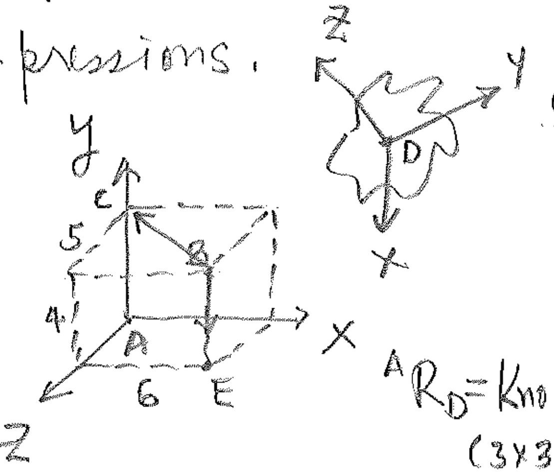
ME 4245E Quiz 1.1 29/8/2012

Answer all Questions. Pls provide complete answers. But you need not evaluate nor simplify the expressions.



$^{A}R_{D}$ = Known (3×3)

Coordinates expressed in Frame A:

D : (10, 20, 30)

B : (6, 4, 5)

C : (0, 4, 0)

E : (6, 0, 5)

$\vec{BC}$ = x axis of Frame B

$\vec{BE}$ = y axis of Frame B

1. Find the coordinates of pt D in B

2. The Body D is rotated around $\vec{BC}$ by 30°.
   Find the new coordinates of D in A.

3. The Body D then translates along its own x, y, z axes by 9, 10, 11 m respectively after the motion in # 2.
   Find the new coordinates of D in A.

$${}^{A}P_D = \begin{pmatrix} 10 \\ 20 \\ 30 \end{pmatrix} \quad {}^{A}x_B = \begin{pmatrix} 0-6 \\ 4-4 \\ 0-5 \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \\ -5 \end{pmatrix} \frac{1}{\sqrt{6^2+5^2}}$$

(normalize)

$$\left.\begin{aligned} {}^{A}y_B &= \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \\ {}^{A}z_B &= {}^{A}x_B \times {}^{A}y_B \end{aligned}\right\} \quad {}^{A}T_B = \begin{pmatrix} {}^{A}x_B & {}^{A}y_B & {}^{A}z_B & {}^{A}P_B \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} {}^{B}P_D \\ 1 \end{pmatrix} = {}^{B}T_A \begin{pmatrix} {}^{A}P_D \\ 1 \end{pmatrix} \quad \text{where } {}^{B}T_A = {}^{A}T_B^{-1}$$

Let ${}^{B}P_{D_0} = {}^{B}P_D$ from #1 (above) = initial pos'n of D before motion

${}^{B}P_{D_1}$ = position after motion = ${}^{B}Mot\ {}^{B}P_{D_0}$, ${}^{B}Mot = Rot(X, 30^\circ)$

$$\text{where } Rot(x, 30^\circ) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ & 0 \\ 0 & \sin 30^\circ & \cos 30^\circ & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

${}^{A}P_{D_1} = {}^{A}T_B\ {}^{B}P_{D_1}$ (${}^{A}T_B$ is from #1)

$${}^{A}T_{D_1} = {}^{A}T_B\ {}^{B}T_{D_1} \qquad {}^{A}T_D = {}^{A}T_B\ {}^{B}T_D$$

$${}^{B}T_{D_1} = Rot(x, 30^\circ)\ {}^{B}T_{D_0}\ ; \quad {}^{B}T_{D_0} = {}^{A}T_B^{-1}\ {}^{A}T_D$$

$${}^{A}T_{D_1} = {}^{A}T_B\ Rot(x, 30^\circ)\ {}^{A}T_B^{-1}\ {}^{A}T_D$$

$$\text{where } {}^{A}T_D = \begin{pmatrix} {}^{A}R_D & {}^{A}P_D \\ 0 & 1 \end{pmatrix}$$

$${}^{D}Mot = Trans(9, 10, 11) = \begin{pmatrix} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 11 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^{A}T_{D_2} = {}^{A}T_{D_1}\ {}^{D}Mot$$

(4th column) $= \left({}^{A}P_{D_2} \right) = {}^{A}T_{D_2} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$