Quiz 1, Robotics, 2000/2001 Solutions

By Inspection is best, or Trans(3,5,2) Rot y(90°) Rot(x, −90°)

1. $${}^{A}T_{B} = \begin{pmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad RPY = \begin{pmatrix} cRcP & cRsPsY - sRcY & cRsPcY + sRsY \\ sRcY & sRsPsY + cRcY & sRsPcY - cRsY \\ -sP & cPsY & cPcY \end{pmatrix}$$

(16)

or Rot(x, −90°) Rot(z, 90°) Trans(−2, −3, 5)

or Trans(3, 5, 2) Rot(x, −90°) Rot(z, 90°) → or Rot(x, −90°) Trans(3, −2, 5) Rot(z, 90°)

or Trans(3, 5, 2) Rot(z, 90) Rot(y, 90°)

(3) Since $n_z = -1 = -\sin P$. $\underline{P = 90°}$ $\cos P = 0$

(3) (3) (6) $Y - R = \text{atan2}(O_x, O_y) = \text{atan2}(-1, 0) = -90°$

$Y + R$ = infinite # of possibilities as long as $Y - R = -90°$

2. ${}^{A}T_{B_0}$ = initial position = ${}^{A}T_{B}$, ${}^{A}T_{D}$ = given , ${}^{B}T_{C}$ = given

$${}^{D}T_{B_0} = {}^{D}T_{A}\,{}^{A}T_{B_0}$$

$${}^{D}T_{B_1} = Rot(x, 10°)\,{}^{D}T_{B_0} = Rot(x, 10°)\,{}^{D}T_{A}\,{}^{A}T_{B_0}$$

$${}^{D}T_{C_1} = {}^{D}T_{B_1}\,{}^{B_1}T_{C_1} \text{ , where } {}^{B_1}T_{C_1} = {}^{B}T_{C}$$

$${}^{D}T_{C_2} = {}^{D}T_{C_1}\,Rot(y, 20°) = {}^{D}T_{B_1}\,{}^{B}T_{C}\,Rot(y, 20°)$$

$${}^{A}T_{C_2} = {}^{A}T_{D}\,{}^{D}T_{C_2} = {}^{A}T_{D}\,{}^{D}T_{B_1}\,{}^{B}T_{C}\,Rot(y, 20°)$$

$${}^{A}T_{C_2} = {}^{A}T_{D}\,\underbrace{Rot(x, 10°)\,{}^{D}T_{A}\,{}^{A}T_{B_0}}_{{}^{D}T_{B_1}}\,{}^{B}T_{C}\,Rot(y, 20°)$$

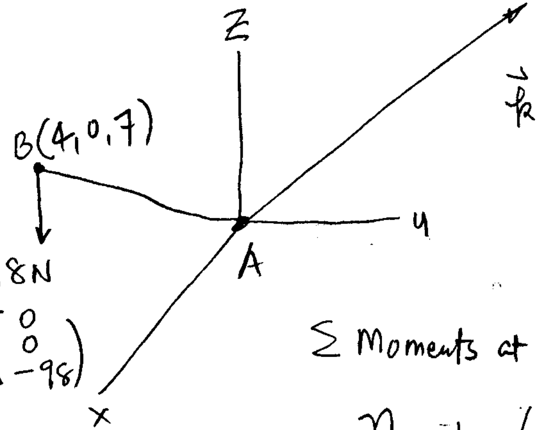
$${}^{A}T_{C_3} = Trans(1, 2, 3)\,{}^{A}T_{C_2}$$

${}^{D}T_{C}$ = initial position + orient of c in D.

$${}^{A}T_{C_3} = Trans(1, 2, 3)\,{}^{A}T_{D}\,\underbrace{Rot(x, 10)\,{}^{D}T_{A}\,{}^{A}T_{B}}_{{}^{D}T_{B_1}}\,{}^{B}T_{C}\,Rot(y, 20°)$$

9 8 8

3.



$F = \begin{pmatrix} 0 \\ 0 \\ -98 \end{pmatrix}$

$$\vec{k} = \begin{pmatrix} 3 \\ 5 \\ 6 \end{pmatrix} \cdot \frac{1}{\sqrt{9+25+36}} = \begin{pmatrix} 3/\sqrt{70} \\ \sqrt{5/14} \\ 3\sqrt{\frac{2}{35}} \end{pmatrix} = \begin{pmatrix} 0.359 \\ 0.598 \\ 0.717 \end{pmatrix} \quad (5)$$

Σ Moments at $A = 0$

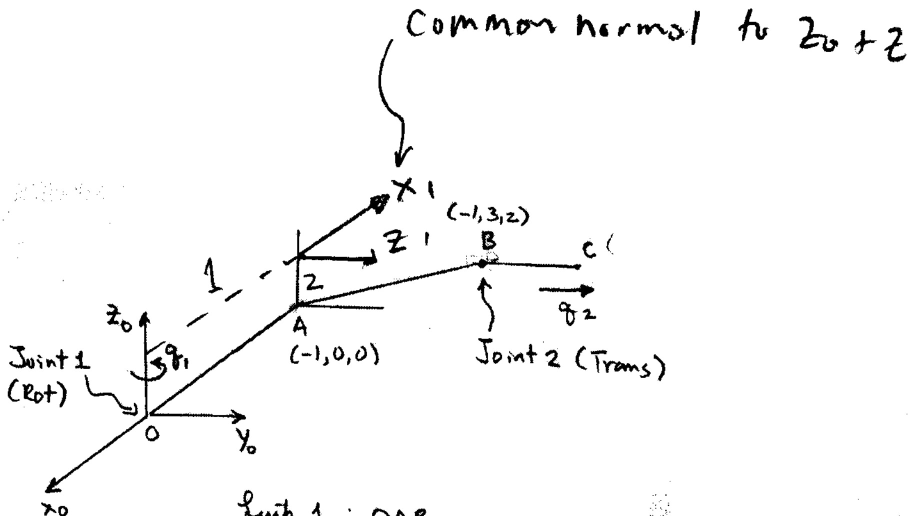
$$n_A + \begin{pmatrix} 4 \\ 0 \\ 7 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -98 \end{pmatrix} = 0$$

$$n_A = - \begin{pmatrix} 4 \\ 0 \\ 7 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -98 \end{pmatrix} = -\begin{pmatrix} 0 \\ 392 \\ 0 \end{pmatrix} \text{ N·m} \quad (10)$$

$$\tau = n_A \cdot \vec{k} = n_A^T k = -28\sqrt{70} = \quad (10)$$

$= -234.265$ N-m.

4)



Link 1 : OAB

Link 2 : BC

$$q_1 = \theta_1 = 180^\circ$$

$$r = 2$$

$$d = 1$$

$$\alpha = 90^\circ$$