National University of Singapore — Department of Mechanical Engineering

ME4245 — Quiz 2 — 19 Sept 2007, 16:00-17:00

Name:______________________________ Matric Number:______________________

(as it appears in your NUS Student card)

Answer all the two questions in this quiz. Please make sure your answers and all expressions are complete. Please note that the 2nd question is at the back of this page. This is an open-book/notes quiz.

1. Fig.1 shows a 2-joint robot with the 1st joint rotational and the 2nd joint translational. The robot operates in the xy plane of Frame U. The first link is of length 1 m and has an axis of motion aligned with $Z_u$. Frame E is attached to the end of the 2nd link which translates along $X_E$. The 2nd link is at an angle of 60 degrees with respect to the 1st link as shown in the figure. The joint variables are β and ρ.

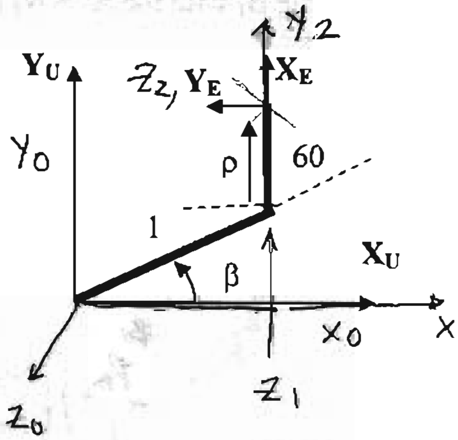


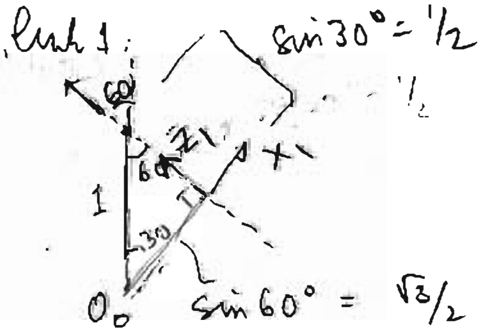

Fig. 1

a. Define and attach frames to according to the Denavit Hartenberg convention given in class.
b. Determine the kinematic parameters that relate the relative position and orientation of adjacent frames. Indicate which parameter is variable and relate it with the joint coordinates β and ρ
c. Derive the expression for ${}^{U}T_{E}$. (You don't need to simplify the expression)
d. Derive the complete manipulator Jacobian and simplify the expression
e. Does this robot have any singularities? Explain the reason for your answer. If it has singularities, describe the singular configuration(s) and corresponding singular direction(s).

note: $x_1$ is not always aligned with $X_0$. $x_1$ moves with link 1

| i | θ | r | d | α |
|---|---|---|---|---|
| 1 | $q_1 = 0$ | 0 | $\sqrt{3}/2$ | $-90^\circ$ |
| 2 | $-90^\circ$ | $1/2 + \rho$ | 0 | $90^\circ$ |

$q_2 = 1/2 + \rho$




$q_1 = \beta - 30^\circ = 0^\circ$ (at the configuration shown in Fig 1, $\beta = 30^\circ$, $Z_1$ parallel to $Y_U$)

${}^{U}T_E = {}^{U}T_0\,{}^{0}T_1\,{}^{1}T_2\,{}^{2}T_E$

where ${}^{U}T_0 = I_{3\times3}$, ${}^{2}T_E = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$,

$${}^{i-1}T_i = \begin{pmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & d_i\cos\theta_i \\ \sin\theta_i & \cos\alpha_i\cos\theta_i & -\sin\alpha_i\cos\theta_i & d_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & r_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

${}^{U}P_E$: $x = \cos\beta + \rho\cos(\beta+60^\circ)$, $z = 0$

$y = \sin\beta + \rho\sin(\beta+60^\circ)$

$u_x = -\sin\beta\,\dot\beta + \dot\rho\cos(\beta+60^\circ) - \rho\sin(\beta+60^\circ)\dot\beta$

$u_y = \cos\beta\,\dot\beta + \dot\rho\sin(\beta+60^\circ) + \rho\cos(\beta+60^\circ)\dot\beta$

${}^{u}\omega_x = {}^{u}\omega_y = {}^{u}u_z = 0$

${}^{u}\omega_z = \dot\beta$

$${}^{u}J_E = \begin{pmatrix} -\sin\beta - \rho\sin(\beta+60^\circ) & \cos(\beta+60^\circ) \\ \cos\beta + \rho\cos(\beta+60^\circ) & \sin(\beta+60^\circ) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}$$

1. con't.

Singularities:

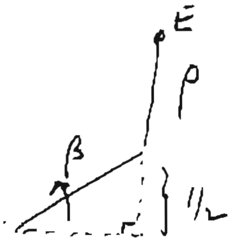
Rows 1 & 2 $(u_x, u_y)$ task

$$\det(J_{12}) = -\sin\beta\sin(\beta+60°) - \rho\sin^2(\beta+60°) + \cos\beta\cos(\beta+60°) - \rho\cos^2(\beta+60°)$$

$$= -\rho - 1/2$$

singularity occurs when $\rho = -1/2$

Does this make sense? → Yes




When $\rho = -1/2$, $\beta$ & $\rho$ produce the same motion in E

Rows 1, 6 $(u_x, \omega_z)$ task

$$\det(J_{16}) = -\cos(\beta+60°)$$

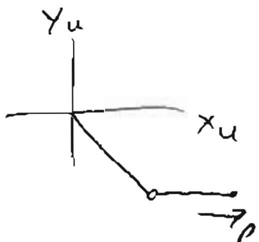
$\cos(\beta+60°) = 0$ $\quad \beta + 60° = 90°$ $\quad \beta = 30°$

(conf shown in Pg 1, $\rho$ does not contribute to $x_u$ motion)

Rows 2, 6 : $(u_y, \omega_z)$ task

$$\det(J_{26}) = -\sin(\beta+60°) = 0$$

$\beta + 60° = 0$ $\quad \beta = -60°$




$\rho$ does not contribute to $u_y$ motion

If task is any general 2 DOF task,

robot is never singular. → Robot can always do a 2DOF task.

→ no configuration exists where all determinants are zero.

2. Figure 2 shows Robot 1 and Robot 2 with $N_1$ and $N_2$ joints respectively. Their manipulator Jacobians are known:

$$J_1 = \begin{pmatrix} J_{P_1} \\ J_{\omega_1} \end{pmatrix} \text{ where } J_{P_1} \text{ and } J_{\omega_1} \text{ are } 3 \times N_1 \text{ matrices}$$

$$J_2 = \begin{pmatrix} J_{P_2} \\ J_{\omega_2} \end{pmatrix} \text{ where } J_{P_2} \text{ and } J_{\omega_2} \text{ are } 3 \times N_2 \text{ matrices}$$

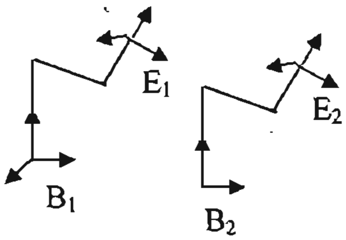



Fig. 2

Frames $B_i$ and $E_i$ represent the base and end-effector frames of each robot. Robot 1 carries Robot 2 according to a known relative position and orientation of $E_1$ and $B_1$. Derive the expression for the complete manipulator Jacobian of the ($N_1+N_2$) robot in terms of the following known expressions:

$$^{B_1}R_{E_1},\ ^{B_1}p_{E_1},\ ^{B_2}R_{E_2},\ ^{B_2}p_{E_2},\ ^{E_1}R_{B_2},\ ^{E_1}p_{B_2},\ J_{P_1},\ J_{\omega_1},\ J_{P_2},\ J_{\omega_2}$$

This manipulator Jacobian relates the velocity of Frame $E_2$ in Frame $B_1$ with the $N_1 + N_2$ joint velocities. Simplify the final expression for the manipulator Jacobian.

$$^{B_1}T_{E_2} = {}^{B_1}T_{E_1}\ {}^{E_1}T_{B_2}\ {}^{B_2}T_{E_2}$$

Differentiating:

$$\left(\begin{array}{c|c} ^{B_1}\hat{\omega}_{E_2}\ ^{B_1}R_{E_2} & ^{B_1}u_{E_2} \\ \hline 0 & 0 \end{array}\right) = \overbrace{\left(\begin{array}{c|c} ^{B_1}\hat{\omega}_{E_1}\ ^{B_1}R_{E_1} & ^{B_1}u_{E_1} \\ \hline 0 & 0 \end{array}\right)}^{^{B_1}\dot{T}_{E_1}} \overbrace{\left(\begin{array}{c|c} ^{E_1}R_{E_2} & ^{E_1}p_{E_2} \\ \hline 0 & 1 \end{array}\right)}^{^{E_1}T_{E_2}} + \overbrace{\left(\begin{array}{c|c} ^{B_1}R_{B_2} & ^{B_1}p_{B_2} \\ \hline 0 & 1 \end{array}\right)}^{^{B_1}T_{B_2}} \overbrace{\left(\begin{array}{c|c} ^{B_2}\hat{\omega}_{E_2}\ ^{B_2}R_{E_2} & ^{B_2}u_{E_2} \\ \hline 0 & 0 \end{array}\right)}^{^{B_2}\dot{T}_{E_2}}$$

$$^{B_1}u_{E_2} = \underbrace{^{B_1}\hat{\omega}_{E_1}}_{^{B_1}\omega_{E_1}}\ ^{B_1}R_{E_1}\ ^{E_1}p_{E_2} + {}^{B_1}u_{E_1} + {}^{B_1}R_{B_2}\ ^{B_2}u_{E_2}$$

$$= J_{w_1}\dot{q}_1 \times {}^{B_1}R_{E_1}\ ^{E_1}p_{E_2} + J_{P_1}\dot{q}_1 + {}^{B_1}R_{B_2}\ J_{P_2}\dot{q}_2$$

$$= -\widehat{^{B_1}R_{E_1}\ ^{E_1}p_{E_2}}\ J_{w_1}\dot{q}_1 + J_{P_1}\dot{q}_1 + {}^{B_1}R_{B_2}\ J_{P_2}\dot{q}_2$$

$$^{B_1}\hat{\omega}_{E_2}\ \cancel{^{B_1}R_{E_2}} = {}^{B_1}\hat{\omega}_{E_1}\ \underbrace{^{B_1}R_{E_1}\ ^{E_1}R_{E_2}}_{\cancel{^{B_1}R_{E_2}}} + {}^{B_1}R_{B_2}\ ^{B_2}\hat{\omega}_{E_2}\ \underbrace{^{B_2}R_{E_2}}_{^{B_2}R_{B_1}\ \cancel{^{B_1}R_{E_2}}}$$

$$^{B_1}\omega_{E_2} = \underbrace{^{B_1}\omega_{E_1}}_{J_{w_1}\dot{q}_1} + {}^{B_1}R_{B_2}\ \underbrace{^{B_2}\omega_{E_2}}_{J_{w_2}\dot{q}_2}$$

$\dot{q}_1 = N_1 \times 1$ (Robot 1)

$\dot{q}_2 = N_2 \times 1$ (Robot 2)

$$\therefore\ J = \left[\begin{array}{c|c} -\widehat{^{B_1}R_{E_1}\ ^{E_1}p_{E_2}}\ J_{w_1} + J_{P_1} & ^{B_1}R_{B_2}\ J_{P_2} \\ \hline J_{w_1} & ^{B_1}R_{B_2}\ J_{w_2} \end{array}\right]$$



$6 \times (N_1 + N_2)$