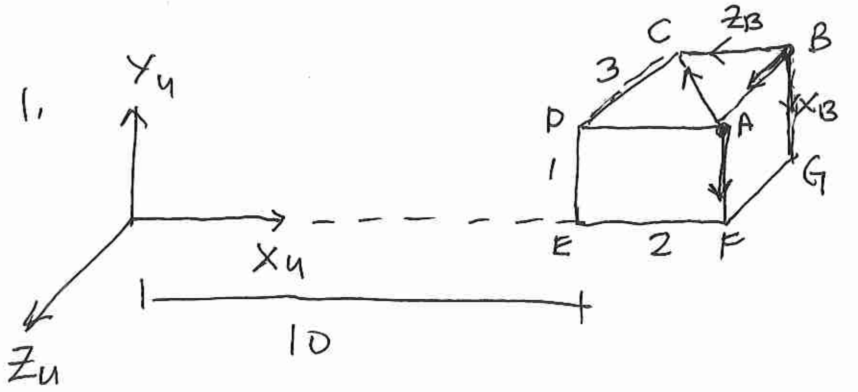
ME4245E QUIZ 1.1 (Chapter 1) 29/8/2014

1.



a) Define orientation of $\vec{AC}$ in U.

b) The block undergoes the ff motion in sequence

1st) Rot (x_u, 60°)

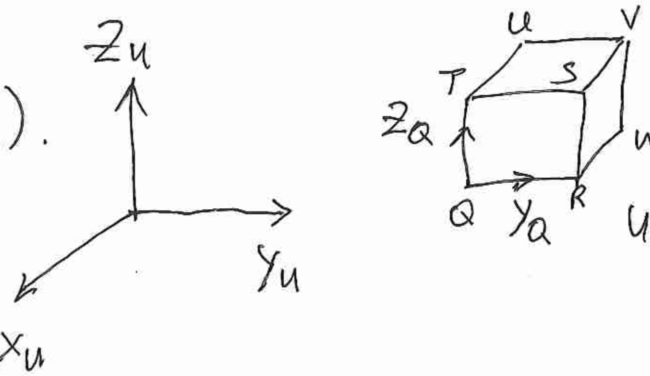
2nd) Rot (AC, 90°)

3rd) Trans ($\vec{BG}$, 6 m)

* Find the new coordinates of C in U

c) Find the coordinates of U in C after the 3 motions.

2).



Block Q is positioned & oriented in U such that

$$ {}^{U}T_{Q} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & \sqrt{3}/2 & -1/2 & 1 \\ 0 & 1/2 & \sqrt{3}/2 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Draw Block Q in U

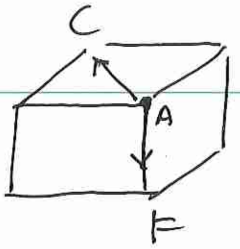
ME4245E QUIZ 1.1 (Chapter 1) 29/8/2014

SOLUTIONS

a)

$$ {}^{u}\vec{AC} = \begin{pmatrix} \delta x \\ \delta y \\ \Delta z \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ -3 \end{pmatrix} \times \frac{1}{\sqrt{(-2)^2 + (-3)^2}} $$

b)



$$ \text{let } \vec{AC} = {}^{u}x_A = {}^{u}\vec{A_C} \text{ (above)} $$

$$ \vec{AF} = {}^{u}y_A = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} $$

$$ {}^{u}z_A = {}^{u}x_A \times {}^{u}y_A $$

$$ {}^{u}T_A = \begin{pmatrix} {}^{u}x_A & {}^{u}y_A & {}^{u}z_A & \begin{matrix} 12 \\ 1 \\ 0 \end{matrix} \\ 0 & 0 & 0 & 1 \end{pmatrix} = {}^{u}T_{A_0} $$

$$ {}^{u}T_{A_1} = Rot(x, 60^\circ)\, {}^{u}T_A, \quad Rot(x, 60^\circ) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 60 & -\sin 60^\circ & 0 \\ 0 & \sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{u}T_{A_2} = {}^{u}T_{A_1}\, Rot(x, 90^\circ), \quad Rot(x, 90^\circ) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{u}T_{B_0} = \begin{pmatrix} {}^{u}x_{B_0} & {}^{u}y_{B_0} & {}^{u}z_{B_0} & {}^{u}p_{B_0} \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{u}x_{B_0} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \quad {}^{u}y_{B_0} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad {}^{u}z_{B} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \quad {}^{u}p_{B_0} = \begin{pmatrix} 12 \\ 1 \\ -3 \end{pmatrix} $$

$$ {}^{A_0}T_{B_0} = {}^{A_0}T_{u}\, {}^{u}T_{B_0} = \left({}^{u}T_{A_0}\right)^{-1} {}^{u}T_{B_0} $$

$$^{A_2}T_{B_2} = {}^{A_0}T_{B_0}$$

$$^{u}T_{B_2} = {}^{u}T_{A_2}\,{}^{A_2}T_{B_2}$$

$$^{u}T_{B_3} = {}^{u}T_{B_2}\,\mathrm{Trans}(6,0,0)$$

where $\mathrm{Trans}(6,0,0) = \begin{pmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$^{u}P_{C_3} = {}^{u}T_{B_3}\,{}^{B_3}P_{C_3} \quad //$$

$$^{B_3}P_{C_3} = {}^{B_0}P_{C_0} = \begin{pmatrix} 0 \\ 0 \\ +2 \\ 1 \end{pmatrix}$$

c). Cannot find coords of u in C since frame C is not defined.

However, we can find coords of u in Frame B since B is defined.

$$^{B_3}T_{u} = {}^{u}T_{B_3}^{-1}$$

$$^{B_3}P_{u} = {}^{B_3}T_{u}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad //$$

2).

$$\text{Since } \sin 60^\circ = \sqrt{3}/2 = \cos 30^\circ$$

$$\cos 60^\circ = 1/2 = \sin 30^\circ$$