ME4245 Robot Kinematics, Dynamics and Control
Quiz 1.2 1 Oct 2012, 16:00 – 17:00.

1. Figure 1 shows a robot with 2 rotational joints followed by 1 translational joint. Frame U is attached to the base of the robot. Its end-effector (E) carries a tool with Frame T attached to the tool at its tip. Frame E is attached to the end-effector and the position and orientation of the tool with respect to the end-effector is known to be ${}^{E}T_{T}$. At the configuration shown:
   - the coordinates of points A, B, C, D & E are indicated in the figure, and all are expressed in Frame U
   - all links of the robot are in the XY plane of Frame U
   - the 1st joint is rotating around Z_U at 3 rad/sec
   - the 2nd joint is rotating along BD at 2 rad/sec, and
   - the 3rd joint is translating along the last link CE at 1 m/sec.

   At the configuration shown in Fig. 1, determine the complete expressions for the following:

   a) translational velocity of the end effector ${}^{U}U_E$
   b) angular velocity of the end effector ${}^{U}\omega_E$
   c) translational velocity of the tool ${}^{U}U_T$
   d) angular velocity of the tool ${}^{U}\omega_T$

2. Figure 2 shows a planar robot with 3 joints (1 rotational followed by 2 translational joints) which operates in the XY plane of Frame U. Frames U and E are attached to the base and end effector, respectively.

   a) Determine the complete manipulator Jacobian that relates the generalized velocity of the end effector (Frame E) in the base frame (U) with the joint velocities $\dot{q}_1, \dot{q}_2, \dot{q}_3$.

   b) If the task is ${}^{U}u_x$ and ${}^{U}u_y$ (end effector translational velocities along xy plane of Frame U), determine the joint positions in which the robot is at a singular configuration.

   c) If the task is ${}^{U}u_x$ (end effector translational velocity along x axis of Frame U) and ${}^{U}\omega_Z$ (end effector angular velocity around an axis parallel to Z axis of Frame U), determine the joint positions in which the robot is at a singular configuration.

   d) The robot's end effector is initially at the joint configuration shown in Fig. 2. It is desired to move the robot's end effector (origin of Frame E) to a position indicated by coordinates (20, 20, 0) in Frame U, such that the sum of the squares of the joint velocities are minimized. Describe what motions the joints should take to achieve this.

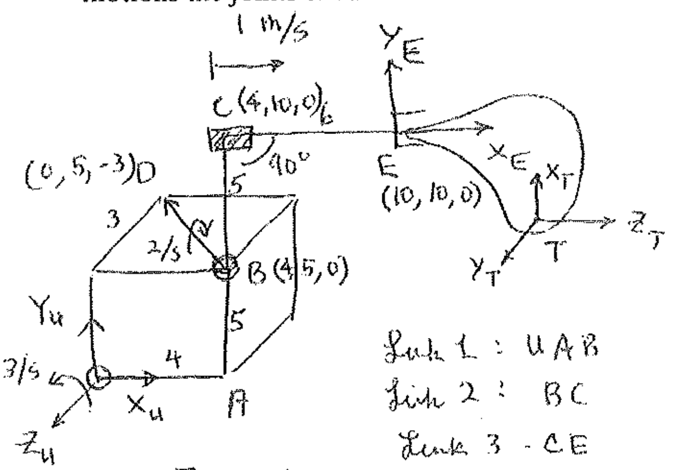


Figure 1

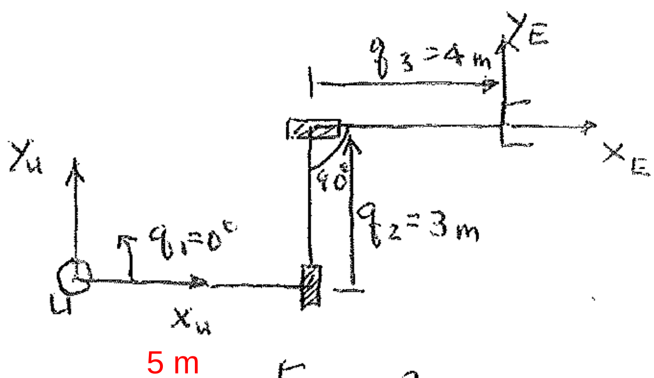


Figure 2

1. $J = [J_1 \ J_2 \ J_3]$

$J_1 = \begin{pmatrix} z_u \times (P_E - P_u) \\ z_u \end{pmatrix}$; $z_u = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $P_u = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$ $P_E = \begin{pmatrix} 10 \\ 10 \\ 0 \end{pmatrix}$

$J_1 = \begin{pmatrix} -10 \\ 10 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$; $J_2 = \begin{pmatrix} \vec{BD} \times (P_E - P_B) \\ \vec{BD} \end{pmatrix}$ $P_B = \begin{pmatrix} 0 \\ 5 \\ -3 \end{pmatrix}$

$\vec{BD} = \begin{pmatrix} -4 \\ 0 \\ -3 \end{pmatrix} \times \frac{1}{\sqrt{25}} = \begin{pmatrix} -4/5 \\ 0 \\ -3/5 \end{pmatrix}$

$J_2 = \begin{pmatrix} 3 \\ -42/5 \\ -4 \\ -4/5 \\ 0 \\ -3/5 \end{pmatrix}$; $J_3 = \begin{pmatrix} \vec{CE} \\ 0 \end{pmatrix}$ $\vec{CE} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$J_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$^u u_E = \begin{pmatrix} -10 & 3 & 1 \\ 10 & -42/5 & 0 \\ 0 & -4 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ //

$^u \omega_E = \begin{pmatrix} 0 & -4/5 & 0 \\ 0 & 0 & 0 \\ 1 & -3/5 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ //

$^u u_T = {}^u u_E + {}^u \omega_E \times ({}^u R_E \, {}^E P_T)$ $\quad {}^E T_T = \begin{pmatrix} {}^E R_T & {}^E P_T \end{pmatrix}$

$^u R_E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

${}^{u}W_T = {}^{u}W_E$ since T & E are on the same rigid body.

2. $J = (J_1\ J_2\ J_3)$

(a)

$$P_x = 5\cos q_1 + q_2 \cos(q_1 + 90^\circ) + q_3 \cos q_1$$

$$= (5+q_3)\cos q_1 + q_2(-\sin q_1) =$$

$$= (5+q_3)\cos q_1 - q_2 \sin q_1 \ //$$

$$P_y = 5\sin q_1 + q_2 \sin(q_1 + 90^\circ) + q_3 \sin q_1$$

$$= (5+q_3)\sin q_1 + q_2 \cos(q_1) \ //$$

$$u_x = (5+q_3)(-\sin q_1)\dot{q}_1 + \dot{q}_3 \cos q_1 - q_2 \cos q_1 \dot{q}_1 - \dot{q}_2 \sin q_1$$

$$= \left(-(5+q_3)\sin q_1 - q_2 \cos q_1\right)\dot{q}_1 - \sin q_1 \dot{q}_2 + \cos q_1 \dot{q}_3$$

$$u_y = (5+q_3)\cos q_1 \dot{q}_1 + \dot{q}_3 \sin q_1 + q_2(-\sin q_1 \dot{q}_1) + \dot{q}_2 \cos q_1$$

$$= \left((5+q_3)\cos q_1 - q_2 \sin q_1\right)\dot{q}_1 + \cos q_1 \dot{q}_2 + \sin q_1 \dot{q}_3$$

$$w_z = \dot{q}_1$$

$$J = \begin{pmatrix} -(5+q_3)\sin q_1 - q_2 \cos q_1 & -\sin q_1 & \cos q_1 \\ (5+q_3)\cos q_1 - q_2 \sin q_1 & \cos q_1 & \sin q_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

ii) Task: u_x, u_y : 1st 2 rows of Jacobian

$$\begin{matrix} u_x \\ u_y \end{matrix} = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix}$$

$a = -(5+q_3)s_1 - q_2 c_1$
$d = (5+q_3)c_1 - q_2 s_1$
$b = -s_1 \quad c = c_1$
$e = c_1 \quad f = s_1$

$$\det\begin{pmatrix} a & b \\ d & e \end{pmatrix} = ae - db$$
$$= \big(-(5+q_3)s_1 - q_2 c_1\big)c_1 - \big((5+q_3)c_1 - q_2 s_1\big)(-s_1)$$
$$= -q_2 c_1^2 - q_2 s_1^2 = -q_2 = 0,$$

$$\det\begin{pmatrix} a & c \\ d & f \end{pmatrix} = af - dc$$
$$= \big(-(5+q_3)s_1 - q_2 c_1\big)s_1 - \big((5+q_3)c_1 - q_2 s_1\big)c_1$$
$$= -(5+q_3) = 0 \qquad q_3 = -5 \;/\!/$$

$$\det\begin{pmatrix} b & c \\ e & f \end{pmatrix} = bf - ec = -s_1 s_1 - c_1 c_1 = 1$$

since there is NO configuration where all determinants are zero, the robot has no singularities.

~ can also be obtained via inspection since there are 2 translational joints on non-parallel axis, End effector can always move along xy plane (c).

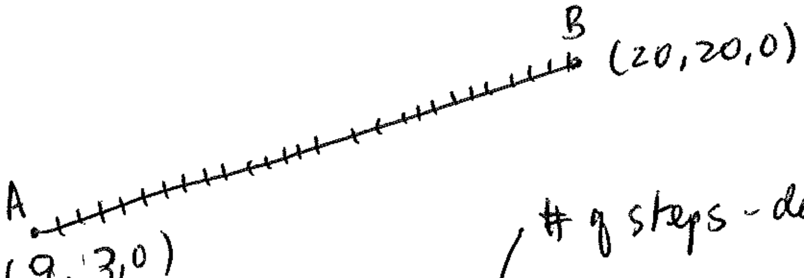
$$\begin{pmatrix} u_x \\ w_z \end{pmatrix} = \begin{pmatrix} a & b & c \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix}$$

$$\det \begin{pmatrix} a & b \\ 1 & 0 \end{pmatrix} = -b = S_1 = 0 \qquad \theta_1 = 0^\circ \text{ or } 180^\circ$$

$$\det \begin{pmatrix} a & c \\ 1 & 0 \end{pmatrix} = -c = C_1 = 0 \qquad \theta_1 = 90^\circ \text{ or } 270^\circ$$

Since both above conditions cannot be simultaneously satisfied, there are NO singularities.

(d).



\# of steps - depends on how accurate you want to be

0) $i = \emptyset$

1) Plan a straight line trajectory from current position (x, y) to B

$$\Delta x = (20 - x)/(1000 - i) = 0.011\text{m}$$
$$\Delta y = (20 - y)/(1000 - i) = 0.017\text{m}$$

$\left.\right\}$ ~~5~~ 1000 steps

2) $$\begin{pmatrix} \Delta q_1 \\ \Delta q_2 \\ \Delta q_3 \end{pmatrix} = J^{\#} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

where $J^{\#} = J^T (J J^T)^{-1}$, right pseudo inverse to minimize $\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2$

$$J = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$$

J is computed with current q_1, q_2, q_3

$(J J^T)^{-1}$ always exists since robot is never singular with respect to $\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$

or compute x, y from q1, q2, and q3.

3) Command robot (move joints):

$$q_1 = q_1 + \Delta q_1$$
$$q_2 = q_2 + \Delta q_2$$
$$q_3 = q_3 + \Delta q_3$$

4) $x = x + \Delta x$

$y = y + \Delta y$

5) $i = i + 1$

6) go to step 2, until $i = 999$

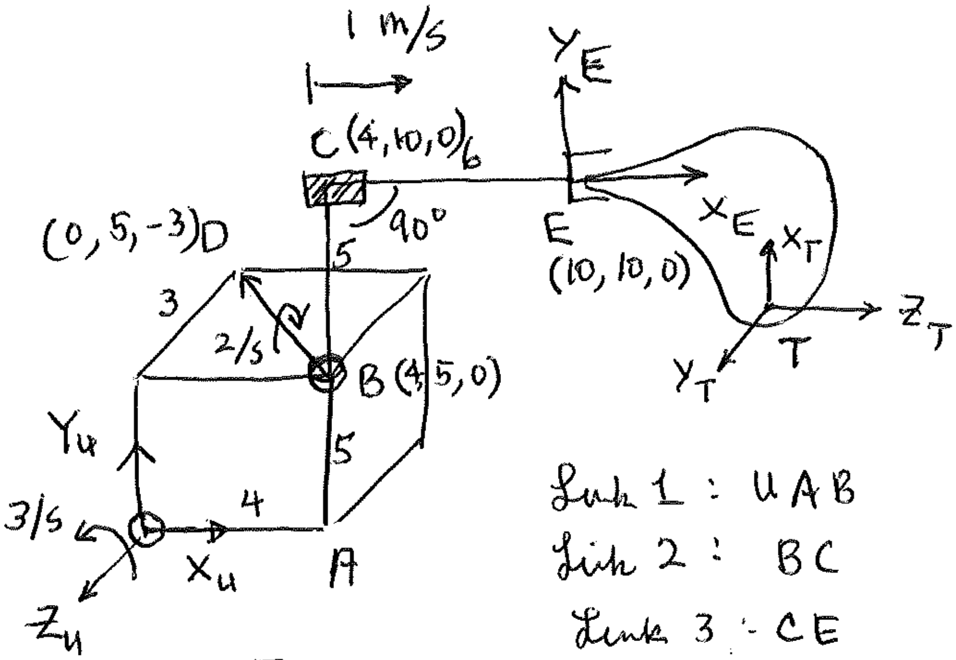


Figure 1

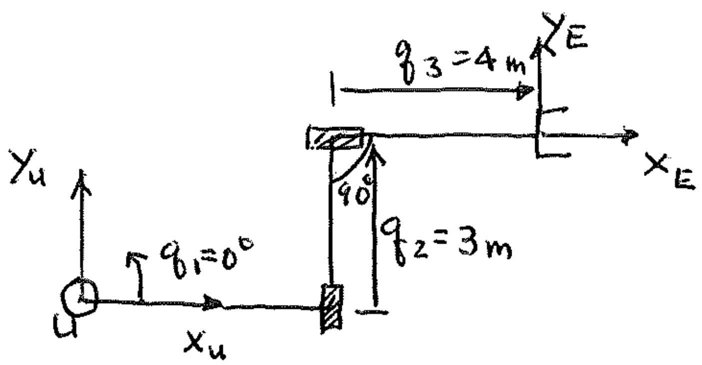


Figure 2