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Adaptive Joint Friction Compensation Using a Model-Based Operational Space Velocity Observer

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***Abstract*— An operational space controller that employs a velocity observer and a friction adaptation law to achieve higher tracking accuracy is presented. Without velocity measurements, the overall observer-controller system can achieve a semi-global asymptotical stability result for the position and velocity tracking errors, and position and velocity estimation errors. The estimated friction coefficients can also approach the actual coefficients asymptotically. Experimental results indicate that, the proposed adaptive observer-controller is able to achieve higher tracking accuracy than the observer-controller without friction compensation.**

I. Introduction

Friction in robot manipulators is one of the major limitations in achieving high precision motion control. If not compensated properly, it may cause stability problems. For these reasons, friction modelling, identification, and compensation have been addressed by a number of researchers. For example, an adaptive controller that consider both static and dynamic friction effects was proposed in [1], a robust adaptive friction compensation in the presence of bounded disturbances and/or modelling uncertainties was addressed in [2], and a variable structure control scheme for the robot with nonlinear friction and dynamic backlash was investigated in [3].

The above mentioned approaches were based on the assumption that actuator velocities were exactly known. In practice, most robot manipulators are only equipped with link position sensors. To overcome this drawback, Lim et al. have proposed a joint space controller that does not rely on velocity measurements [4]. Since in many robotic applications, tasks are defined in operational space [5], an observer-controller for operational space trajectory tracking was proposed in [6]. The experimental results done on PUMA 560 robot manipulator verified its better position tracking performance over the same controller but employing filtered velocity [7].

To make use of the merits of the “cleaner” observed velocity obtained from our proposed observer-controller, we developed an adaptive controller that combined the adaptive friction control law with the observer-controller presented in [7] to achieve higher tracking accuracy. Experimental results using PUMA 560 indicated that the the proposed observer-controller incorporating adaptive friction compensation could achieve higher tracking accuracy than the one without friction compensation.

This paper is organized as follows: in Section II, the dynamic models of a robot in joint space and operational space are given, and some properties of a robot are also presented; in Section III, we present the structure of the proposed adaptive observer-controller; in Section IV, the overall adaptive observer-controller system stability analysis is given; in Section V, the implementation of the friction adaptation law is given; in Section VI, experimental results and some comments are presented; and in the last section, conclusions have been made on the proposed adaptive observer-controller.

II. Dynamic Model with Friction

Consider the following joint space dynamic model for the n degree-of-freedom robot:

$$A(q)\ddot{q} + B(q,\dot{q})\dot{q} + g(q) + \tau_f = \Gamma \tag{1}$$

where Γ is the $n \times 1$ vector of joint torques, q is the $n \times 1$ vector of joint positions, $A(q)$ is the $n \times n$ inertial matrix, $B(q,\dot{q})$ is the $n \times n$ centrifugal and Coriolis matrix expressed in joint space, $g(q)$ is the $n \times 1$ vector of gravitational torques, τ_f is the $n \times 1$ vector of friction torques.

The following friction model is used in this paper [8]:

$$\tau_f = \tau_{vis}\dot{q} + \left[\tau_{cou} + \tau_{sti}\exp(-\tau_{dec}\dot{q}^2)\right]\mathrm{sgn}(\dot{q}) \tag{2}$$

where τ_{vis} denotes the diagonal coefficient matrix of viscous friction; τ_{cou} denotes the Coulomb friction-related diagonal coefficient matrix; τ_{sti} denotes the static friction-related diagonal coefficient matrix; τ_{dec} is a positive diagonal coefficient matrix corresponding to Stribeck effect; and the signum function $\mathrm{sgn}(\cdot)$ is defined as:

$$\mathrm{sgn}(\dot{q}) = \begin{cases} +1, & \dot{q} > 0 \\ 0, & \dot{q} = 0 \\ -1, & \dot{q} < 0 \end{cases} \tag{3}$$

For a non-redundant robot, the corresponding end-effector equation of motion in operational space can be expressed as [5]:

$$\Lambda(x)\ddot{x} + \Psi(x,\dot{x})\dot{x} + p(x) + f = F \tag{4}$$

where F is the $n \times 1$ operational forces vector, x is the $n \times 1$ vector describing the position and orientation of the end-effector, $\varLambda(x)$ is the $n \times n$ kinetic energy matrix, $\varPsi(x, \dot{x})$ is the $n \times n$ centrifugal and Coriolis matrix expressed in operational space, $p(x)$ is the $n \times 1$ vector of gravitational forces, and f is the $n \times 1$ friction vector expressed in operational space.

The relationships between the components of the joint space dynamic model and those of the operational space dynamic model can be expressed as:

$$\begin{array}{lcl} \varLambda(x) & = & J^{-T}(q)A(q)J^{-1}(q) \\ \varPsi(x,\dot{x}) & = & J^{-T}(q)\left[B - A(q)J^{-1}(q)\dot{J}(q,\dot{q})\right]J^{-1}(q) \\ p(x) & = & J^{-T}(q)g(q) \\ f & = & J^{-T}(q)\tau_f \\ \varGamma & = & J^{T}(q)F \end{array} \tag{5}$$

where $J(q)$ is the basic Jacobian of the robot.

For the proposed observer-controller stability analysis, the following properties of the robot dynamic model need to be used:

Property 1-The $n \times n$ kinetic energy matrix $\varLambda(x)$ defined in (4) satisfies the following inequality [9]:

$$m_1\|z\|^2 \leq z^T\varLambda(x)z \leq m_2\|z\|^2 = \|\varLambda(x)\|_{i2}\|z\|^2 \quad \forall z \in \Re^n \tag{6}$$

where m_1 and m_2 are known positive scalar constants. $\|\cdot\|$ represents the standard Euclidean norm, and $\|\cdot\|_{i2}$ represents the matrix induced two norm [10].

Property 2-In joint space dynamic model (1), the centrifugal and Coriolis matrix satisfies the following relationship [11]:

$$V_m(q,y)z = V_m(q,z)y \quad \forall y,z \in \Re^n$$

Property 3-In operational space dynamic model (4), the centrifugal and Coriolis matrix $\varPsi(x,\dot{x})$ satisfies the following relationships:

$$z^T\left[\frac{1}{2}\dot{\varLambda}(x) - \varPsi(x,\dot{x})\right]z = 0 \quad \forall z \in \Re^n \tag{7}$$

$$\varPsi(x,y)z = \varPsi(x,z)y \quad \forall y,z \in \Re^n \tag{8}$$

and

$$\|\varPsi(x,\dot{x})\|_{i\infty} \leq \zeta_c\|\dot{x}\| \tag{9}$$

where ζ_c is a known positive scalar constant and $\|\cdot\|_{i\infty}$ represents the matrix induced infinity norm [10].

Property 4-Properties of friction model:

We assume that the friction term τ_f in (2) is uncoupled among the joints, so that

$$\tau_f = \text{vec}\{\tau_{f_i}(\dot{q}_i)\} \equiv \begin{bmatrix} \tau_{f_1}(\dot{q}_1) \\ \cdot \\ \cdot \\ \cdot \\ \tau_{f_n}(\dot{q}_n) \end{bmatrix} \tag{10}$$

with $\tau_{f_i}(\cdot)$ known scalar functions that may be determined for any given arm. Here we have defined the vec$\{\cdot\}$ function for future use.

We assume that the viscous frictions have the form:

$$\tau_{vis}\dot{q} = \text{vec}\{\tau_{vis_i}\dot{q}_i\} \tag{11}$$

with τ_{vis_i} constant coefficients. Then τ_{vis} =diag$\{\tau_{vis_i}\}$, a diagonal matrix with entries τ_{vis_i}.

The viscous friction term has the following property [12]:

$$\|\tau_{vis}\dot{q}\| \leq \zeta_v\|\dot{q}\| \tag{12}$$

where ζ_v is a positive scalar.

The relationship between a robot end-effector velocity $\dot{x}$ and joint velocity $\dot{q}$ can be expressed as:

$$\dot{x} = J(q)\dot{q}$$

For a non-redundant robot, in the non-singular region, the joint velocity $\dot{q}$ can be obtained by:

$$\dot{q} = J^{-1}(q)\dot{x} \tag{13}$$

and from (13) we can get [10]:

$$\|\dot{q}\| \leq \|J^{-1}\|_{i2}\|\dot{x}\|$$

thus the following result can be obtained:

$$\|\tau_{vis}\dot{q}\| \leq \zeta_e\|\dot{x}\| \tag{14}$$

where ζ_e is defined as:

$$\zeta_e = \zeta_v\|J^{-1}\|_{i2} \tag{15}$$

We will use this property for our controller development.

Assume that the Coulomb friction has the form:

$$\tau_{cou}\text{sgn}(\dot{q}) = \text{vec}\{\tau_{cou_i}\text{sgn}(\dot{q}_i)\} \tag{16}$$

with τ_{cou_i} constant coefficients, and τ_{cou} =diag$\{\tau_{cou_i}\}$.

The friction term $\tau_{sti}\exp(-\tau_{dec}\dot{q}^2)\text{sgn}(\dot{q})$ in (2) is the combination of static friction and Stribeck effect, and we assume that it can be written in the following form [8]:

$$\tau_{sti}\exp(-\tau_{dec}\dot{q}^2)\text{sgn}(\dot{q}) = \text{vec}\{\tau_{sti_i}\exp(-\tau_{dec_i}\dot{q}_i^2)\text{sgn}(\dot{q}_i)\} \tag{17}$$

with τ_{sti_i} and τ_{dec_i} constant coefficients, and τ_{sti} =diag$\{\tau_{sti_i}\}$, τ_{dec} =diag$\{\tau_{dec_i}\}$.

The joint space robot friction model (2) can be written in the following linearity-in-the-parameters form:

$$\tau_f = W_j(\dot{q})\theta \tag{18}$$

where $W_j(\dot{q})$ is the $n \times 3n$ regression vector given by:

$$\begin{array}{lcl} W_j(\dot{q}) & = & \left[\begin{array}{ccc} w_{j1}(\dot{q}) & w_{j2}(\dot{q}) & w_{j3}(\dot{q}) \end{array} \right] \\ w_{j1}(\dot{q}) & = & \text{diag}(\dot{q}) \\ w_{j2}(\dot{q}) & = & \text{diag}(\text{sgn}(\dot{q})) \\ w_{j3}(\dot{q}) & = & \text{diag}(\text{sgn}(\dot{q})\exp(-\tau_{dec}\dot{q}^2)) \end{array} \quad (19)$$

and θ is the $3n \times 1$ vector of constant parameters defined as:

$$\theta = \left[\begin{array}{ccc} \text{vec}^T\{\tau_{vis_i}\} & \text{vec}^T\{\tau_{cou_i}\} & \text{vec}^T\{\tau_{sti_i}\} \end{array} \right]^T \quad (20)$$

Here we assume that the coefficients τ_{vis}, τ_{cou} and τ_{sti} are unknown constants, but the Stribeck parameters τ_{dec} are assumed to be known.

III. Adaptive Observer-Controller Formulation

Our proposed adaptive observer-controller consists of a model-based velocity observer, an operational space controller, plus friction adaptation law.

A. Formulation of Operational Space Velocity Observer

In our velocity observer formulation, the following second order velocity observer is utilized to estimate the end-effector velocity:

$$\dot{\hat{x}} = y + k\tilde{x}, y(0) = -k\tilde{x}(0) \quad (21)$$

$$\dot{y} = \Lambda(x)^{-1}\left[F - \Psi(x, \dot{\hat{x}})\dot{\hat{x}} - p(x) - \hat{f} + k_i\tilde{x}\right] \quad (22)$$

where

$$\tilde{x} = x - \hat{x} \quad (23)$$

y is a $n \times 1$ auxiliary variable, F is the force control input to the observer, it is the force generated by the controller indicated in (32). $\hat{f}$ is the estimated friction term given later in (31). k_i is a positive scalar constant to be decided. k is a positive scalar constant defined by:

$$k = \frac{1}{m_1}\left[\zeta_c\zeta_d + \zeta_c k_0 + \zeta_c k_s k_0 + k_s + 2k_n + \zeta_e\right] \quad (24)$$

where k_0, k_s and k_n being positive scalar control gains, ζ_c is declared in (9), ζ_e is defined in (15), and ζ_d is a known positive scalar constant defined by:

$$\|\dot{x}_d\| \leq \zeta_d \quad (25)$$

where $\dot{x}_d$ represents the desired end-effector velocity.

B. Formulation of Friction Adaptation Law

The friction parameter estimate vector $\hat{\theta}$ is updated using the following adaptation algorithm:

$$\dot{\hat{\theta}} = -K_{ad}W_j^T(\dot{\hat{q}})\dot{\tilde{q}} \quad (26)$$

where K_{ad} is a $3n \times 3n$ diagonal, positive-definite, adaptation gain matrix; the joint velocity observation error $\dot{\tilde{q}}$ is defined as the difference between the actual joint velocity and the observed joint velocity:

$$\dot{\tilde{q}} = \dot{q} - \dot{\hat{q}} \quad (27)$$

In the non-singular region of a robot, the observed joint velocity $\dot{\hat{q}}$ can be obtained by:

$$\dot{\hat{q}} = J^{-1}(q)\dot{\hat{x}} \quad (28)$$

where the observed end-effector velocity $\dot{\hat{x}}$ is calculated by (21).

$W_j(\dot{\hat{q}})$ is the $n \times 3n$ regression vector given by:

$$\begin{array}{lcl} W_j(\dot{\hat{q}}) & = & \left[\begin{array}{ccc} \hat{w}_{j1}(\dot{\hat{q}}) & \hat{w}_{j2}(\dot{\hat{q}}) & \hat{w}_{j3}(\dot{\hat{q}}) \end{array} \right] \\ \hat{w}_{j1}(\dot{\hat{q}}) & = & \text{diag}(\dot{\hat{q}}) \\ \hat{w}_{j2}(\dot{\hat{q}}) & = & \text{diag}(\text{sgn}(\dot{\hat{q}})) \\ \hat{w}_{j3}(\dot{\hat{q}}) & = & \text{diag}(\text{sgn}(\dot{\hat{q}})\exp(-\tau_{dec}\dot{\hat{q}}^2)) \end{array} \quad (29)$$

and the estimated joint frictions are obtained by:

$$\hat{\tau}_f = W_j(\dot{\hat{q}})\hat{\theta} \quad (30)$$

From (30), the estimated frictions in operational space can be obtained by:

$$\hat{f} = J^{-T}\hat{\tau}_f \quad (31)$$

C. Formulation of Operational Space Controller

By using the estimated velocity $\dot{\hat{x}}$ proposed in section III-A, the following model-based controller is formulated to generate the required driving force:

$$F = (k_s + k_{nd})\eta_p + w_e - k_i\tilde{x} \quad (32)$$

where k_{nd} is a positive controller gain defined as:

$$k_{nd} = 2k_n + \zeta_c k_0 + (k_s m_2 + k m_2)^2 k_n \quad (33)$$

the $n \times 1$ observed filtered tracking error signal η_p is defined as:

$$\eta_p = \dot{x}_d + k_s e - \dot{\hat{x}} \quad (34)$$

and the $n \times 1$ auxiliary vector w_e is defined as:

$$w_e = \Lambda(x)[\ddot{x}_d + k_s(\dot{x}_d - \dot{\hat{x}})] + \Psi(x, \dot{\hat{x}})(\dot{x}_d + k_s e) + p(x) + \hat{f} \quad (35)$$

where the $n \times 1$ end-effector position and orientation tracking error e is defined as:

$$e = x_d - x \quad (36)$$

The force command F will be used in the observer indicated by (22). And the torque commands for driving the robot can be obtained by:

$$\Gamma = J^T(q)F$$

IV. Overall System Stability Result and Analysis

Theorem 1: Under the assumption that the exact model of a robot except friction is known, if the observer-controller gains satisfy the following sufficient conditions :

$$\begin{array}{ccc} k_s & > & 1/k_n + \eta_e \\ k_0 & > & \|err(0)\| \end{array} \tag{37}$$

the closed-loop tracking error system is asymptotically stable as illustrated by:

$$\lim_{t\to\infty} e(t) = 0, \lim_{t\to\infty} \dot{e}(t) = 0 \tag{38}$$

$$\lim_{t\to\infty} \tilde{x}(t) = 0, \lim_{t\to\infty} \dot{\tilde{x}}(t) = 0 \tag{39}$$

and

$$\lim_{t\to\infty} \hat{\theta}(t) = \theta, \tag{40}$$

where

$$err = \begin{bmatrix} \eta_p^T & e^T & \dot{\tilde{x}}^T & \tilde{x}^T \end{bmatrix}^T \in \Re^{4n} \tag{41}$$

We will now present the proof using Lyapunov stability analysis. To determine the stability of the overall closed-loop control system, we define the following Lyapunov function:

$$V = V_0 + V_1 + V_2 \tag{42}$$

where the three sub-Lyapunov functions V_0, V_1, and V_2 are defined as:

$$V_0 = \frac{1}{2}\dot{\tilde{x}}^T \Lambda(x)\dot{\tilde{x}} + \frac{1}{2}\tilde{x}^T k_i \tilde{x} + \frac{1}{2}\tilde{\theta}^T K_{ad}^{-1}\tilde{\theta} \tag{43}$$

where $\tilde{\theta}$ is the difference between the actual θ and the estimated $\hat{\theta}$.

$$V_1 = \frac{1}{2}e^T e \tag{44}$$

$$V_2 = \frac{1}{2}\eta_p^T \Lambda(x)\eta_p \tag{45}$$

Hence, $\dot{V}$ can be obtained by:

$$\dot{V} = \dot{V}_0 + \dot{V}_1 + \dot{V}_2 \tag{46}$$

We will formulate the bound of $\dot{V}_0$, $\dot{V}_1$ and $\dot{V}_2$ separately, and then combine them together to get the bound of $\dot{V}$.

A. Lyapunov Function for Observation Error $\tilde{x}$, $\dot{\tilde{x}}$ and $\tilde{\theta}$

In section IV, V_0 was defined as:

$$V_0 = \frac{1}{2}\dot{\tilde{x}}^T \Lambda(x)\dot{\tilde{x}} + \frac{1}{2}\tilde{x}^T k_i \tilde{x} + \frac{1}{2}\tilde{\theta}^T K_{ad}^{-1}\tilde{\theta}$$

where the velocity observation error $\dot{\tilde{x}}$ is obtained by differentiating (23) with respect to time:

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}} \tag{47}$$

To form the bound of $\dot{V}_0$, first, take the time derivative of (21) and then substitute (22) into the resulting expression to yield:

$$\Lambda(x)\ddot{\hat{x}} + \Psi(x,\dot{\hat{x}})\dot{\hat{x}} + p(x) + \hat{f} - k\Lambda(x)\dot{\tilde{x}} - k_i\tilde{x} = F \tag{48}$$

Subtract (48) from (4), use (8) and (47) to yield the following closed-loop observer error system:

$$\Lambda(x)\ddot{\tilde{x}} + \Psi(x,\dot{x})\dot{\tilde{x}} + \Psi(x,\dot{\hat{x}})\dot{\tilde{x}} + k\Lambda(x)\dot{\tilde{x}} + k_i\tilde{x} + f - \hat{f} = 0 \tag{49}$$

Differentiate V_0 along (49) to get:

$$\begin{aligned} \dot{V}_0 = \; & \dot{\tilde{x}}^T \left[-\Psi(x,\dot{x})\dot{\tilde{x}} - \Psi(x,\dot{\hat{x}})\dot{\tilde{x}} - k\Lambda(x)\dot{\tilde{x}} - k_i\tilde{x} \right] \\ & + \frac{1}{2}\dot{\tilde{x}}^T \dot{\Lambda}(x)\dot{\tilde{x}} + \dot{\tilde{x}}^T k_i \tilde{x} - \dot{\tilde{x}}^T (f - \hat{f}) \end{aligned} \tag{50}$$

Utilizing (7) to get:

$$\dot{V}_0 = -\dot{\tilde{x}} \left[\Psi(x,\dot{\hat{x}}) + k\Lambda(x) \right] \dot{\tilde{x}} - \dot{\tilde{x}}^T (f - \hat{f}) \tag{51}$$

Then utilize (6) and (9) to get the upper bound of $\dot{V}_0$:

$$\dot{V}_0 \leq \left(\zeta_c \|\dot{\hat{x}}\| - km_1 \right) \|\dot{\tilde{x}}\|^2 + \zeta_e \|\dot{\tilde{x}}\|^2 \tag{52}$$

Substitute for $\dot{\hat{x}}$ from (34) into (52), and utilize (25) to get the new upper bound for $\dot{V}_0$:

$$\dot{V}_0 \leq \left(\zeta_c\zeta_d + \zeta_c\|\eta_p\| + \zeta_c k_s \|e\| - km_1 \right) \|\dot{\tilde{x}}\|^2 + \zeta_e \|\dot{\tilde{x}}\|^2 \tag{53}$$

B. Lyapunov Function for Tracking Error e

In section IV, V_1 was defined as:

$$V_1 = \frac{1}{2}e^T e$$

The position tracking error system can be formed by differentiating (36) with respect to time to yield:

$$\dot{e} = \dot{x}_d - \dot{x}$$

Since $\dot{x}$ is not measurable, we use the estimated term $\dot{\hat{x}}$ to eliminate $\dot{x}$ and get the following equation:

$$\dot{e} = \dot{x}_d - \dot{\hat{x}} - \dot{\tilde{x}} \tag{54}$$

Simplify (54) by utilizing (34) to get:

$$\dot{e} = -k_s e + \eta_p - \dot{\tilde{x}} \tag{55}$$

The upper bound for the time derivative of V_1 along (55) is given by:

$$\dot{V}_1 \leq -k_s\|e\|^2 + \|e\|\|\eta_p\| + \|e\|\|\dot{\tilde{x}}\| \tag{56}$$

C. Lyapunov Function for η_p

The tracking error system for η_p can be formed by differentiating (34) with respect to time, multiplying both sides of the resulting expression by $\Lambda(x)$, and substituting the right-hand side of (48) for $\ddot{\tilde{x}}$ to yield:

$$\begin{aligned} \Lambda(x)\dot{\eta}_p &= \Lambda(x)\ddot{x}_d + k_s\Lambda(x)(\dot{x}_d - \dot{x}) - k\Lambda(x)\dot{\tilde{x}} \\ &\quad -k_i\tilde{x} + \Psi(x,\dot{\hat{x}})\dot{\hat{x}} + p(x) - F \end{aligned} \tag{57}$$

Substitute the force input given by (32) into (57), use the definitions of w_e and η_p to get:

$$\begin{aligned} \Lambda(x)\dot{\eta}_p &= -(k_s + k_{nd})\eta_p - (k + k_s)\Lambda(x)\dot{\tilde{x}} \\ &\quad -\Psi(x,\dot{\hat{x}})\eta_p \end{aligned} \tag{58}$$

Rewrite the term $\Psi(x,\dot{\hat{x}})\eta_p$ on the right-hand side of (58) in terms of $\dot{\tilde{x}}$, and utilize (8) and (47) to yield:

$$\begin{aligned} \Lambda(x)\dot{\eta}_p &= -\Psi(x,\dot{x})\eta_p - (k_s + k_{nd})\eta_p \\ &\quad -(k_s + k_{nd})\Lambda(x)\dot{\tilde{x}} + \Psi(x,\dot{\tilde{x}})\eta_p \end{aligned} \tag{59}$$

In section IV, V_2 was defined as:

$$V_2 = \frac{1}{2}\eta_p^T\Lambda(x)\eta_p$$

Differentiate V_2 along (59), and utilize (7) to get:

$$\begin{aligned} \dot{V}_2 &= -(k_s + k_{nd})\eta_p^T\eta_p - (k + k_s)\eta_p\Lambda(x)\dot{\tilde{x}} \\ &\quad +\eta_p^T\Psi(x,\dot{\tilde{x}})\eta_p \end{aligned} \tag{60}$$

From (60), utilize (6) and (9), we can obtain the following upper bound for $\dot{V}_2$:

$$\begin{aligned} \dot{V}_2 &\leq -(k_s + k_{nd})\|\eta_p\|^2 + (k + k_s)m_2\|\|\eta_p\|\dot{\tilde{x}}\| \\ &\quad +\zeta_c\|\eta_p\|^2\|\dot{\tilde{x}}\| + \zeta_e\|\dot{\tilde{x}}\|^2 \end{aligned} \tag{61}$$

D. Overall System Stability Analysis

Use the upper bound of $\dot{V}_0$, $\dot{V}_1$ and $\dot{V}_2$, and utilize (24), (33), and (41), we can form the upper bound on $\dot{V}$:

$$\begin{aligned} \dot{V} \leq\ & -k_s\|e\|^2 - k_s\|\eta_p\|^2 - k_s\left\|\dot{\tilde{x}}\right\|^2 \\ & + \|\eta_p\|\left(\|e\| - 2k_n\|\eta_p\|\right) + \left\|\dot{\tilde{x}}\right\|\left(\|e\| - 2k_n\left\|\dot{\tilde{x}}\right\|\right) \\ & + (k + k_s)\, m_2\|\eta_p\|\left(\left\|\dot{\tilde{x}}\right\| - (k + k_s)\, m_2 k_n\|\eta_p\|\right) \\ & + \left(\zeta_c\zeta_d\left\|\dot{\tilde{x}}\right\|^2 - \zeta_c\zeta_d\left\|\dot{\tilde{x}}\right\|^2\right) + \zeta_e\|\dot{\tilde{x}}\|^2 \\ & - (k_o - \|err\|)\left(\zeta_c\left\|\dot{\tilde{x}}\right\|^2 + \zeta_c k_s\left\|\dot{\tilde{x}}\right\|^2 + \zeta_c\|\eta_p\|^2\right) \end{aligned} \tag{62}$$

where we have used the fact derived from (41) that $\|err\| \geq \|e\|, \|\eta\|$, and $\|\dot{\tilde{x}}\|$.

By applying the nonlinear damping tool [13] on the terms in the second and third lines on the right hand side of (62), a new upper bound on $\dot{V}$ can be formed as:

$$\begin{aligned} \dot{V} \leq\ & -(k_s - \tfrac{1}{k_n})\|e\|^2 - k_s\|\eta_p\|^2 - (k_s - \tfrac{1}{k_n} - \zeta_e)\left\|\dot{\tilde{x}}\right\|^2 \\ & - (k_o - \|err\|)\left(\zeta_c\left\|\dot{\tilde{x}}\right\|^2 + \zeta_c k_s\left\|\dot{\tilde{x}}\right\|^2 + \zeta_c\|\eta_p\|^2\right) \end{aligned} \tag{63}$$

From (63) we can see that, if $k_s > 1/k_n + \zeta_e$ and $k_0 \geq ||err||$, we can get:

$$\dot{V} \leq 0 \tag{64}$$

From (64), we can get the conclusion that, friction coefficients estimation error $\tilde{\theta}$, position tracking errors e, position estimation errors $\tilde{x}$, velocity estimation errors $\dot{\tilde{x}}$, observed filtered tracking error signal η_p of the observer-controller are all asymptotically stable, i.e. $\lim\limits_{t\to\infty}\hat{\theta} = \theta$, $\lim\limits_{t\to\infty} e(t) = 0$, $\lim\limits_{t\to\infty}\tilde{x}(t) = 0$, $\lim\limits_{t\to\infty}\dot{\tilde{x}}(t) = 0$ and $\lim\limits_{t\to\infty}\eta_p(t) = 0$. Furthermore, the end-effector velocity tracking error is also asymptotically stable, i.e. $\lim\limits_{t\to\infty}\dot{e}(t) = 0$. In fact, after adding and subtracting $\dot{x}$ to the right-hand side of (34) and rearranging the terms, we can formulate the following inequality:

$$\|\dot{e}\| = \|\dot{x}_d - \dot{x}\| \leq \|\eta_p\| + k_s\|e\| + \|\dot{x}\| \tag{65}$$

Since each of the terms on the right-hand side of the above equation is asymptotically stable, it is easy to see that $\|\dot{e}\|$ is also asymptotically stable. This yields the result indicated by *Theorem 1*.

V. Implementation of Friction Adaptation Law

The friction adaptation law indicated by (26) needs to use the difference between the actual joint velocity and the observed joint velocity. Since we assume that the actual joint velocity is unknown, in order to implement this friction adaptation law, both sides of (26) are integrated, giving:

$$\hat{\theta}(t) = \hat{\theta}(t - \Delta t) - K_{ad}\int_{t-\Delta t}^{t} W_j^T(\dot{\hat{q}})(dq/dt - \dot{\hat{q}})dt \tag{66}$$

where Δt represents the sampling time of the system.

From (66) we can get the following form:

$$\begin{aligned} \hat{\theta}(t) &= \hat{\theta}(t - \Delta t) - K_{ad}W_j^T(\dot{\hat{q}})(t - \Delta t)D_q \\ D_q &= \left[q(t) - q(t - \Delta t) - \dot{\hat{q}}(t - \Delta t)\Delta t\right] \end{aligned} \tag{67}$$

We will use (67) as our friction adaptation algorithm.

VI. Experimental Results

The experiments were performed using PUMA 560 robot, and the sampling time is selected to be 1ms.

The defined trajectory is to move the end-effector in XYZ direction with the desired position trajectory indicated by (68), while maintaining the initial end-effector orientation constant all the time.

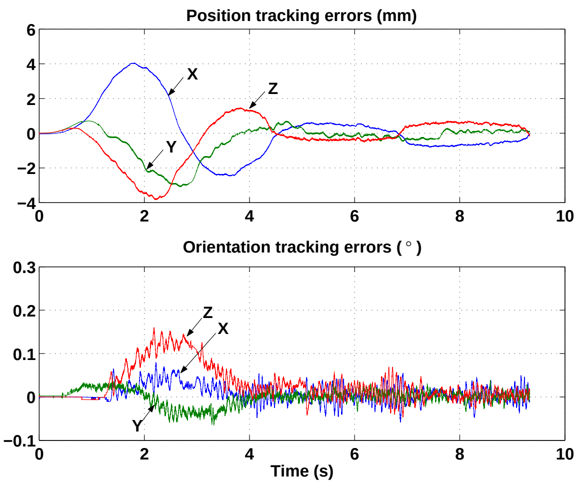

Fig. 1. Tracking errors with adaptive friction compensation

TABLE I
MAXIMUM TRACKING ERRORS WITH ADAPTIVE FRICTION COMPENSATION

e_x	e_y	e_z	e_{ϕ_x}	e_{ϕ_y}	e_{ϕ_z}
0.38mm	0.44mm	0.28mm	0.06^o	0.04^o	0.06^o

$$
\begin{aligned}
p_{x_d} &= p_{x_0} + 50.0\sin(0.4\pi t)\left(1 - e^{-0.05t^3}\right)\text{ mm} \\
p_{y_d} &= p_{y_0} + 50.0\cos(0.4\pi f t)\left(1 - e^{-0.05t^3}\right)\text{ mm} \\
p_{z_d} &= p_{z_0} + 50.0\cos(0.4\pi f t)\left(1 - e^{-0.05t^3}\right)\text{ mm}
\end{aligned} \tag{68}
$$

where p_{x_0}, p_{y_0} and p_{z_0} are the initial positions of the robot. The exponential terms are to ensure that the initial desired velocities and accelerations are all zeros.

The controller gains were selected as diagonal gains matrices as following:

$$
\begin{aligned}
k_{nd} &= \text{diag}\{120, 120, 120, 35, 35, 35\} \\
k &= \text{diag}\{108, 108, 108, 32, 32, 32\} \\
k_s &= \text{diag}\{97, 97, 97, 30, 30, 30\} \\
k_i &= \text{diag}\{2000, 2000, 2000, 3000, 3000, 3000\}
\end{aligned} \tag{69}
$$

All the diagonal terms of the 18×18 friction adaptation gains K_{ad} were selected to be 500, all the diagonal terms of τ_{dec}were selected to be 1, and all the initial estimated friction coefficients are set to zeros.

Using the trajectory defined by (68), the experimental result is shown in Fig. 1, and Tables I and II show the tracking errors and the identified friction coefficients after the robot ran for about two minutes. J_i stands for Joint i, and e_x, e_y, and e_z are the position tracking errors along X, Y, and Z axis, and e_{ϕ_x}, e_{ϕ_y}, and $e_{\phi_z}$are the orientation tracking errors about X, Y, and Z axis, respectively.

Under the same conditions, using the same controllers gains listed in (69) but without friction compensation, the result is shown in Tabel III

The results indicate that the tracking errors of the controllers with adaptive friction compensation is about 2 to 5 times smaller than the controller without friction compensation, which verify the effectiveness of the proposed adaptive controller.

TABLE II
IDENTIFIED FRICTION COEFFICIENTS OF EACH JOINT

	J_1	J_2	J_3	J_4	J_5	J_6
τ_{vis_i} (N.m.s/rad)	5.8	3.7	6.0	0.1	1.9	0.7
τ_{cou_i} (N.m)	4.9	4.8	3.0	1.0	0.6	1.2
τ_{sti_i} (N.m)	4.1	4.3	0.6	1.5	0.4	1.0

TABLE III
MAXIMUM TRACKING ERRORS WITHOUT FRICTION COMPENSATION

e_x	e_y	e_z	e_{ϕ_x}	e_{ϕ_y}	e_{ϕ_z}
1.84mm	1.58mm	1.30mm	0.11^o	0.09^o	0.19^o

VII. CONCLUSIONS

In this paper, we proposed an operational space observer-controller with adaptive friction compensation capability. The friction adaptation algorithm is designed to make use of the merits of the "cleaner" observed velocity to achieve better performance. Experimental results using PUMA 560 indicate that, the proposed adaptive controller is able to achieve higher tracking accuracy than the observer-controller without friction compensation, which verifies the effectiveness of the control algorithm.

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