National University of Singapore
Faculty of Engineering

Drill Problem Set 3: ME4245/EE4304: Robotics Term 1, 1996/1997

1. At an orientation of

$$R = \begin{pmatrix} -0.5 & -0.433 & 0.75 \\ 0.866 & -0.25 & 0.433 \\ 0 & 0.866 & 0.5 \end{pmatrix},$$

a rigid body is rotating at the Euler angle rates of

$$\dot{\alpha} = \dot{\beta} = \dot{\gamma} = 1^o \text{ / sec}.$$

Determine the angular velocity of the rigid body. The Euler angles α, β, and γ are defined as $R = Rot(z, \alpha)Rot(y, \beta)Rot(z, \gamma)$. (*Hint: You may need to first solve for the Euler angles corresponding to this orientation.)*

Two Possible Answers:

$$\omega = \begin{pmatrix} 0.25 \\ 1.3 \\ 1.5 \end{pmatrix} \text{deg/ sec} \qquad or \qquad \omega = \begin{pmatrix} 1.25 \\ -0.433 \\ 1.5 \end{pmatrix} \text{deg/ sec}$$

2 Fig. 2 shows a connecting (bent) rod (BJ) rigidly connecting the rotational joint J to the cuboid (ABCDEFGO). Let

1> Frame U be fixed and serve as the universe frame of reference.
2> Frame H be **translating** with respect to Frame U.
3> Frame H is attached rigidly to the rotational joint J.
3> Axis **k** be fixed onto Frame H.
4> Frame M be fixed rigidly onto the cuboid.
5> The cuboid (ABCDEFGO), bent connecting rod (BJ), and joint J be all connected as one rigid assembly.
6> Joint J be a rotational joint that rotates the cuboid-connecting rod rigid assembly about axis **k**. This assembly is rotating about axis **k.**

At a certain instant of time, the origin of Frame H (joint J) is translating at a velocity of ${}^U u_H = (1 \quad 2 \quad 3)^T$ m/sec, and the cuboid-connecting rod assembly is rotating about axis **k** at an angular velocity of 10 rad/sec. At this same instant of time the assembly and frames are at the configuration indicated in Fig. 1.

At that same instant of time, determine:

i. The angular velocity of the cuboid-connecting rod assembly with respect to Frame U.
ii. The translational velocity of point C (on the cuboid) with respect to Frame U.

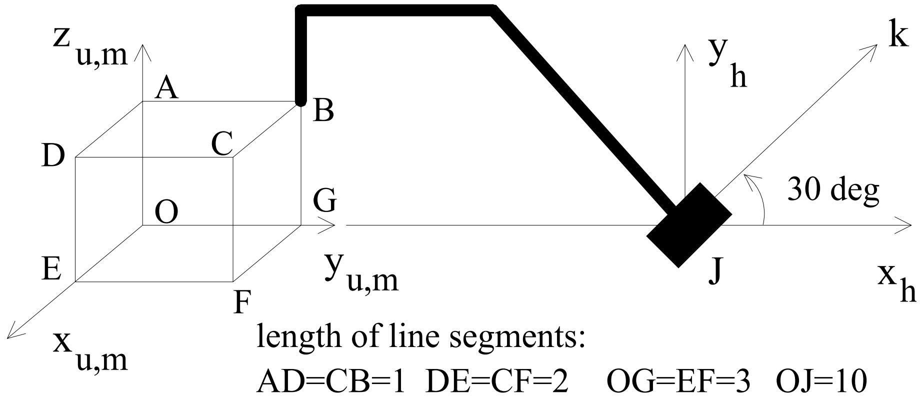

Fig. 2

Ans: $u = \begin{pmatrix} 53.32 \\ 7 \\ -5.66 \end{pmatrix} m/\sec \qquad \omega = \begin{pmatrix} 0 \\ 8.66 \\ 5 \end{pmatrix} rad/\sec$

3. Fig.3 shows a 4-axis robot with all rotational joints. The first joint rotates about a vertical axis while the next three joints rotate about a horizontal axis parallel to the xy plane of Frame U. Frame E is attached to the robot end-effector. A six axis force-torque sensor provides 3 force and 3 torque readings along and about the x, y, and z axes of the sensor frame S. Frame S is coincident to the fixed frame of reference U. If the robot end-effector is carrying a payload of 20 kg when it is at a configuration indicated by

$$^{U}T_{E} = \begin{pmatrix} ^{U}R_{E} & ^{U}p_{E} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.0 & -1.0 & 0 & 10 \\ 0.866 & 0 & 0.5 & 5 \\ -0.5 & 0 & -0.866 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

Determine the six readings of the six-axis force-torque sensor. Assume that the robot is weightless less. The gravitational force is pointing downward along the negative z axis direction of Frame U. Assume all the links are of length 1 m, determine the joint forces/torques required.

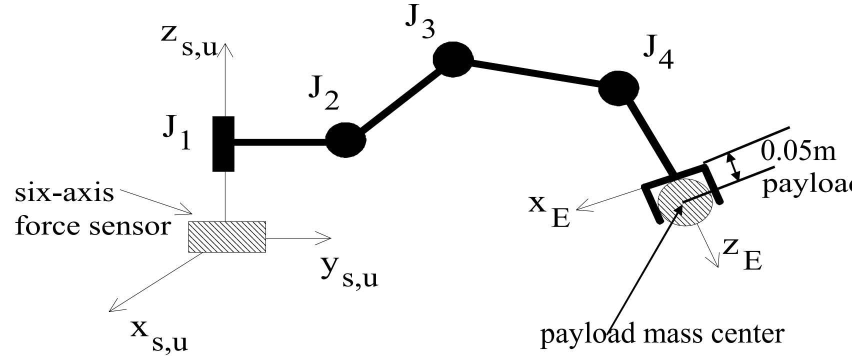

Figure 3

Ans: $F = \begin{pmatrix} 0 \\ 0 \\ -196 \end{pmatrix} N \qquad T = \begin{pmatrix} -1029 \\ 1960 \\ 0 \end{pmatrix} N.m$

4. There are three moving frames A,B, and C. At a certain time instant, they are at:

$$ {}^{A}T_{B} = \begin{pmatrix} 0.866 & -0.5 & 0 & 10 \\ 0.5 & 0.866 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad {}^{B}T_{C} = \begin{pmatrix} -1 & 0 & 0 & 3 \\ 0 & 0 & -1 & 4 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{pmatrix}; $$

and are moving with the following generalized velocity vectors (the first three components represent the translational velocity while the last three components represent the angular velocity):

$$ {}^{A}V_{B} = \begin{pmatrix} 0.0 \\ 2.0 \\ -3.0 \\ 1.414 \\ 1.414 \\ 0.0 \end{pmatrix} \quad ; \quad {}^{B}V_{C} = \begin{pmatrix} 1.0 \\ 4.0 \\ -5.0 \\ 0.5 \\ 1.0 \\ 2.0 \end{pmatrix} $$

Find the generalized velocity ${}^{A}V_{C}$ of Frame C as seen from Frame A.

Ans: $$ {}^{A}V_{C} = \begin{pmatrix} 5.936 \\ -1.106 \\ -1.826 \\ 1.347 \\ 2.53 \\ 2 \end{pmatrix} $$