Name:

1. Figure 1 shows the location of adjacent joint axes z_0 and z_1 . The coordinates shown are all with respect to frame 0. Complete the frame assignment for Frame 1 according to the Denavit-Hartenberg (DH) convention discussed in class. Identify the four DH parameters (θ , r, d, α) that relate the two frames.



2. Figure 2 shows a planar robot with the 1st joint rotational (about z_0) and the 2nd joint translational. The first joint has no joint limits while the 2nd joint has a motion range 100 mm $\leq q_2 \leq 1000$ mm. A welding torch is rigidly attached to the end-effecor as shown. The tip of the torch is at location (0, -10, 30). The torch weighs 20 N with its center of gravity at (0, -5, 15). All these coordinates are with respect to Frame E.



- a) Derive the expression for the position and orientation of the end effector (Frame E) in Frame 0 as function of q_1 and q_2 . Express this as a homogenous transformation matrix ${}^{0}T_{E}$.
- b) Derive the inverse kinematic expressions that compute the joint positions as functions of the end-effector position in Frame 0.
- c) Derive the 2 x 2 Jacobian that relates the translational velocity of the endeffector in Frame 0 with the joint velocities.
- d) Are there singularities for this robot. Explain.
- e) At a certain instant of time, $q_1 = 30^\circ$, $q_2 = 500$ mm, and the joint velocities are 60 $^\circ$ /sec and 100 mm/sec respectively. What is the translational velocity of the tip of the welding torch with respect to Frame 0 at this instant of time.
- f) Determine the joint torque and forces required at joints 1 and 2 to carry the torch at position $q_1 = 30^{\circ}$, $q_2 = 500$ mm. Assume the links to be weightless.