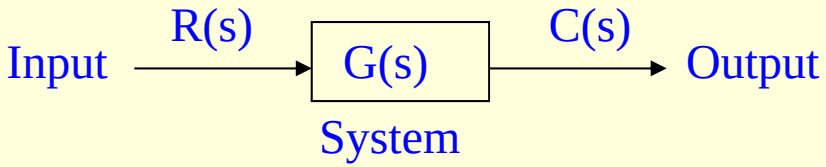
NUS National University of Singapore

# Block Diagram Algebra

# Block Diagram Representation

*A block diagram is a graphical tool* to help us ***visualize the model*** of a system and ***evaluate the mathematical relationships among their elements***, using their transfer functions.

## The Transfer Function Block



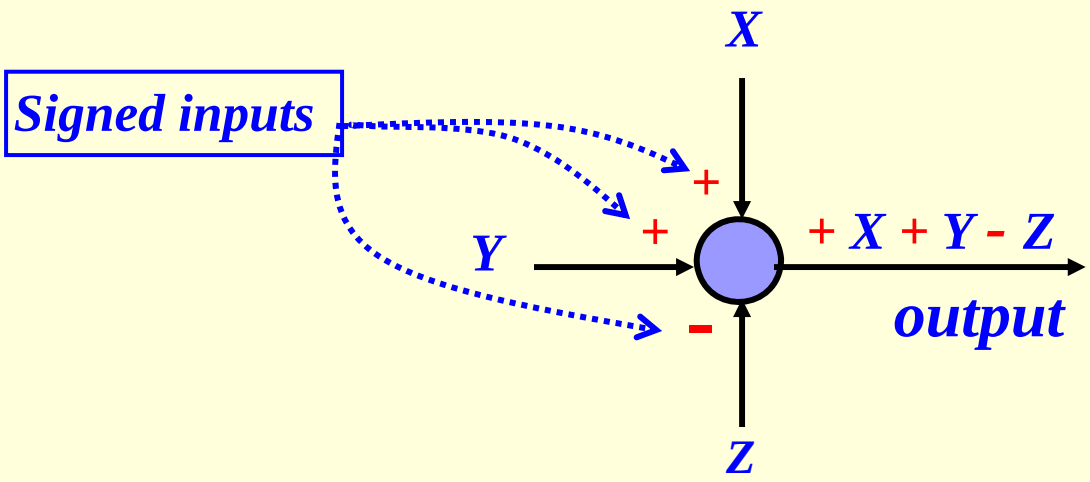

$$G(s) = \frac{C(s)}{R(s)}$$

The transfer function G(s) is

- defined only for a linear time-invariant system and not for nonlinear systems.
- Is a property of the system and is independent of the input to the system.
- Commutative $G_1 G_2 = G_2 G_1$
- Associative $G_1 + G_2 = G_2 + G_1$

# Block Diagram Elements

## The Summing Point



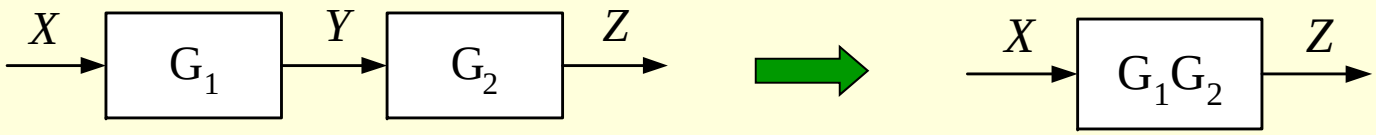

- Any number of inputs. Only one output

# Block Diagram Algebra

*The block diagram must be a true representation of the mathematical equations describing the system.*

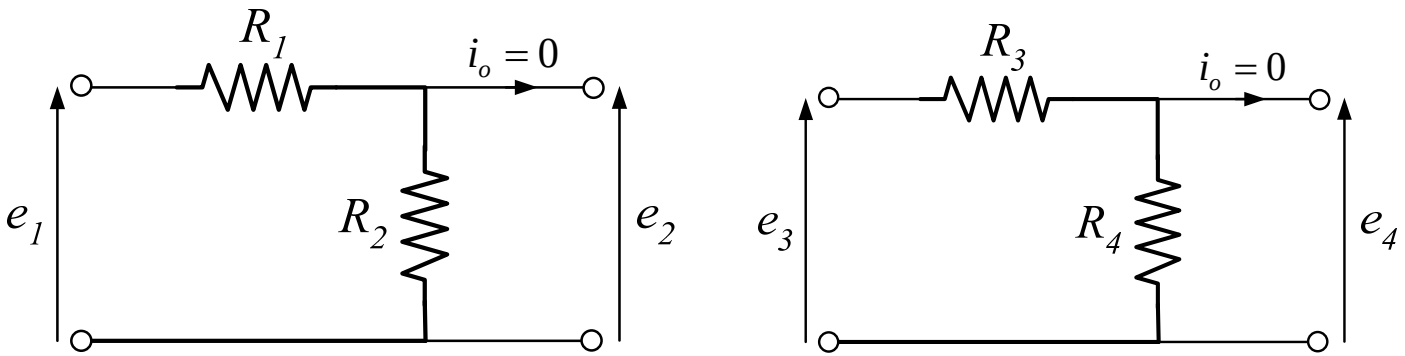
*When manipulating block diagrams, the original relationships, or equations, relating the various variables must remain the same.*

## Blocks in series or cascaded blocks

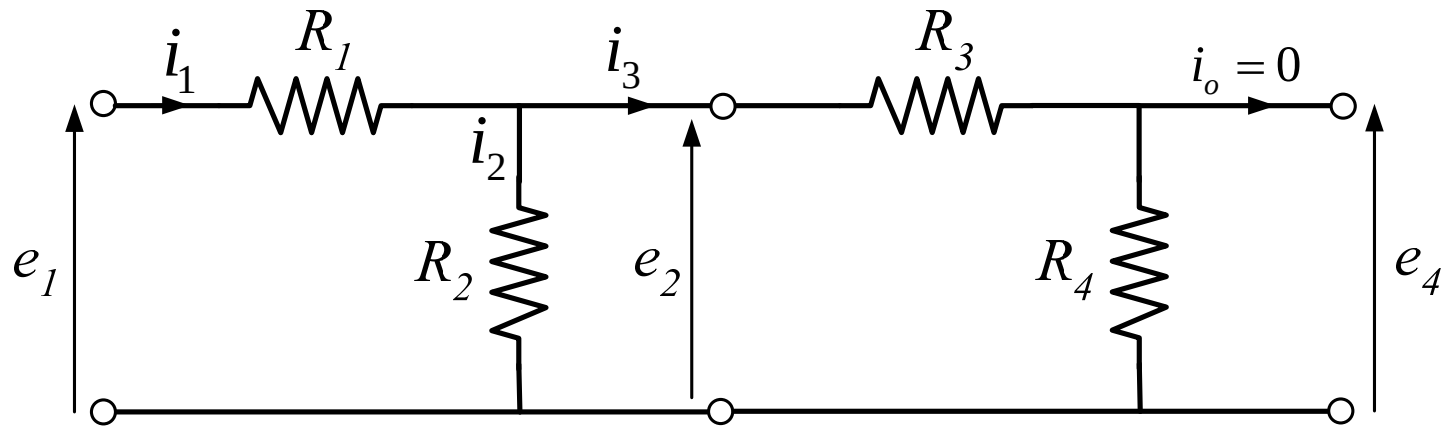



- When blocks are connected in series, there must be no loading effect.

# Example of Loading Effect

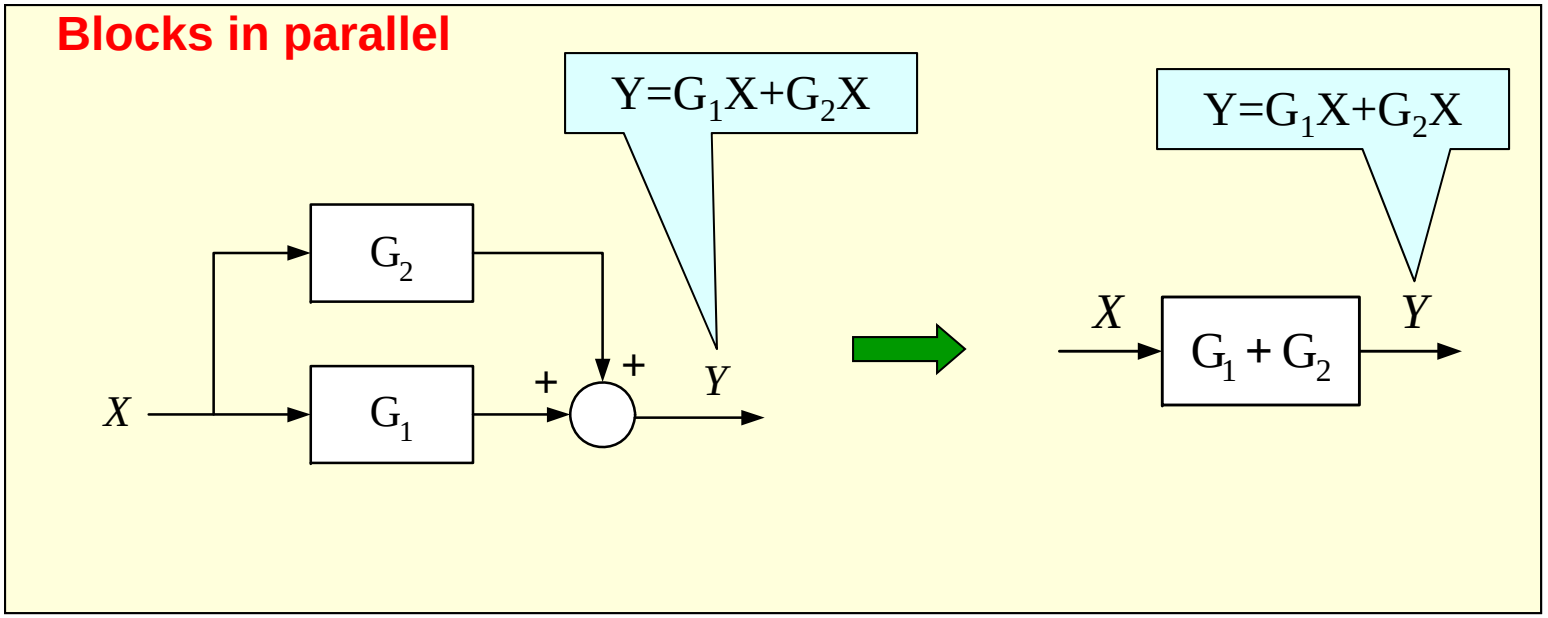



When the output of the first circuit is connected as input to the second input, the voltage e2 will drop because of loading effect.

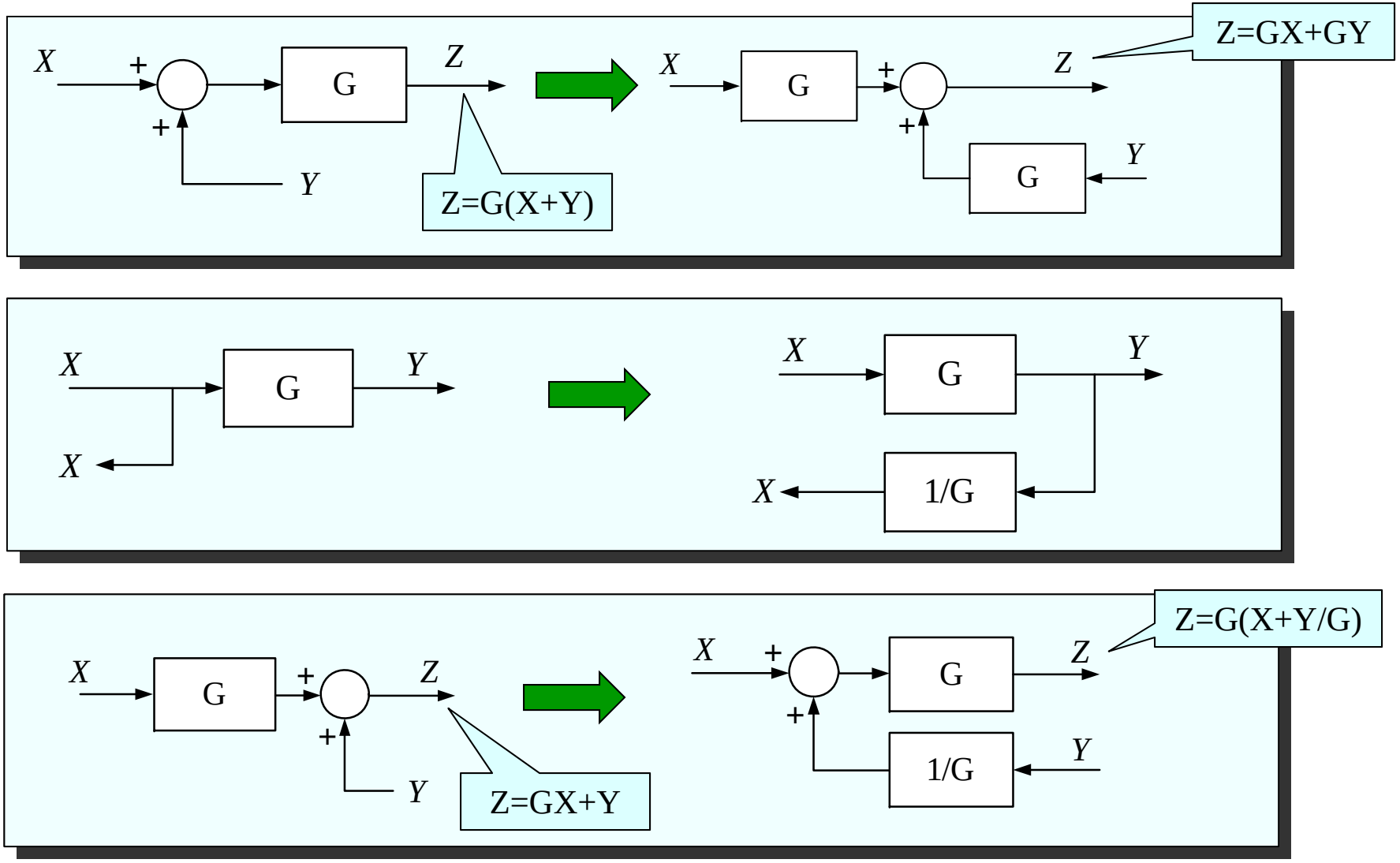


# Block Diagram Algebra

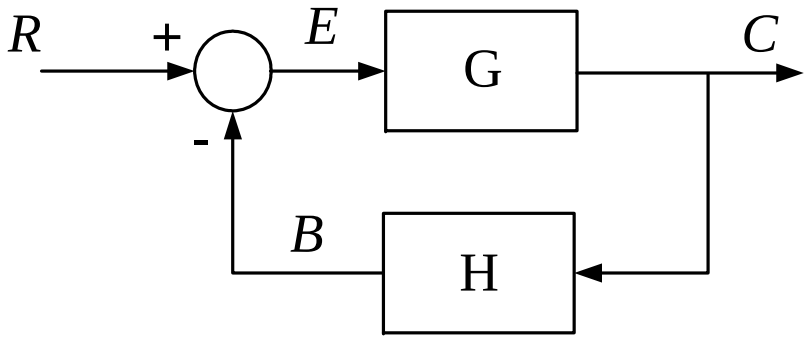
## Blocks in parallel



# Block Diagram Algebra



# Closed-Loop Feedback System

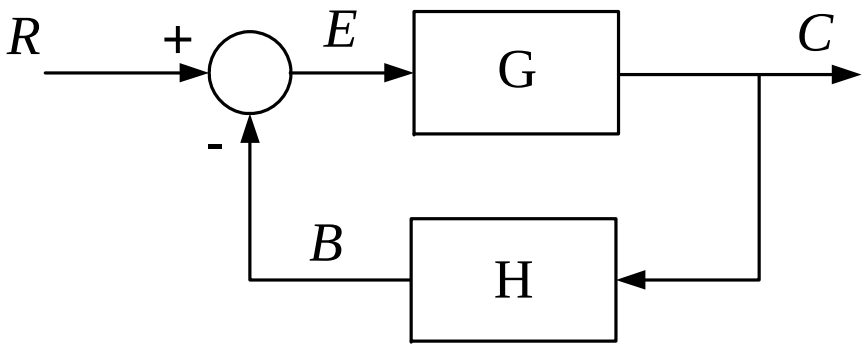



**R** is called the reference input
**C** is the output or controlled variable
**B** is the feedback
**E = (R – B)** is the error

$\frac{C}{E} = G$ is called the feedforward transfer function

$\frac{B}{E} = GH$ is called the open-loop transfer function

# Closed-Loop Feedback System




$$C = GE$$
$$= G(R - B)$$
$$= G(R - HC)$$
$$C(1 + GH) = GR$$

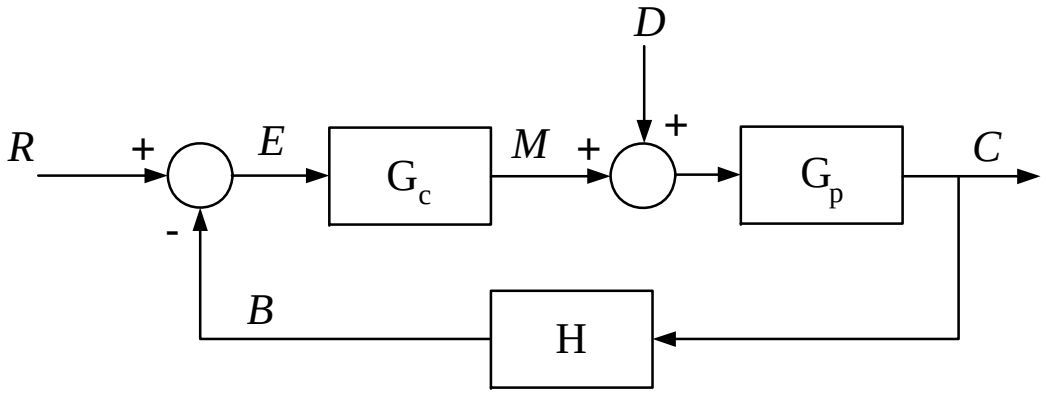
$$\boxed{\frac{C}{R} = \frac{G}{1 + GH}}$$

$\frac{C}{R}$ is the closed-loop transfer function

Also $E = \frac{C}{G}$ and $\boxed{\frac{E}{R} = \frac{1}{G}\frac{C}{R} = \frac{1}{1 + GH}}$

$\frac{E}{R}$ is called the error transfer function

# Closed-Loop Control Feedback System

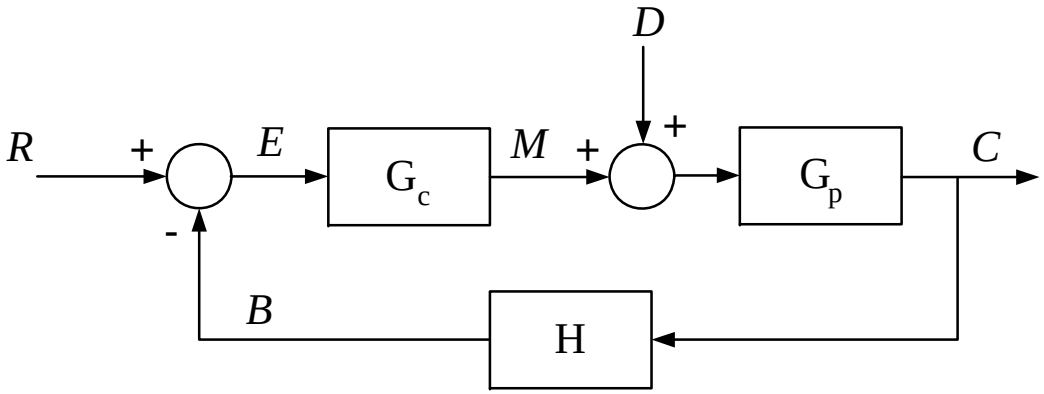



**$G_c$** is the controller transfer function
**$G_p$** is the plant transfer function
**M** is the manipulated variable
D is the external disturbance

$\frac{C}{E} = G_c G_p$ is the feedforward transfer function

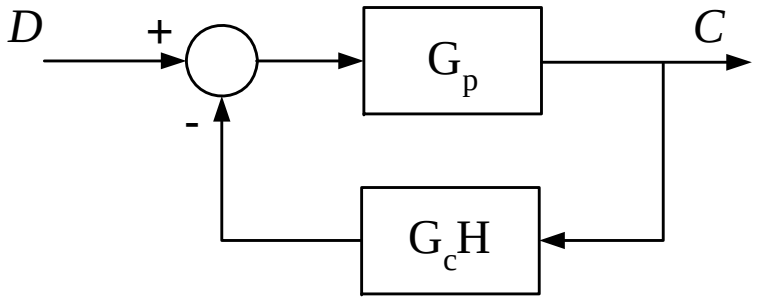
$\frac{B}{E} = G_c G_p H$ is the open-loop transfer function

# Closed-Loop Control Feedback System




$$\frac{C}{R} = \frac{G}{1+GH} = \frac{G_c G_p}{1+G_c G_p H}$$

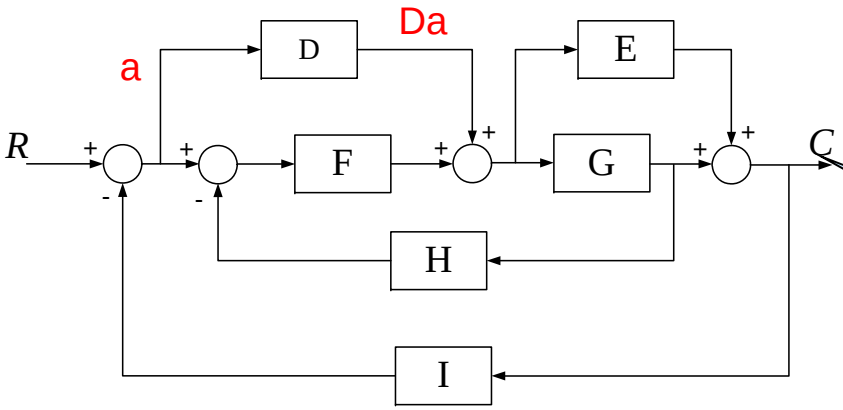
Assuming R = 0, we can re-draw




$$\frac{C}{D} = \frac{G}{1+GH} = \frac{G_p}{1+G_p G_c H}$$

# Block Diagram Manipulation

**Example:** Determine C(s)/R(s)




**When manipulating blocks, must ensure C(s) does not change, so that C(s)/R(s) remains same.**

# Block Diagram Manipulation

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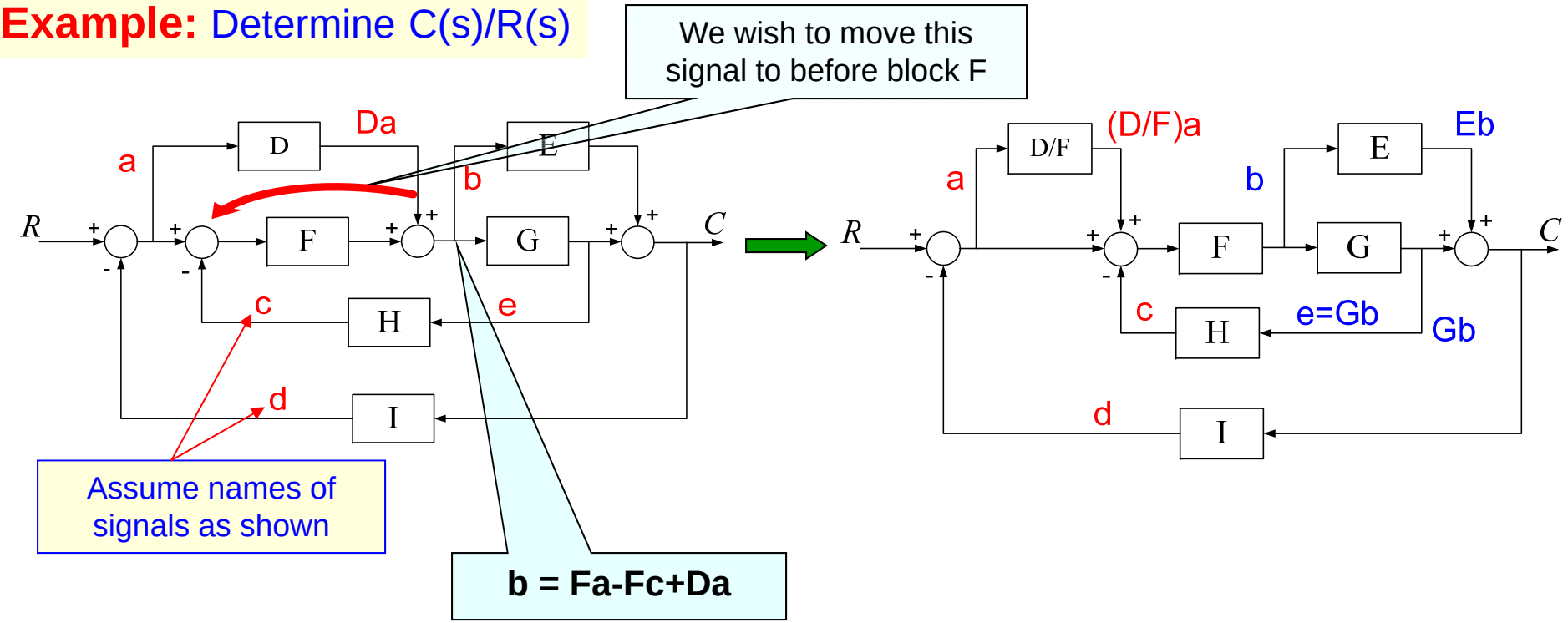

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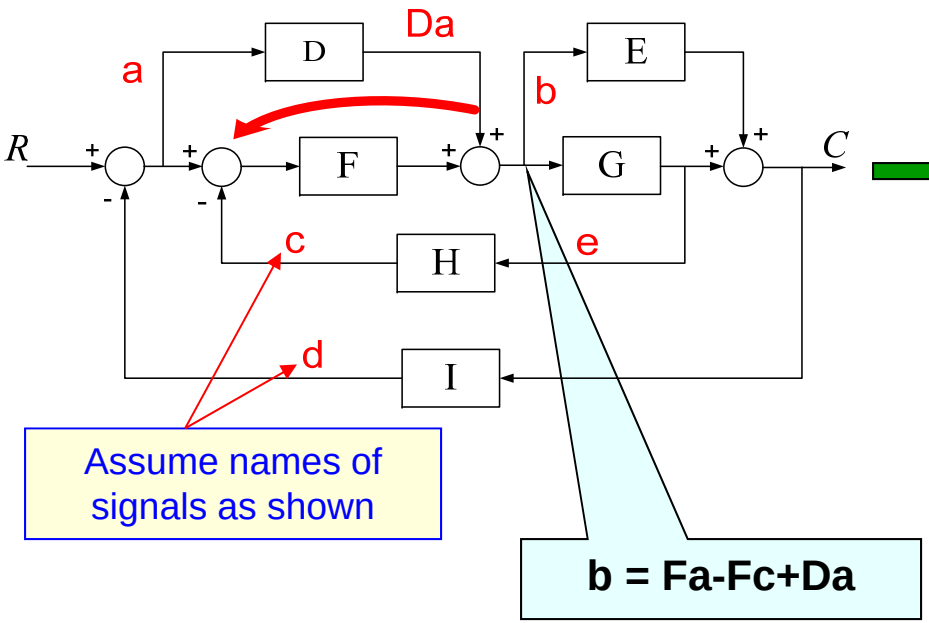


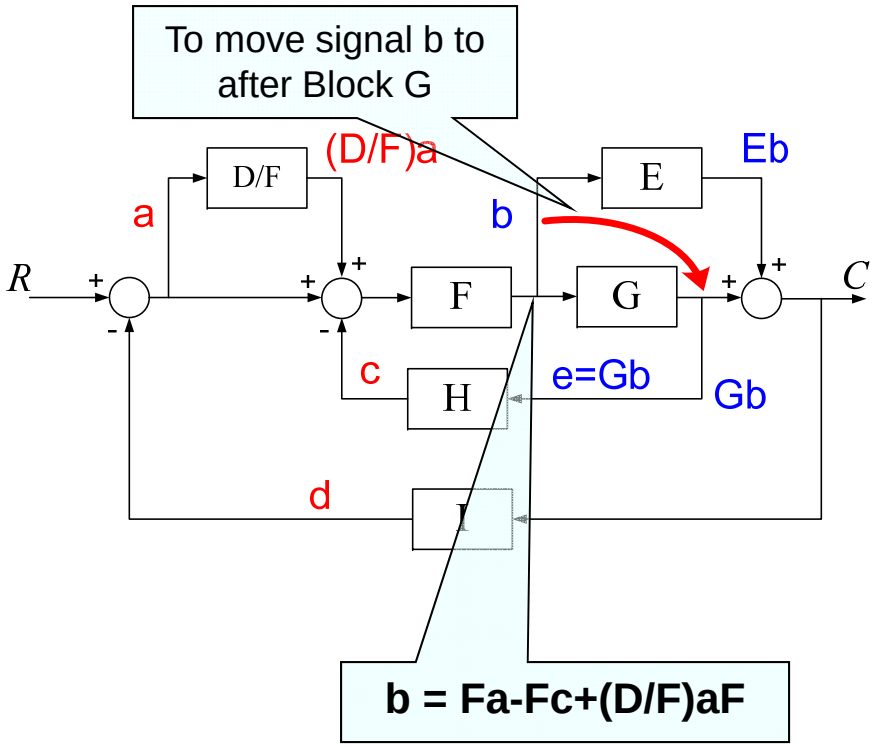

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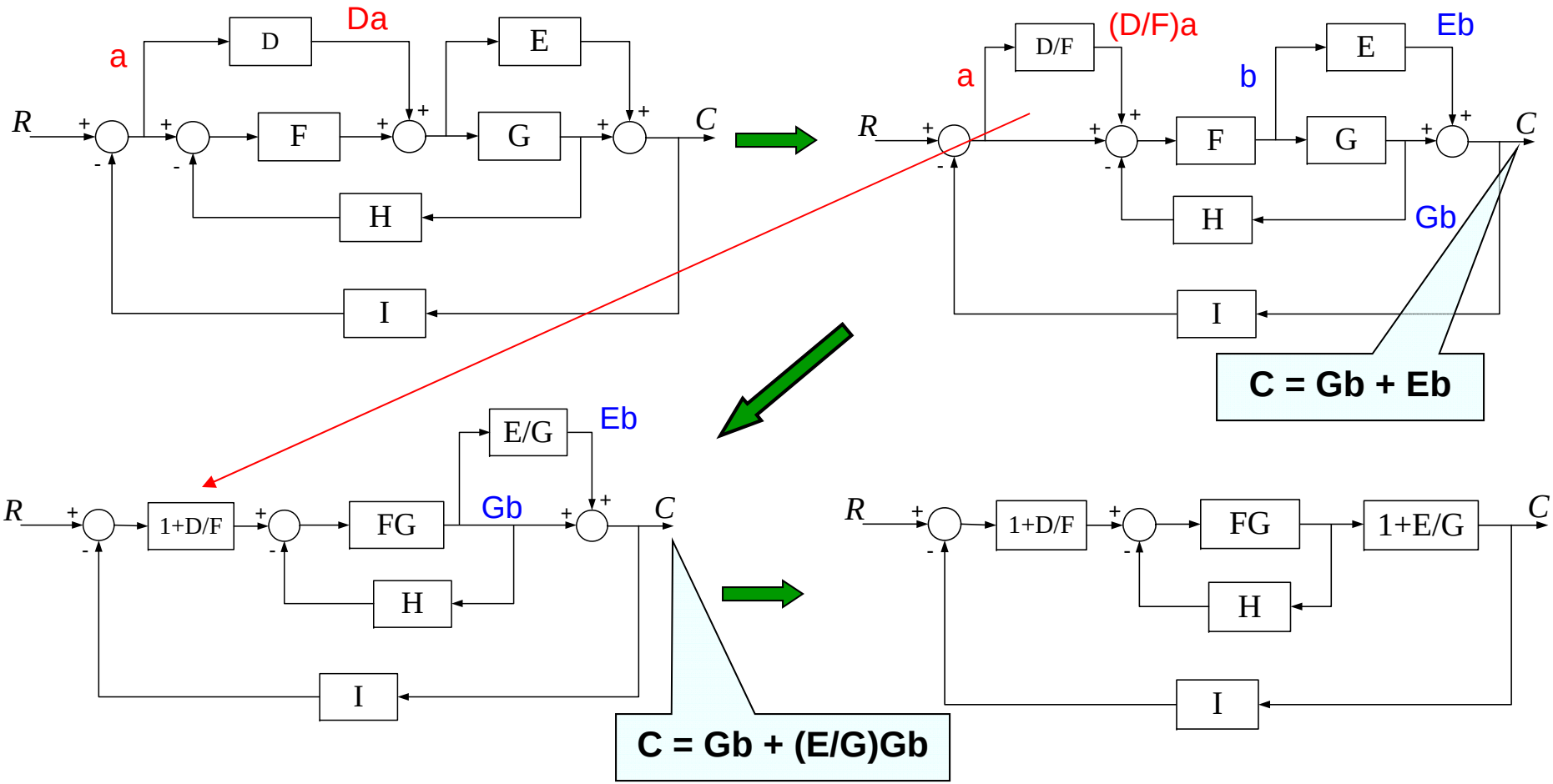
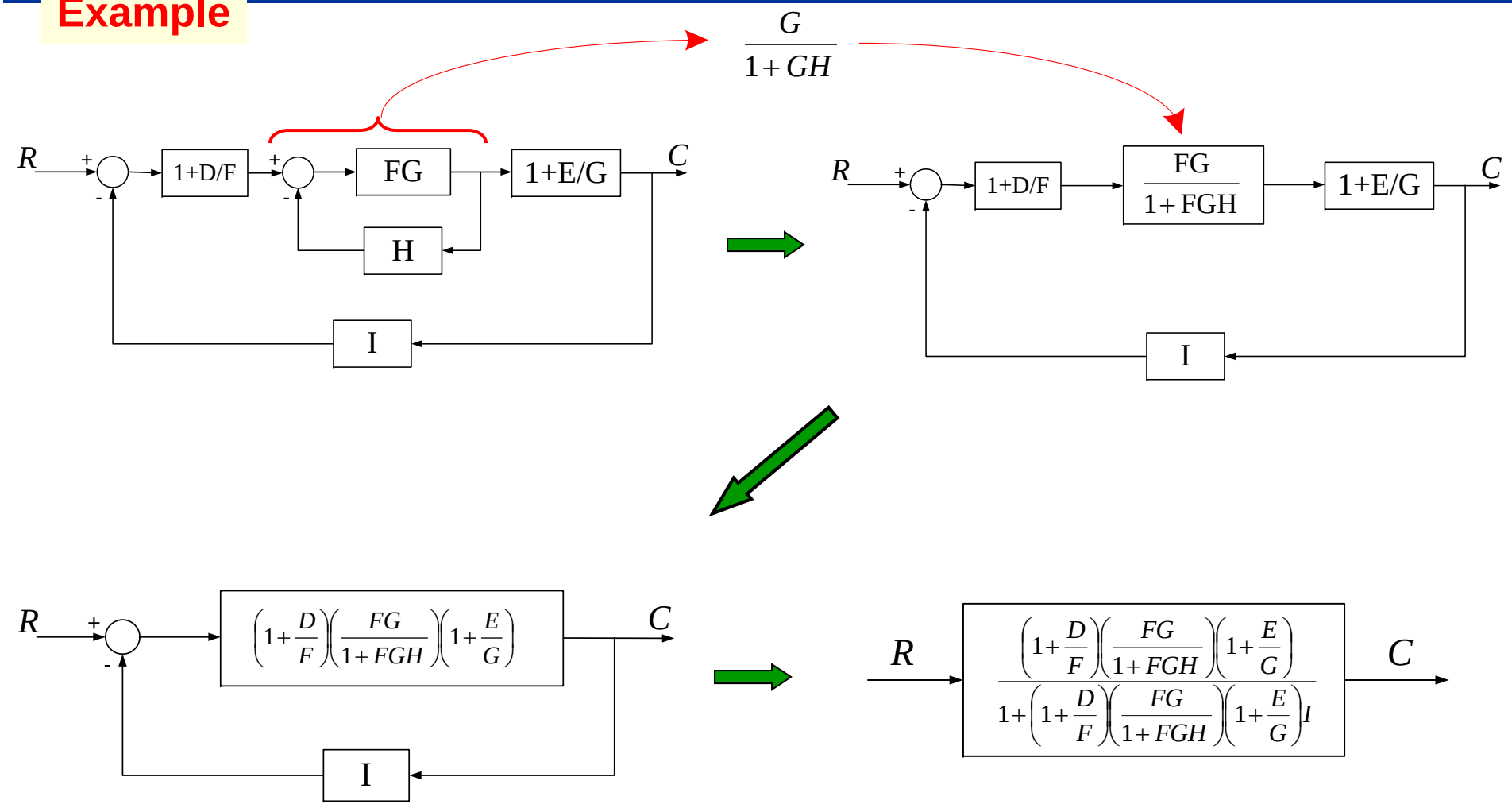


# Block Diagram Manipulation

## Example



End

End