ME4245 Quiz 1.1 (Chapter 1) 31 Aug 2016, 16:00-17:00

This is an open book/notes exam. But there is strictly no sharing of books and materials. Solve all problems. Unless otherwise specified, you don't need to simplify nor evaluate the expressions. But please make sure all expressions are complete, with all detailed steps leading to the final answer.

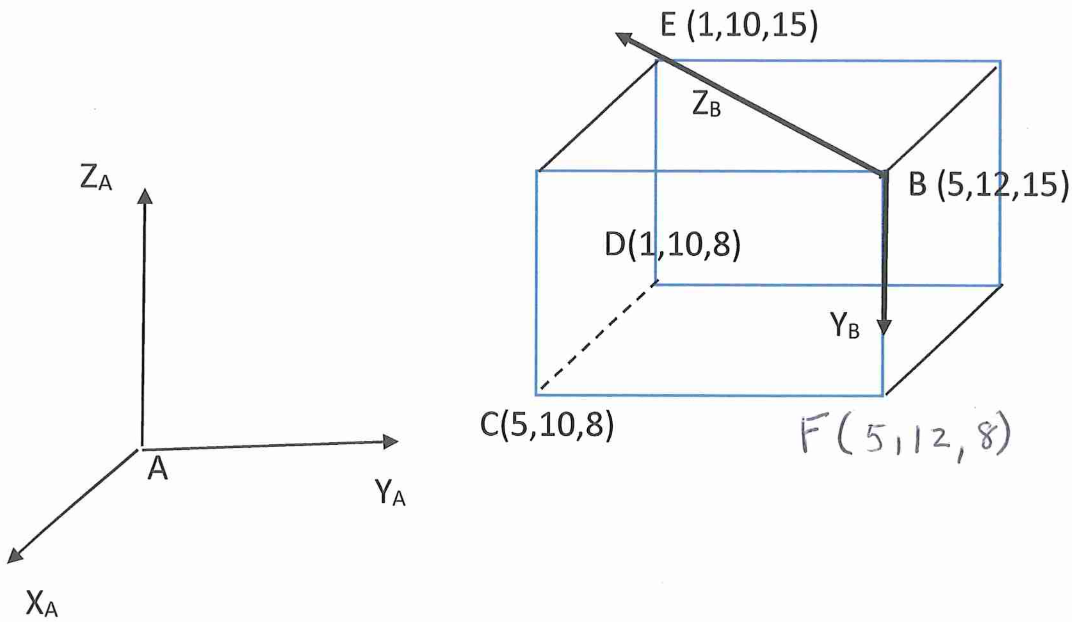
Frame B is attached to a rectangular block as shown in the figure below. All coordinates indicated are expressed in Frame A.

1. Determine the position and orientation of Frame B in A. Express this as a homogenous transformation matrix.

2. Determine the complete set of roll, pitch and yaw angles that describe the orientation of Frame B in A.

3. Find the coordinates of corner D in Frame B.

4. The block undergoes the following sequence of motions:
   1st> Rotation about Z_A by 20 degrees
   2nd> Translation along Frame B by (1,2,3) cm
   a) What is the new position and orientation of Frame B in Frame A?
   b) What are the new coordinates of corner D in A?
   c) What is the orientation of $\overrightarrow{CD}$ in A?



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1. $^AT_B = \begin{pmatrix} ^Ax_B & ^Ay_B & ^Az_B & ^Ap_B \\ 0 & 0 & 0 & 1 \end{pmatrix}$ where $^Ap_B = \begin{pmatrix} 5 \\ 12 \\ 15 \end{pmatrix}$ $^Ay_B = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$

$$^Az_B = \begin{pmatrix} 1-5 \\ 10-12 \\ 15-15 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 0 \end{pmatrix}, \text{normalized to} \begin{pmatrix} -4 \\ -2 \\ 0 \end{pmatrix} \times \frac{1}{\sqrt{4^2+2^2}}$$

$$^Az_B = \begin{pmatrix} -4/\sqrt{20} \\ -2/\sqrt{20} \\ 0 \end{pmatrix}$$

$$^Ax_B = {}^Ay_B \times {}^Az_B = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \times \begin{pmatrix} -4/\sqrt{20} \\ -2/\sqrt{20} \\ 0 \end{pmatrix} = \begin{pmatrix} -2/\sqrt{20} \\ 4/\sqrt{20} \\ 0 \end{pmatrix}$$

2. RPY: $\begin{pmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{pmatrix} = \begin{pmatrix} -2/\sqrt{20} & 0 & -4/\sqrt{20} \\ 4/\sqrt{20} & 0 & -2/\sqrt{20} \\ 0 & -1 & 0 \end{pmatrix}$

for Pitch, θ

$\theta = ATAN2\left(\frac{-n_z}{+\sqrt{n_x^2+n_y^2}}\right) = ATAN2(0/+1) = 0°$ ($\cos\theta$ = positive)

Roll, ϕ = $ATAN2\left(\frac{n_y}{n_x} = \frac{4/\sqrt{20}}{-2/\sqrt{20}}\right) = 116.5°$

Yaw, ψ = $ATAN2\left(\frac{o_z}{a_z} = \frac{-1}{0}\right) = -90°$

} RPY = (116.5, 0°, -90°)

Fi / $\cos\theta$ = negative

Pitch, θ = $ATAN2\left(\frac{-n_z}{-\sqrt{n_x^2+n_y^2}}\right) = ATAN2\left(\frac{0}{-1}\right) = 180°$

Roll, ϕ = $ATAN2\left(\frac{-n_y}{-n_x} = \frac{-4/\sqrt{20}}{2/\sqrt{20}}\right) = -63.5°$

Yaw, ψ = $ATAN2\left(\frac{o_z}{-a_z} = \frac{+1}{0}\right) = 90°$

RPY = (-63.5, 180°, 90°)

3. ${}^B P_D = {}^B T_A \, {}^A P_D$, ${}^A P_D = \begin{pmatrix} 1 \\ 10 \\ 8 \\ 1 \end{pmatrix}$

${}^B T_A = {}^A T_B^{-1}$ where ${}^A T_B$ is from #1

Coords = (x, y, z) when ${}^B P_D = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$

4. 1st) ${}^A T_{B_1} = {}^1Mot \; {}^A T_B$, ${}^1Mot = Rot(z, 20^\circ) = \begin{pmatrix} \cos 20^\circ & -\sin 20^\circ & 0 & 0 \\ \sin 20^\circ & \cos 20^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

2nd) ${}^A T_{B_2} = {}^A T_{B_1} \; {}^2Mot$,

${}^2Mot = Trans(1, 2, 3) = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

a) ${}^A T_{B_2}$

b) ${}^A P_{P_2} = {}^A T_{B_2} \; {}^{B_2} P_{D_2}$, where ${}^{B_2} P_{D_2} = {}^B P_D$ (#3 above)

c) ${}^A \overset{\rightharpoonup}{CD}_2 = {}^A T_{B_2} \; {}^{B_2} CD_2$, ${}^A T_{B_2}$ = 2nd) #4

${}^{B_2} CD_2 = {}^{B_0} CD_0 = {}^B T_A \; {}^A CD_0$

where ${}^A CD_0 = \begin{pmatrix} 1-5 \\ 10-10 \\ 8-8 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 0 \\ 0 \end{pmatrix} \times \frac{1}{4} = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ orientation Vector

→ scaling factor is ϕ

${}^B T_A = {}^A T_B^{-1}$, where ${}^A T_B$ is from #1