1. Frame B is initially coincident to frame A in Figure 1(a). Frame B is then rotated 30 degrees about the vector described by the directed line segment from P to Q (following the right-hand rule). Determine the position and orientation of the new frame B with respect to frame A. Express your answer in the form of a homogeneous transformation matrix.

![Figure 1 (a)](image1.png)

(b) Referring to Figure 1(b), determine the homogeneous transformation matrix that describes frame C in frame A. Also determine the homogeneous transformation matrix that describes frame A in frame C.

![Figure 1(b)](image2.png)
2. Frames M and C are attached rigidly to a cuboid as shown in Fig. 3. Frame U is fixed and serves as the universe frame of reference. The cube undergoes the following motion in the indicated sequence:

1> Rotation about the z axis of Frame C by \(30^\circ\), then
2> Translation of \((1, 2, 3)\) along Frame C, then
3> Rotation about the x axis of Frame M by \(45^\circ\), and then
4> Rotation about the y axis of Frame U by \(60^\circ\).

Let \(\mathbf{T}_{C_i}\) and \(\mathbf{T}_{M_i}\) be the \(4 \times 4\) homogeneous transformation matrices that describes the position and orientation of Frames C and M, respectively, in U after motion \(i\).

Find

i. \(\mathbf{T}_{C_1}\)
ii. \(\mathbf{T}_{C_2}\)
iii. \(\mathbf{T}_{C_3}\)
iv. \(\mathbf{T}_{C_4}\)
v. \(\mathbf{T}_{M_4}\)

Ans:

\[
\mathbf{T}_{C_1} = \begin{pmatrix}
-0.866 & 0.5 & 0 & 1 \\
-0.416 & -0.721 & -0.555 & 3 \\
-0.277 & -0.48 & 0.832 & 2 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
\mathbf{T}_{C_2} = \begin{pmatrix}
-0.866 & 0.354 & 0.354 & 1.662 \\
-0.416 & -0.117 & -0.902 & -2.696 \\
-0.277 & -0.928 & 0.249 & 4.872 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
\mathbf{T}_{C_3} = \begin{pmatrix}
-0.6732 & -0.627 & 0.392 & 5.05 \\
-0.416 & -0.117 & -0.902 & -2.696 \\
0.611 & -0.77 & -0.182 & 0.997 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
\mathbf{T}_{M_4} = \begin{pmatrix}
0.6732 & 0.304 & 0.674 & 2.117 \\
0.416 & 0.598 & -0.685 & -3.535 \\
-0.611 & 0.742 & 0.276 & -1.169 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

Figure 3

Line segment lengths:

- \(AD=1\)
- \(DC=3\)
- \(DE=2\)
Frame C is firmly attached to a corner of the rigid cube with $z_C$ parallel $x_A$ and $y_C$ parallel to $z_A$, as shown in the figure below. Frame B is located at a fixed position and orientation with respect to Frame A with $x_B$ parallel to $z_A$ and the angle 60 degrees represents a rotation about $x_B$. The following ordered sequence of motions is applied to the cube:

I) rotation about $y_B$ by 45 degrees, followed by
II) rotation about $x_C$ by 30 degrees.

Find the new position and orientation of Frame C expressed in Frame A.

Ans:

$$^A\mathbf{T}_C = \begin{bmatrix}
-0.127 & -0.140 & 0.982 & 1.855 \\
0.927 & -0.370 & 0.067 & 2.339 \\
0.354 & 0.918 & 0.177 & 2.190 \\
0 & 0 & 0 & 1
\end{bmatrix}$$