A = \begin{pmatrix} +1 \\ 0 \\ 0 \end{pmatrix}$

$$ {}^{O}T_A = \begin{pmatrix} {}^{O}X_A & {}^{O}Y_A & {}^{O}Z_A & {}^{O}P_A \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ = \begin{pmatrix} 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

1 (b)

$${}^{0}T_{A_1} = Rot(z, 30^\circ)\ {}^{0}T_{A_0} \quad \text{where } {}^{0}T_{A_0} = {}^{0}T_A \text{ from question 1 (a)}$$

where
$$Rot(z, 30^\circ) = \begin{pmatrix} \cos 30^\circ & -\sin 30^\circ & 0 & 0 \\ \sin 30^\circ & \cos 30^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^{0}T_{A_2} = {}^{0}T_{A_1}\ Trans(2,3,4) \quad \text{where } Trans(2,3,4) = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

By inspection:

$${}^{A}T_B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} = {}^{A_2}T_{B_2} = {}^{A_0}T_{B_0} = {}^{A_1}T_{B_1}$$

$${}^{0}T_{B_2} = {}^{0}T_{A_2}\ {}^{A_2}T_{B_2} = {}^{0}T_{B_2}$$

$${}^{0}T_{B_3} = {}^{0}T_{B_2}\ Rot(x, 60^\circ) \quad \text{where } Rot(x, 60^\circ) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 60^\circ & -\sin 60^\circ & 0 \\ 0 & \sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2. $\quad {}^0\omega_A = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} 1/s = {}^0\omega_B$

$${}^0u_A = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} m/s \overset{?}{=} {}^0u_B \qquad {}^0u_A \neq {}^0u_B$$

~~you assume~~.   Note: ${}^0u_A \neq {}^0v_B$

$${}^0u_A = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \text{ then } {}^0u_B = ?$$

$${}^0p_B = {}^0T_A \, {}^Ap_B \qquad {}^0T_B = {}^0T_A \, {}^AT_B$$

$${}^0u_B = {}^0\dot{T}_A \, {}^Ap_B + \cancel{{}^0T_A \, {}^A\dot{p}_B}$$

OR 0

$$= \left(\begin{array}{c|c} {}^0\hat{\omega}_A \, {}^0R_A & {}^0u_A \\ \hline 0 & 0 \end{array} \right) \begin{pmatrix} {}^Ap_B \\ 1 \end{pmatrix}$$

$${}^0u_B = {}^0\hat{\omega}_A \, {}^0R_A \, {}^Ap_B + {}^0u_A$$

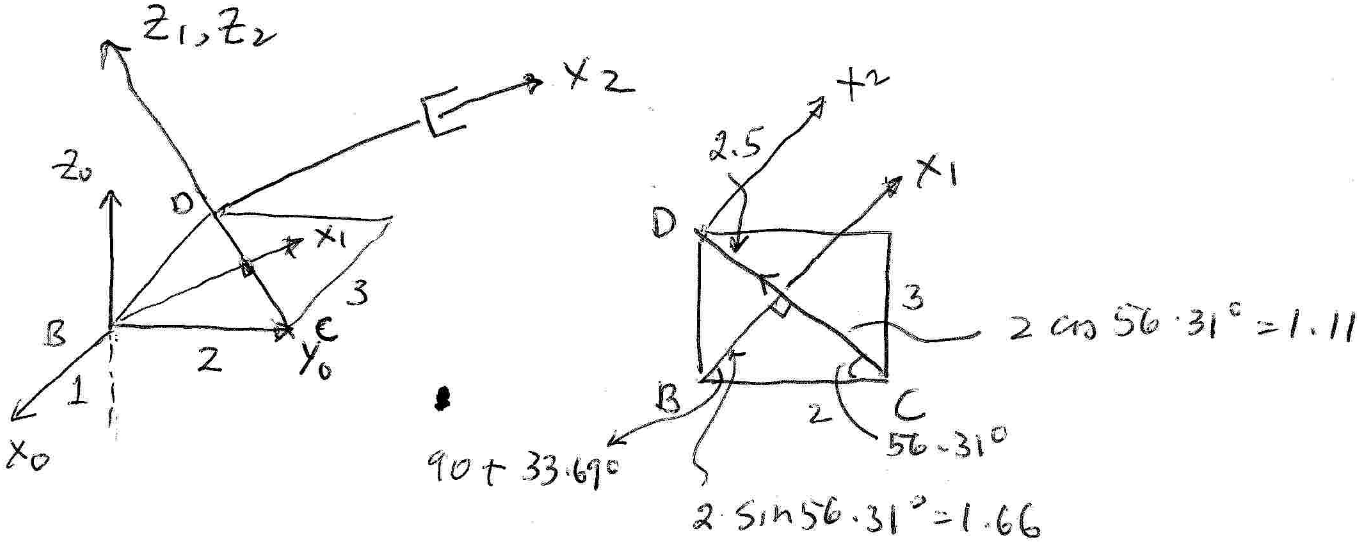
$${}^0\hat{\omega}_A = \begin{pmatrix} 0 & -6 & 5 \\ 6 & 0 & -4 \\ -5 & 4 & 0 \end{pmatrix}$$

${}^0R_A = $ ______ from 1(a)

${}^Ap_B = $ ______ from 1(a)

3.

(a)



(b)

θ	r	d	α	
$q_1 = 33.69° + 90°$	0	1.66	$-90°$	$\rightarrow {}^0T_1$
$q_2 = 0°$	2.5	0	$0°$	$\rightarrow {}^1T_2$

(c) $J = 6 \times 2 \quad = (J_1 \; J_2)$

$$J_1 = \begin{pmatrix} {}^0Z_0 \times ({}^0P_2 - {}^0P_0) \\ {}^0Z_0 \end{pmatrix} \qquad J_2 = \begin{pmatrix} {}^0Z_1 \times ({}^0P_2 - {}^0P_1) \\ {}^0Z_1 \end{pmatrix}$$

$${}^{i-1}T_i = \begin{pmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & d_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & d_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & r_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where $\theta_i, r_i, d_i, \alpha_i \rightarrow$ from DH Table.

$${}^0Z_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad {}^0P_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$${}^0P_1 = {}^0T_1 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \qquad {}^0P_2 = {}^0T_1 \, {}^1T_2 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$${}^0Z_1 = {}^0T_1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$