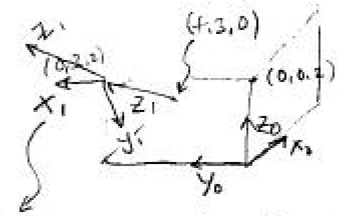
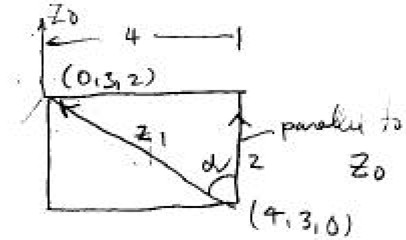
1st Semester 98/99

Quiz 2

1.



Common normal of Z_0 + Z_1



$\alpha = ATAN2(4,2)$
$= 63.43°$

$\theta = 90°$

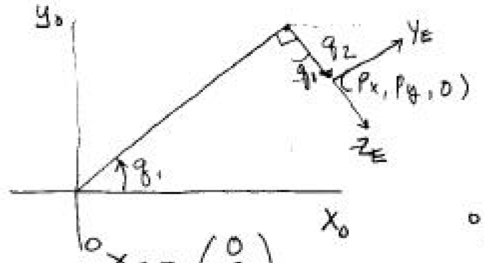
$r = 2$

$d = 3$

$\alpha = 63.43°$

2.

(a)



$$P_x = 1200 C_1 + q_2 S_1$$
$$P_y = 1200 S_1 - q_2 C_1$$
$$P_z = 0$$

$${}^0X_E = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \quad {}^0Z_E = \begin{pmatrix} S_1 \\ -C_1 \\ 0 \end{pmatrix}, \quad {}^0Y_E = \begin{pmatrix} C_1 \\ S_1 \\ 0 \end{pmatrix}$$

$$\therefore \; {}^0T_E = \begin{pmatrix} 0 & C_1 & S_1 & 1200C_1 + q_2 S_1 \\ 0 & S_1 & -C_1 & 1200S_1 - q_2 C_1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad //$$

(b) I.K., i.e. Express q_1, q_2 as function of P_x, P_y.

$$P_x = 1200C_1 + q_2 S_1 \qquad (1)$$
$$P_y = 1200S_1 - q_2 C_1 \qquad (2)$$

$$(1) * C_1 \Rightarrow \quad P_x C_1 = 1200\, C_1^2 + q_2 S_1 C_1$$
$$(2) * S_1 \Rightarrow + \quad P_y S_1 = 1200\, S_1^2 - q_2 S_1 C_1$$

$$P_x C_1 + P_y S_1 = 1200 \quad \rightsquigarrow \text{Case 4}$$
$$a \cos\theta + b \sin\theta = c$$

→ If $P_x P_y \neq 0$

$$q_1 = \text{ATAN2}(P_y, P_x) + \text{ATAN2}\left(\pm\sqrt{P_x^2 + P_y^2 - 1200^2},\ 1200\right)$$

for $P_x \neq 0$ & $P_y \neq 0$.

Note that P_x & P_y cannot be both zero.

q_1 = 2 solutions

→ If $P_x = 0$, $P_y \neq 0$ $P_y S_1 = 1200$

$$q_1 = \text{ATAN2}\left(1200/P_y,\ \pm\sqrt{1 - (1200/P_y)^2}\right)$$

∴ 2 sol'ns.

→ If $P_y = 0$, $P_x \neq 0$ $P_x C_1 = 1200$

$$q_1 = \text{ATAN2}\left(\pm\sqrt{1 - \left(\frac{1200}{P_x}\right)^2},\ \frac{1200}{P_x}\right)$$

∴ 2 sol'ns.

Now for q_2

$$\begin{aligned}
(1) * S_1 &\Rightarrow & P_x S_1 &= 1200\, S_1 C_1 + q_2 S_1^2 \\
(2) * C_1 &\Rightarrow^{(-)} & P_y C_1 &= 1200\, S_1 C_1 - q_2 C_1^2 \\
\hline
& & P_x S_1 - P_y C_1 &= q_2
\end{aligned}$$

$$\boxed{q_2 = P_x S_1 - P_y C_1}$$

∴ 2 sol'ns. Note sol'ns should be checked against the joint limits.

c). $\dot{p}_x = -1200 S_1 \dot{q}_1 + q_2 C_1 \dot{q}_1 + S_1 \dot{q}_2$

$\dot{p}_y = 1200 C_1 \dot{q}_1 + q_2 S_1 \dot{q}_1 - C_1 \dot{q}_2$

$$\begin{pmatrix} \dot{p}_x \\ \dot{p}_y \end{pmatrix} = \underbrace{\begin{pmatrix} -1200 S_1 + q_2 C_1 & S_1 \\ 1200 C_1 + q_2 S_1 & -C_1 \end{pmatrix}}_{J} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix}$$

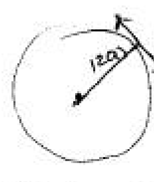
d) $\det(J) = 1200 S_1 C_1 - q_2 C_1^2 - 1200 S_1 C_1 - q_2 S_1^2$

$= -q_2$

Singular at $q_2 = 0$

, at this configuration, robot can only move in one direction. But since $100 \le q_2 \le 1000$,

∴ $q_2 = 0$ is not a singularity,



But there are singularities at $q_2 = 100$, at $q_2 = 1000$, because at these boundaries, robot can only move in 1 direction

e) First, find velocity of Frame E.

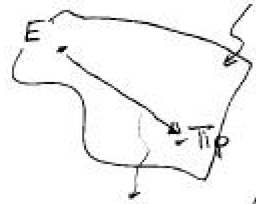
$$\begin{pmatrix} \dot{p}_x \\ \dot{p}_y \end{pmatrix} = \begin{pmatrix} -1200 \sin 30^\circ + 500 \cos 30 & \sin 30 \\ 1200 \cos 30^\circ + 500 \sin 30 & -\cos 30 \end{pmatrix} \begin{pmatrix} 60 \times \pi/180 \\ 100 \end{pmatrix}$$

$$= \begin{pmatrix} -166.99 & 0.5 \\ 1289.23 & -0.866 \end{pmatrix} \begin{pmatrix} 1.047 \\ 100 \end{pmatrix} = \begin{pmatrix} -124.84 \\ 1263.22 \end{pmatrix} \text{m/sec}$$

$\dot{p}_z = 0$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} 60 \times \pi/180 = \begin{pmatrix} 0 \\ 0 \\ 1.047 \end{pmatrix} \text{rad/sec}$$

Now to find velocity of tip.



$$ {}^{0}\omega_E = {}^{0}\omega_{tip} = \begin{pmatrix} 0 \\ 0 \\ 1.047 \end{pmatrix} $$

$$ {}^{E}P_{Tip} = \begin{pmatrix} 0 \\ -10 \\ 30 \end{pmatrix} $$

$$ {}^{0}u_{Tip} = {}^{0}u_E + \omega \times ({}^{0}P_{Tip} - {}^{0}P_E) $$

$$ = {}^{0}u_E + \omega \times ({}^{0}R_E \, {}^{E}P_{Tip}) $$

$$ = \begin{pmatrix} -124.84 \\ 1263.22 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1.047 \end{pmatrix} \times \begin{pmatrix} 0 & \cos 30^\circ & \sin 30^\circ \\ 0 & \sin 30^\circ & -\cos 30^\circ \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ -10 \\ 30 \end{pmatrix} $$

$$ = \begin{pmatrix} -124.84 \\ 1263.22 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1.047 \end{pmatrix} \times \begin{pmatrix} 6.34 \\ -20.98 \\ 0 \end{pmatrix} $$

$$ = \begin{pmatrix} -124.84 \\ 1263.22 \\ 0 \end{pmatrix} + \begin{pmatrix} 21.967 \\ 6.64 \\ 0 \end{pmatrix} $$

$$ {}^{0}U_{Tip} = \begin{pmatrix} -102.873 \\ 1269.86 \\ 0 \end{pmatrix} \text{mm/sec} \; // $$

(4)

$$\tau = J^T F$$

Since this is a planar problem, lets simplify J to be a 3×2 matrix, i.e.

$$\begin{pmatrix} \dot{P}_x \\ \dot{P}_y \\ \omega_z \end{pmatrix} = J_{3\times2} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} \quad \text{where } J_{3\times2} = \begin{pmatrix} -1200S_1 + q_2C_1 & S_1 \\ 1200C_1 + q_2S_1 & -C_1 \\ 1 & 0 \end{pmatrix}$$

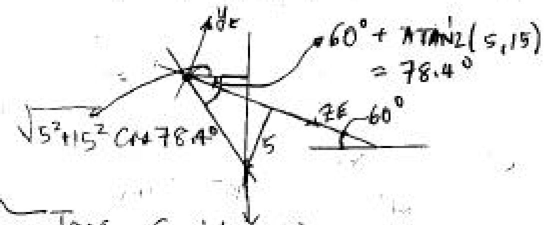
$$\text{since } \omega_z = \dot{q}_1$$

Then

$$\tau_{2\times1} = J^T_{3\times2} F_{3\times1} \quad \text{where } F = \text{generalized Force exerted on environment}$$

In the Case,

$${}^0F = \begin{pmatrix} 0 \\ +20 \\ +20\sqrt{5^2+15^2}\cos 78.4^\circ \end{pmatrix}$$



$$= \begin{pmatrix} 0 \\ 20\text{ N} \\ 63.59\text{ N-mm} \end{pmatrix}$$

Torque Exerted 20N by E-E on torch } also equal to

$$\begin{pmatrix} 0 & 0.866 & 0.5 \\ 0 & 0.5 & -0.866 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ -5 \\ 15 \end{pmatrix} \times \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 63.59 \end{pmatrix}$$

∴

$$\tau = J^T F$$

$$= \begin{pmatrix} -1200\sin 30^\circ + 500\cos 30^\circ & 1200\cos 30^\circ + 500\sin 30^\circ & 1 \\ \sin 30^\circ & -\cos 30^\circ & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 20 \\ 63.59 \end{pmatrix}$$

$$= \begin{pmatrix} -166.99 & 1289.23 & 1 \\ 0.5 & -0.866 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 20 \\ 63.59 \end{pmatrix}$$

$$\begin{matrix} \tau_1 \\ \tau_2 \end{matrix} = \begin{pmatrix} 25848.19\text{ N-mm} \\ -17.32\text{ N} \end{pmatrix}$$

Note τ_2 = Units of Force Bec Joint is Translat!