1. Fig. 1 shows a planar robot with 1st and 3rd joint rotational and the 2nd joint translational. With its 3 joints, the robot is able to position its end effector (E) at a location (x, y) and orient its end-effector at an angle $\beta$ with respect to the positive x axis.

$$X = c_1 + q_2 \cos(q_1 + q_3) + \cos(q_1 + q_3)$$

$$Y = c_1 - q_2 \sin(q_1 + q_3) + \sin(q_1 + q_3)$$

By inspection, singular configuration is when $q_2 = 0$.

This can also be verified mathematically:

We take only 1st 2 rows and last row of the Jacobian (corresponding to the task).
c. If the task of the robot is \((x, y)\), are there singularities? If so, indicate the singular configuration(s), i.e., describe the robot joint coordinates in which the robot is singular.

By inspection, singular configuration is when \((\cos[q3]=0 \text{ and } q2 = 0)\)

![Diagram showing fully folded and fully stretched configurations]

We can also verify or derive this mathematically. For this task, we take only the 1st two rows of the Jacobian

\[
J = \begin{pmatrix}
-\frac{q_2}{2} c_1 - \frac{q_3}{2} - s_1 & -s_1 & -c_{13} \\
-\frac{q_2}{2} s_1 - s_{13} + c_1 & c_1 & -s_{13} \\
\end{pmatrix}
\]

**Determinant**

\[
\det(J) = q_2 \\
\det(J) = 0 \rightarrow q_2 = 0
\]

Singularities occur when Rank of \(J\) becomes less than 2.

When rank is less than 2, 2 joints are not able to provide two degrees of freedom for the task.

So we need to take all possible subsets of 2 joints to find singularities. Take all possible 2x2 matrices, singularities occur at configurations where the determinant of these matrices are all zero.
Singularities occur at joint coordinates where the following 3 equations are satisfied:

\[
\begin{align*}
&q_2 - C_3 = 0 \\
&C_3 + q_2 S_3 = 0 \\
&C_3 = 0
\end{align*}
\]

Mathematically, we take only the 1st and last rows of the Jacobian, and evaluate determinants of the three 2x2 matrices:

\[
\begin{align*}
\text{Joint } 1+2 & \quad \det(J) = -q_2 - C_3 \\
\text{Joint } 1+3 & \quad \det(J_{2x2}) = C_3 + q_2 S_3 \\
\text{Joint } 2+3 & \quad \det(J_{2x2}) = C_3
\end{align*}
\]

The singular configuration(s) are indicated by the above equations.

By inspection, when \(q_1 = 0\), or 180 deg \textbf{and} \(q_2 = 0\)

Mathematically, we take only the 1st and last rows of the Jacobian, and evaluate determinants of the three 2x2 matrices:

\[
\begin{align*}
\text{Joint } 1+2 & \quad \det(J) = S_1 = 0 \\
\text{Joint } 1+3 & \quad \det(J_{2x2}) = -q_2 C_1 - S_1 = 0 \\
\text{Joint } 2+3 & \quad \det(J_{2x2}) = -S_1 = 0
\end{align*}
\]

The singular configuration(s) are indicated by the above equations.

e. If the task of the robot is \((y, \beta)\), are there singularities? If so, indicate the singular configuration(s), i.e., describe the robot joint coordinates in which the robot is singular.

Again, by inspection, when \(\cos[q_1] = 0\) \textbf{and} \(q_2 = 0\) \textbf{or} \([q_1 = 90 \text{ or } -90] \textbf{and} q_2 = 0\]

Mathematically, we take only the 2nd and last rows of the Jacobian, and evaluate determinants of the following three 2x2 matrices:
2. Fig 2 shows two robotic manipulators whose kinematic models have been derived. The kinematic models include the derivations of their respective manipulator Jacobians. \(O_1\) and \(O_2\) are the “base frames” of each robot, while \(N_1\) and \(N_2\) are the end-effector frames. Robot #1 has \(N_1\) joints while Robot #2 has \(N_2\) joints. Robot #2 is rigidly attached to the end-effector of Robot #1, with \(^N_T_{O_2}\) known (this 4 x 4 matrix describes the relative position and orientation of the end-effector of Robot #1 and base of Robot #2, this is constant). Derive an expression for the Jacobian of the entire manipulator system containing \(N_1 + N_2\) joints. This Jacobian is used to compute the 6 x 1 velocity of Frame \(N_2\) with respect to Frame \(O_1\) given the \(N_1 + N_2\) joint velocities.

\[
\begin{align*}
\text{Joint } (1+2) & : \quad \det(J) = -c_1 = 0 \\
\text{Joint } (1+3) & : \quad \det(J) = c_1 - q_2 s_1 = 0 \quad \text{and} \\
\text{Joint } (2+3) & : \quad \det(J) = c_1 = 0 \\
\end{align*}
\]
\[ 0^1 U_{N_2} = 0^1 U_{N_1} + 0^1 R_{0^2} U_{N_2} + 0^1 W \times (0^1 R_{N_1}, 0^1 R_{N_2}) \]

\[ = 0^1 U_{N_1} - 0^1 R_{N_1} P_{N_2} \times 0^1 W_{N_1} \]

\[ 0^1 W_{N_2} = 0^1 W_{N_1} + 0^1 R_{0^2} W_{N_2} + J_2 q_2 \]

Considering:

\[ \begin{pmatrix} 0^1 U_{N_2} \\ 0^1 W_{N_2} \end{pmatrix} = \begin{pmatrix} 0 \quad R_{N_1} P_{N_2} \\ 0 \\ \end{pmatrix} \begin{pmatrix} 0^1 U_{N_1} \\ 0^1 W_{N_1} \end{pmatrix} + \begin{pmatrix} 0 \\ 0^1 R_{0^2} \\ \end{pmatrix} \begin{pmatrix} 0^2 U_{N_2} \\ 0^2 W_{N_2} \end{pmatrix} \]

\[ J = \begin{bmatrix} A J_1 \\ B J_2 \end{bmatrix} \quad (N_1 + N_2) \]