max marks: 70

ME4245 Quiz 1, 26/8/2013

16:00 – 17:00

Answer all Questions. You need not simplify your answers but make sure they are complete.

1. (15 marks)

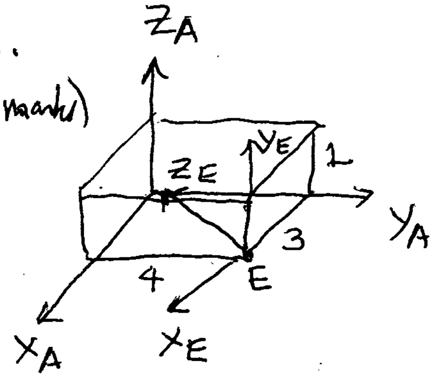

Fig 1

a) Find the position & orientation of Frame E in A. (Fig 1)

b) describe the position & orientation of E in A ~~as~~ as a 4×4 homogeneous transformation matrix, i.e. ${}^{A}T_E = ?$

2.

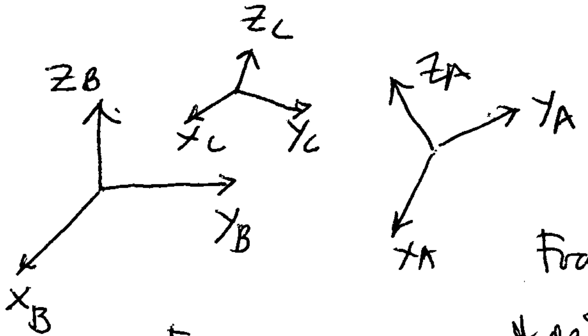

Fig. 2

Let Frame A be attached to a block as shown in Fig. 1.

Frame A (the block) is at a position & orientation in Frame B, ⓐ ${}^{B}T_A$ Known.

(10 marks) a) Find the position coordinates of the corner E of the block in Frame B

(45 marks) b) The Block undergoes the following motion in sequence:

① $Rot(Z_B, 30°)$

② $Rot(y_c, 40°)$ with ${}^{B}T_C$ Known.

③ $Rot(x_A, 50°)$

④ ~~Rot~~ Trans (1, 2, 3) along Frame E where E Frame is attached to the block too.

Find new position coordinates of E in Frame B.

(Assume this is the question)

1. $y_E = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\quad z_E = \begin{pmatrix} -3 \\ -4 \\ 0 \end{pmatrix} \dfrac{1}{\sqrt{3^2+4^2}} = \begin{pmatrix} -3/5 \\ -4/5 \\ 0 \end{pmatrix}$

(15)

$$^A x_E = {}^A y_E \times {}^A z_E = \begin{pmatrix} 4/5 \\ -3/5 \\ 0 \end{pmatrix}$$

a) $^A P_E = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$ //

b) $\therefore$ $^A T_E = \begin{pmatrix} \underbrace{\begin{matrix} 4/5 & 0 & -3/5 \\ -3/5 & 0 & -4/5 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{matrix}}_{\substack{\text{orientation} \\ ^A R_E}} & \begin{matrix} 3 \\ 4 \\ 0 \\ 1 \end{matrix} \end{pmatrix}$ //

Position $^A P_E$

(10) 2) a) $^B P_E = {}^B T_A \, {}^A P_E$, where $^A P_E = \begin{pmatrix} 3 \\ 4 \\ 0 \\ 1 \end{pmatrix}$ 4×1 version

b) $^B P_{E_1} = Rot(Z, 30°)\, {}^B P_{E_0}$ $\quad {}^B P_{E_0} = {}^B P_E$ from (a).

$Rot(Z, 30°)$ = 4×4 version

(10) ① (1st motion) or $^B T_{E_1} = Rot(Z, 30°)\, {}^B T_{E_0}$, $\quad {}^B T_{E_0} = {}^B T_A \, {}^A P_E$

need this 4×4 version for the 4th motion (later)

(10) ② (2nd motion) $^C P_{E_1} = {}^C T_B \, {}^B P_{E_1}$ // where $^C T_B = {}^B T_C^{-1}$ $\quad {}^B T_C$ = given (known)

or

$^C T_{E_1} = {}^C T_B \, {}^B T_{E_1}$

$$Rot(Z, 30°) = \begin{pmatrix} \cos 30° & -\sin 30° & 0 & 0 \\ \sin 30° & \cos 30° & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$^C T_{E2} = Rot(y, 40°)\, {}^C T_{E_1}$

(10) ③ (3rd motion) $^C T_{A_2} = {}^C T_{E2} \, {}^E T_A$; $\quad {}^E T_A = {}^A T_E^{-1}$ (#1) – fixed

$^C T_{A_3} = {}^C T_{A_2} \, Rot(x, 150°)$

$$Rot(y, 40°) = \begin{pmatrix} \cos 40° & 0 & \sin 40° & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 40° & 0 & \cos 40° & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(15) ④ (4th motion) $^C T_{E3} = {}^C T_{A3} \, {}^A T_E$

$^C T_{E4} = {}^C T_{E_3} \, Trans(1, 2, 3)$

$$Trans(1,2,3) = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Rot(X, 150°) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 50 & -\sin 50 & 0 \\ 0 & \sin 50 & \cos 50 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$^B T_{E4} = {}^B T_C \, {}^C T_{E4}$

$^B P_E = {}^B T_{E4} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ //

$${}^{B}T_{E4} = {}^{B}T_{C}\ \underbrace{{}^{C}T_{E3}\ Trans(1,2,3)}$$

$$= {}^{B}T_{C}\ {}^{C}T_{A3}\ {}^{A}T_{E}\ Trs(1,2,3)$$

$$= {}^{B}T_{C}\ \overbrace{{}^{C}T_{A2}\ Rot(x,50^\circ)}\ {}^{A}T_{E}\ Trs(1,2,3)$$

$$= {}^{B}T_{C}\ \overbrace{{}^{C}T_{E2}}\ {}^{E}T_{A}\ Rot(x,50^\circ)\ {}^{A}T_{E}\ Ts(1,2,3)$$

$$= {}^{B}T_{C}\ \overbrace{Rot(y,40^\circ)\ {}^{C}T_{B}\ Rot(z,50^\circ)\ {}^{B}T_{E0}}\ {}^{E}T_{A}\ Rot(x,50)$$

$${}^{A}T_{E}\ Tras(1,2,3)$$