NATIONAL UNIVERSITY OF SINGAPORE

FINAL EXAMINATION FOR THE DEGREE OF B.ENG

ME 444 - DYNAMICS AND CONTROL OF ROBOTIC SYSTEMS

October/November 1994 - Time Allowed: 3 Hours

INSTRUCTIONS TO CANDIDATES:

1. This examination paper contains SIX (6) questions and comprises TEN (10) pages.
2. Answer any FOUR (4) questions.
3. All questions carry equal marks.
4. This is an *open-book* examination. You may open any number of books and notes.

1. a. Fig. 1 shows a connecting (bent) rod (BJ) rigidly connecting the rotational joint J to the cuboid (ABCDEFGO). Let

1> Frame U be fixed and serve as the universe frame of reference.
2> The cuboid (ABCDEFGO), bent connecting rod (BJ), and joint J be all connected as one rigid assembly.
3> Frame H be attached rigidly to the rotational joint J and Frame M be fixed rigidly onto the cuboid. That is, Frames H and M are attached rigidly to the cuboid -connecting-rod assembly body.
4> Frame H be **translating** with respect to Frame U.
5> Axis **k** be fixed onto Frame H.
6> The cuboid-connecting-rod rigid assembly is rotating about axis **k.**

At a certain instant of time, the origin of Frame H (joint J) is translating at a velocity of ${}^{U}u_H = (1 \quad 2 \quad 3)^T$ m/sec, and the cuboid-connecting-rod assembly is rotating about axis **k** at an angular velocity of 10 rad/sec. At this same instant of time the assembly and frames are at the configuration indicated in Fig. 1.

At that same instant of time, determine:

i. The angular velocity of the cuboid-connecting-rod assembly with respect to Frame U.
ii. The translational velocity of point C (on the cuboid) with respect to Frame U.

(15 marks)

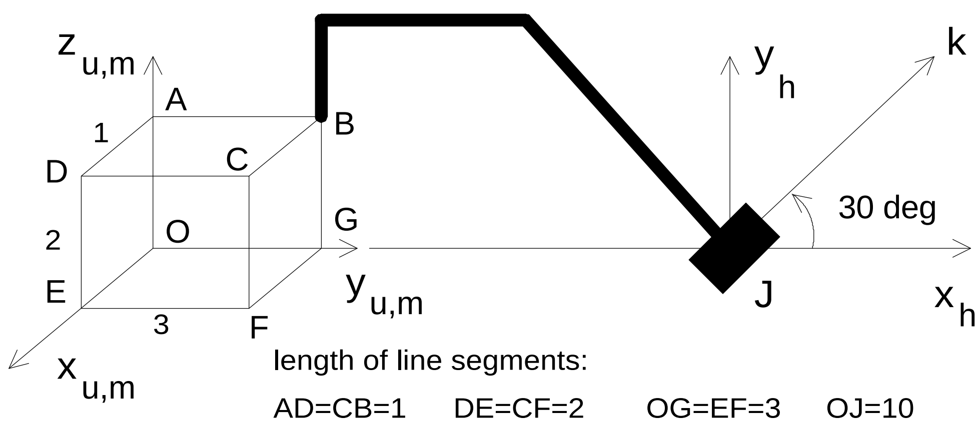

Fig. 1

b. The *Intelledex* robot is a six-axis manipulator with all rotational joints. The first three axis intersect at a common point. Fig. 2 is a schematic diagram of the Intelledex robot showing the six rotational joint axes (indicated by the line segments protruding out of the links). We wish to solve the inverse kinematics of the position problem for this robot.

Can the *decoupling* principle be used?

If so, identify the decoupled subsystem by showing the equations relating the decoupled task subspace and the decoupled subset of joints. Indicate the number of degrees of freedom of this subsystem.

If not, explain why?

(10 marks)

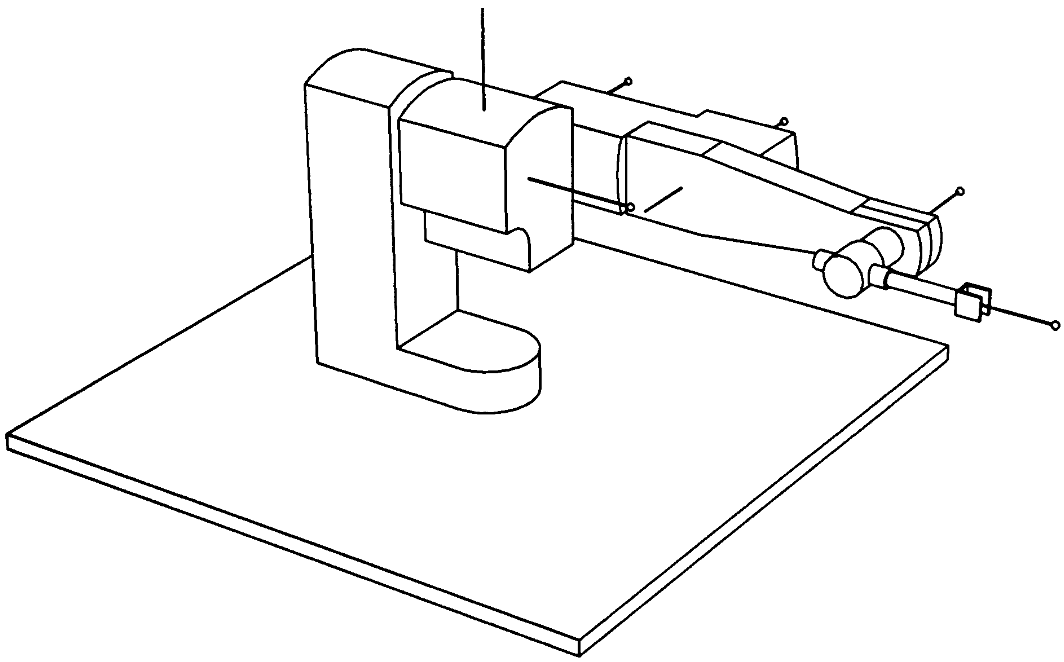

Fig. 2

2. a. Frames M and C are attached rigidly to a cuboid as shown in Fig. 3. Frame U is fixed and serves as the universe frame of reference. The cube undergoes the following motion in the indicated sequence:

1> Rotation about the z axis of Frame C by 30°, then
2> Translation of (1, 2, 3) along Frame C, then
3> Rotation about the x axis of Frame M by 45°, and then
4> Rotation about the y axis of Frame U by 60°.

Let ${}^{U}T_{C_i}$ and ${}^{U}T_{M_i}$ be the 4 × 4 homogeneous transformation matrices that describe the position and orientation of Frames *C* and M, respectively, in *U* after motion *i*.

Find

i. ${}^{U}T_{C_1}$

ii. ${}^{U}T_{C_2}$

iii. ${}^{U}T_{C_3}$

iv. ${}^{U}T_{C_4}$

v. ${}^{U}T_{M_4}$

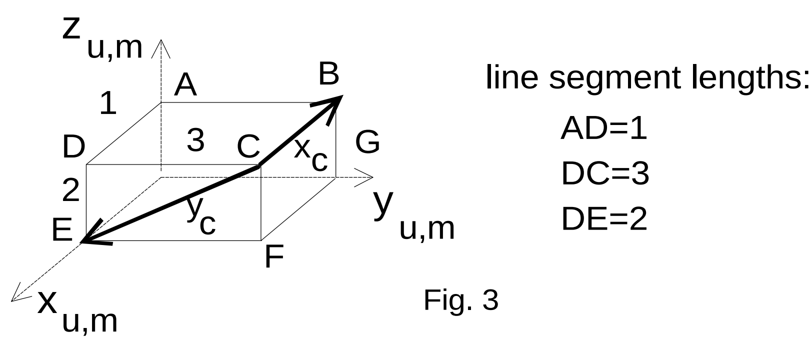

Fig. 3

(18 marks)

b. Fig.4 shows a 4-axis robot with all rotational joints. The first joint rotates about a vertical axis while the next three joints rotate about horizontal axes parallel to the xy plane of Frame U. Frame E is attached to the robot end-effector. A six axis force-torque sensor provides 3 force and 3 torque readings along and about the x, y, and z axes of the sensor frame S. Frame S is coincident to the fixed frame of reference U. At the following stationary configuration:

$$ {}^{U}T_{E} = \begin{pmatrix} {}^{U}R_{E} & {}^{U}p_{E} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.5 & 0 & 0 & 10 \\ 0.612 & 0.436 & -0.660 & 5 \\ -0.61 & 0.789 & -0.047 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}, $$

the robot end-effector is carrying a cubic (0.1 m cube) payload of 20 kg. Determine the six readings of the six-axis force-torque sensor. Assume that the robot links are weightless. The gravitational force is pointing downward along the negative z axis direction of Frame U. The end-effector frame is at the base of the gripper. The center of mass of the payload is at its center and lies on the z axis of the end-effector. The acceleration due to gravity is 9.8 m/s^2.

(7 marks)

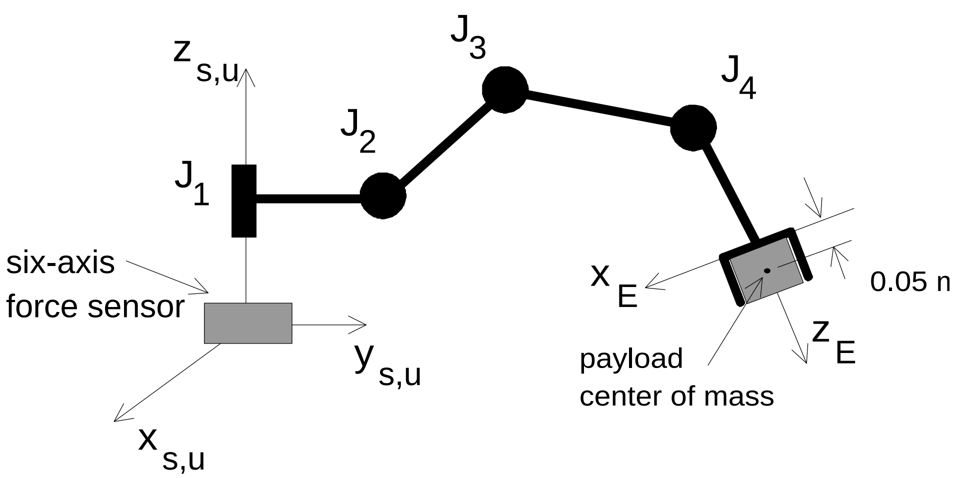

Fig. 4

3. a. Frame C is attached rigidly to one corner of a cuboid as shown in Fig. 5. The z axis of frame C is directed from point C to E. The y axis of frame C is directed from point C to B. Determine the 4 × 4 homogeneous transformation matrix ${}^{U}T_{C}$ that describes the position and orientation of frame C in frame U.

(5 marks)

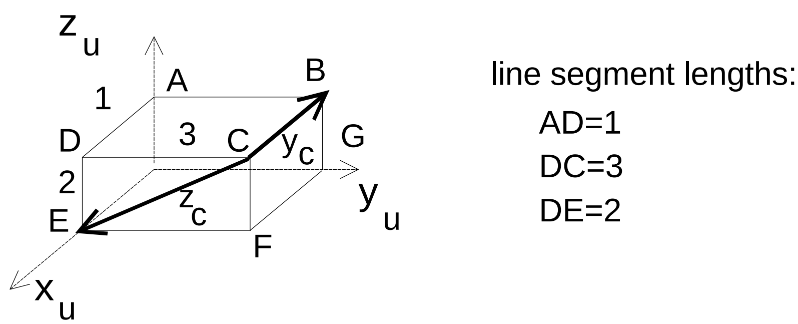

Fig. 5

b. Fig. 6 shows a robot with two rotational joints J_1 and J_2. Frame 0 is fixed to ground. The first link (the rigid bent rod J_1ABJ_2 which connectes joints J_1 and J_2) rotates about the z-axis of frame 0. The bent rod J_1AB always lies on the xy plane of frame 0. Frame 2 is attached to the end-effector as indicated in Fig. 6. The z and x axes of frame 2 are normal and along the longitudinal axis, respectively, of link 2. The robot is in motion, and at a certain time instant the following are known:

1> The robot is at the configuration shown in Fig. 6.
2> The axis of motion of joint J_2 is parallel to the x axis of frame 0.
3> The coordinates of J_2 in frame 0 are (2, 4, 6).

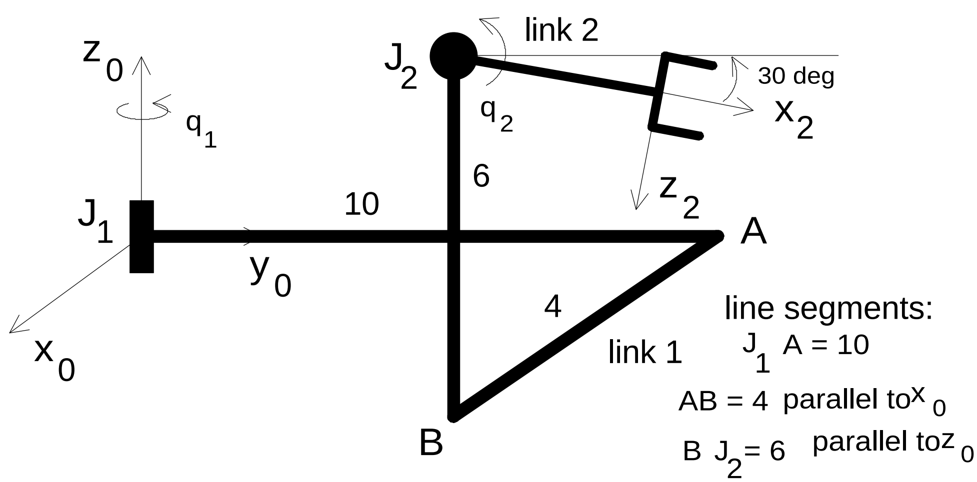

Fig. 6

i. Assign a Cartesian coordinate frame to link 1 (the bent rod) according to the Denavit-Hartenberg Convention. Sketch the robot with the coordinate frames on your examination scripts.

ii. Fill in the the following table of kinematic parameters:

Link	θ_i	r_i	d_i	α_i
1				
2				

iii. Draw the robot at the configuration $q_1 = 0°$ and $q_2 = 45°$.

iv. Determine the position and orientation of the end-effector (frame 2) in frame 0 as a function of the 2 joint variables. Express this position and orientation as a 4×4 homogeneous transformation matrix ${}^0T_2$.

(20 marks)

4. A two degree-of-freedom manipulator shown in Fig. 7 moves an object of mass *m* in the vertical plane. The manipulator starts at rest from *Position 1* at time t = 0 (i.e. $q(0) = q_1, r(0) = r_1, \dot{q}(0) = 0, \dot{r}(0) = 0$), and moves to *Position 2* at time t = *T* seconds (i.e. $q(T) = q_2, r(T) = r_2, \dot{q}(T) = 0, \dot{r}(T) = 0$) as shown in Fig. 8.

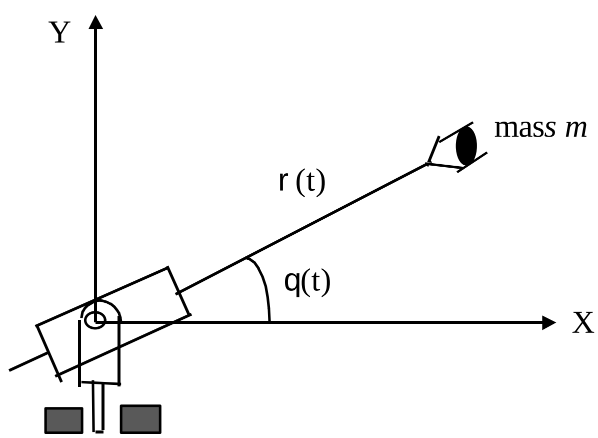

Fig. 7

a. Derive the Lagrange dynamic equations of the manipulator by taking into account the object mass *m* and neglecting the masses of the manipulator links.

(12 marks)

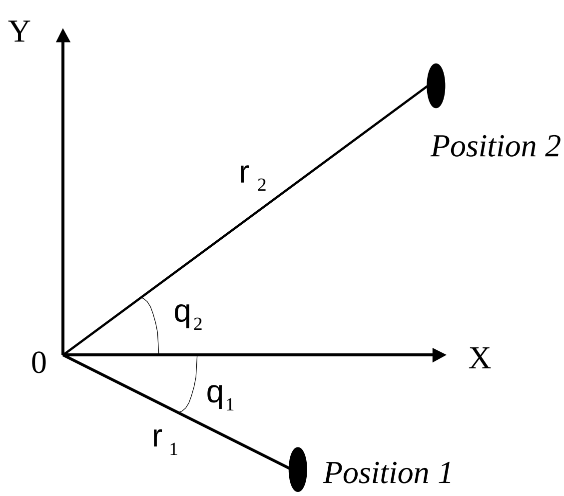

Fig. 8

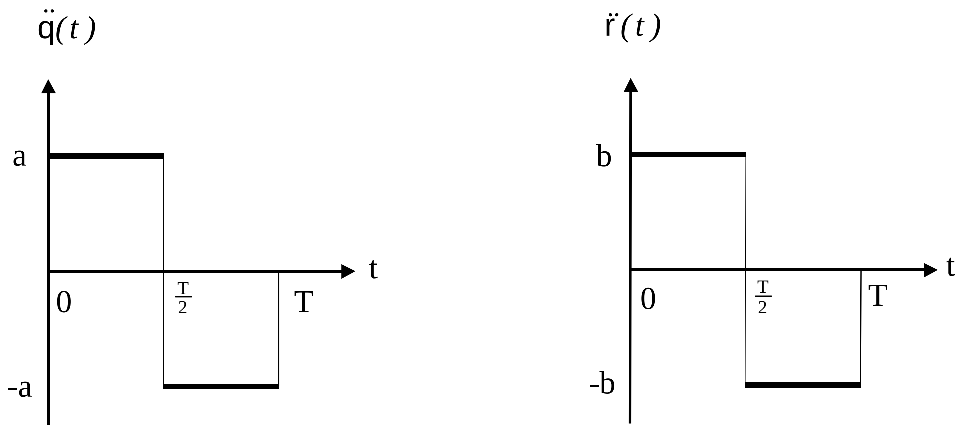

Fig. 9

b. Assume that the joint motions are performed with constant accelerations as shown in Fig. 9. For $q_1 = -15$ degrees, $q_2 = 15$ degrees, $r_1 = 2$ m, $r_2 = 2$ m, $T = 2$ sec, determine the following:

i. the magnitudes *a* and *b* of the joint accelerations $\ddot{q}(t)$ and $\ddot{r}(t)$ respectively.

ii. the trajectories $q(t)$ and $r(t)$.

iii. the actuator torque $t(t)$ and the actuator force *f(t)* required for this motion.

(13 marks)

5. **a.** Consider the coupled nonlinear system

$$\ddot{y}_1 + 3y_1y_2 + y_2^2 = u_1 + y_2u_2,$$
$$\ddot{y}_2 + \dot{y}_2 \cos y_1 + 3(y_1 - y_2) = u_2 - 3(\cos y_1)^2 y_2u_1,$$

where u_1, u_2 are the inputs and y_1, y_2 are the outputs.

i. What is the dimension of the state space of the system?

ii. Choose state variables and write the system as a system of first order differential equations in state space.

iii. Find an inverse dynamics control so that the closed loop system is linear and decoupled, with each subsystem having natural frequency 10 radians and damping ratio 0.5.

(12 marks)

b. Consider the nonlinear dynamic equation of a manipulator given by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u,$$

where $M(q)$ denotes the inertia matrix, $C(q,\dot{q})\dot{q}$ denotes the coriolis and centrifugal terms, $g(q)$ denotes the gravitational terms, and u denotes the generalized inputs.Use Lyapunov analysis to show that the PD control law

$$u = -K_p q - K_d \dot{q},$$

where K_p, K_d are diagonal matrices of (positive) proportional and derivative gains respectively, yields a stable nonlinear system. Give an expression for the steady state value of q.

(13 marks)

6. a. Consider the simplified system shown in Fig. 10 representing an end effector contacting the environment along the direction labeled *x* where

f	:	input force exerted by the end effector
f_{dist}	:	a disturbance force
m_e	:	equivalent inertia of environment (positive)
b_e	:	damping constant (positive)
k_e	:	stiffness constant (positive).

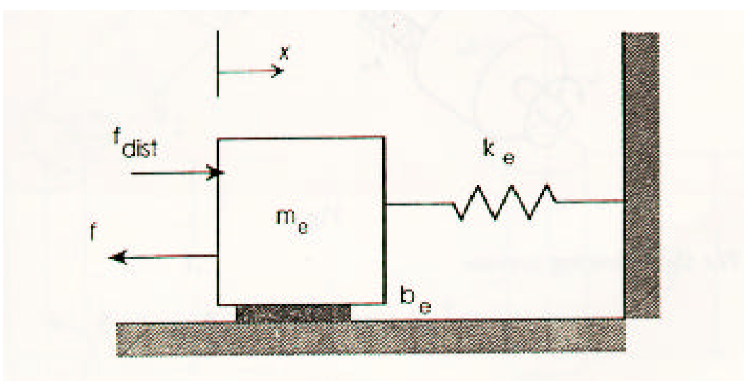

Fig. 10

The contact force which is given by

$$f_e = k_e x$$

is required to track a constant desired force trajectory f_d.

i. Choosing $x, \dot{x}$ as state variables, and the force error as the output variable y, i.e.

$$y = f_d - f_e,$$

express the dynamic equations of the system in state space form.

ii. If the force input is chosen as

$$f = -k_1 x - k_2 \dot{x} + k_3 f_d + f_{dist} \; ,$$

write down all the conditions involving k_1, k_2, k_3, k_e, b_e such that the steady state force error y_{ss} is zero when f_d is a constant (i.e. a step input).

(12 marks)

b. The Lagrange dynamic equations of a two degree of freedom cartesian manipulator is given by

$$\ddot{x} = f_x - f_{xe},$$
$$\ddot{y} = f_y - f_{ye},$$

where x, y are the joint variables, f_x, f_y are the input forces, and f_{xe}, f_{ye} are the cartesian components of the contact force at the end effector. Consider the constraint surface shown in Fig. 11 which is described as

$$0.8x + 0.6y - 2.4 = 0 .$$

The end effector is initially in contact with the constraint surface and initial conditions are given by

$$x(0) = 3, y(0) = 0, \dot{x}(0) = 0, \dot{y}(0) = 0 .$$

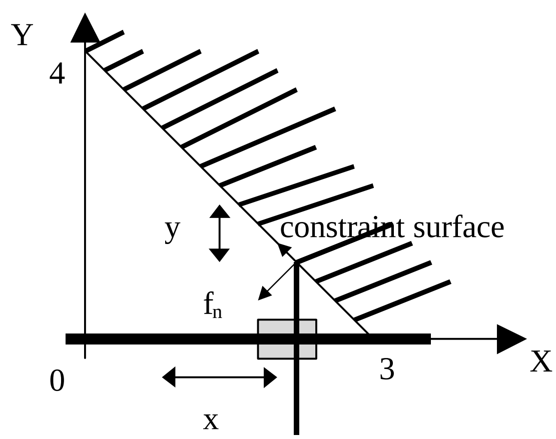

Fig. 11

Assume that the constraint surface is frictionless and the contact force is always normal to the constraint surface. Find the input forces f_x, f_y which would maintain a constant normal contact force f_n of magnitude 2 N, and move the end effector of the manipulator along the constraint surface with a tangential velocity t m/s, where t is the time in seconds.

(13 marks)

END OF PAPER