National University of Singapore Department of Mechanical Engineering
ME4245 Quiz 2 10 Sept 2008, 16:00-17:30

Name:______________________________ Matric Number:______________________
(as it appears in your NUS Student card)

Answer all the 3 questions in this quiz. You need not simplify your answers. But, please make sure all expressions are complete. Please write your answers here and please feel free use the back pages and/or add additional pages.

1. *(15 marks)* Frame U is a frame fixed to the ground. A body "A" (with Frame A attached to it) is in motion while carrying another body "B" (with Frame B attached to it). Body B is also in motion with respect to A. At some instant of time, the following quantities are known:

    - Motion of A with respect to U: translational velocity of ${}^{U}u_{A} \in R^{3x1}$ and angular velocity of ${}^{U}\omega_{A} \in R^{3x1}$
    - Position (${}^{U}p_{A} \in R^{3x1}$) and Orientation (of ${}^{U}R_{A} \in R^{3x3}$) of A with respect to U.
    - Body B is translating along the Z axis of body A at 10 m/s

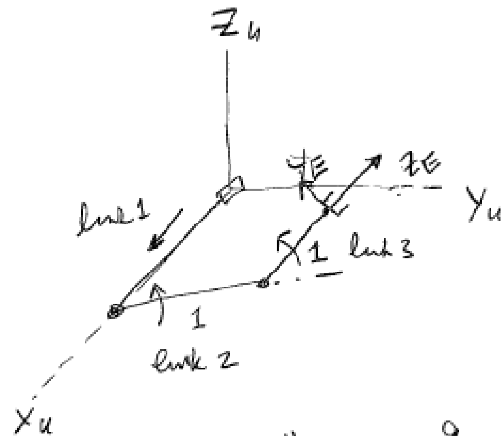
    Determine the expressions for the following velocities in terms of the known quantities above:

    a. translational velocity of B with respect to U.

    b. angular velocity of B with respect to U.

Ans:

2. *(25 marks)* The figure on the right shows a robot with three joints. The first link translates along the X axis of Frame U which is fixed to the ground. The second and third links rotate about two axes that are parallel to Frame U. Links 2 and 3 always move in a plane parallel the YZ plane of Frame U. Frame E is attached to the end-effector (link 3) such that its Y and Z axes are always parallel to the YZ plane of Frame U.



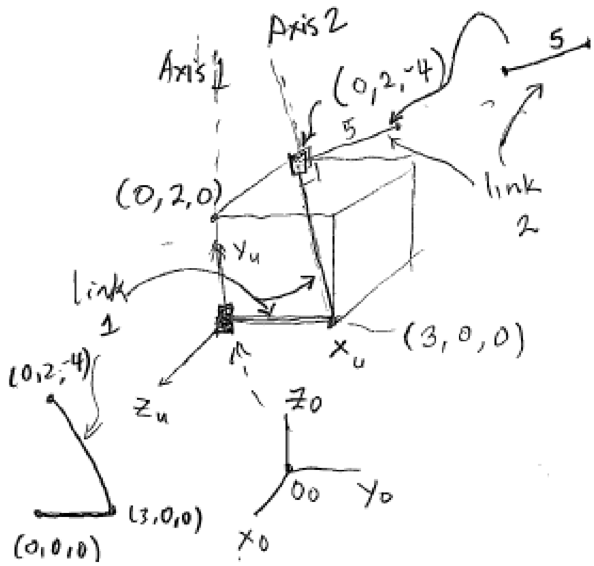
   a. Determine the full manipulator Jacobian that relates the 6 x 1 generalized (translational (u_x, u_y, u_z) and angular (ω_x, ω_y, ω_z)) velocity of the end-effector (Frame E) with the joint velocities.

   b. If the task of interest is (u_x, u_y, u_z), determine the singular configurations, if any, of the robot. If applicable, describe the lost degrees of freedom for each singular configuration of the robot.

   c. If the task of interest is (u_x, u_y), determine the singular configurations, if any, of the robot. If applicable, describe the lost degrees of freedom for each singular configuration of the robot.

   d. If the task of interest is (u_x, ω_z), determine the singular configurations, if any, of the robot. If applicable, describe the lost degrees of freedom for each singular configuration of the robot.

3. (60 marks) The figure below shows a robot with two rotational joints. Frame U is fixed to the ground and the first link rotates about its Y axis (Axis 1). The second link is of length 5 and rotates about Axis 2. All Cartesian coordinates of the points are expressed in Frame U.



a. Assign frames to the robot according to the D-H convention discussed in class.

b. Determine all the kinematic parameters (i.e., fill in the D-H table of kinematic parameters) and identify which of the parameters are the joint variables.

c. If possible, identify the values of the joint variables of the robot at the configuration shown in the figure above.

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(#1) $$ {}^{u}T_B = {}^{u}T_A \, {}^{A}T_B $$

$$ {}^{u}\dot{T}_B = {}^{u}T_A \, {}^{A}\dot{T}_B + {}^{u}\dot{T}_A \, {}^{A}T_B $$ (5)

translation only

$$ \begin{pmatrix} {}^{u}\hat{\omega}_B \, {}^{u}R_B & {}^{u}u_B \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} {}^{u}R_A & {}^{u}p_B \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cancel{{}^{A}\hat{\omega}_B \, {}^{A}R_B} & {}^{A}u_B \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} {}^{u}\hat{\omega}_A \, {}^{u}R_A & {}^{u}u_A \\ 0 & 0 \end{pmatrix} $$

$$ {}^{A}u_B = \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix} \qquad \begin{pmatrix} {}^{A}R_B & {}^{A}p_B \\ 0 & 1 \end{pmatrix} $$

$$ {}^{u}u_B = {}^{u}R_A \, {}^{A}u_B + \underbrace{{}^{u}\hat{\omega}_A \, {}^{u}R_A \, {}^{A}p_B}_{{}^{u}\omega_A \times ({}^{u}p_B - {}^{u}p_A)} + {}^{u}u_A $$ // (5)

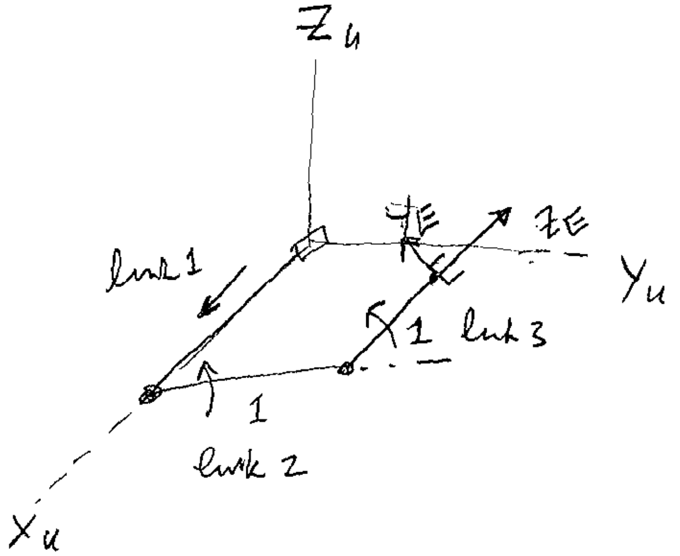
$$ {}^{u}\hat{\omega}_B \, {}^{u}R_B = \cancel{{}^{u}R_A (\phi)} + {}^{u}\hat{\omega}_A \, {}^{u}R_A \, {}^{A}R_B $$

ϕ

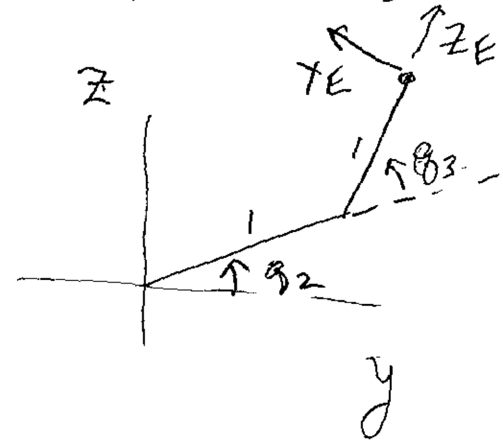
$$ {}^{u}\hat{\omega}_B = {}^{u}\hat{\omega}_A $$

$$ {}^{u}\omega_B = {}^{u}\omega_A $$ // (5)

#2



On a plane parallel to YZ plane of Frame U



$$
{}^{u}P_E = \begin{matrix} q_1 \\ \cos q_2 + \cos(q_2+q_3) \\ \sin q_2 + \sin(q_2+q_3) \end{matrix} = \begin{pmatrix} q_1 \\ C_2 + C_{23} \\ S_2 + S_{23} \end{pmatrix}
$$

$$
\frac{d}{dt}\,{}^{u}P_E = \begin{matrix} \dot{q}_1 \\ -S_2\dot{q}_2 - S_{23}(\dot{q}_2+\dot{q}_3) \\ C_2\dot{q}_2 + C_{23}(\dot{q}_2+\dot{q}_3) \end{matrix} = \begin{matrix} \dot{q}_1 \\ (-S_2 - S_{23})\dot{q}_2 - S_{23}\dot{q}_3 \\ (C_2+C_{23})\dot{q}_2 + C_{23}\dot{q}_3 \end{matrix}
$$

$$
{}^{u}V_E = {}^{u}J_E\,\dot{q} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -S_2-S_{23} & -S_{23} \\ 0 & C_2+C_{23} & C_{23} \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix}
$$

$$
= \begin{matrix} u_x \\ u_y \\ u_z \\ \omega_x \\ \omega_y \\ \omega_z \end{matrix}
$$

Task (u_x, u_y, u_z)

$$
\det \begin{pmatrix} 1 & 0 & 0 \\ 0 & -S_2-S_{23} & -S_{23} \\ 0 & C_2+C_{23} & C_{23} \end{pmatrix} = (-S_2 - S_{23})C_{23} + (C_2 + C_{23})S_{23}
$$

$$
S_{23}C_2 - C_{23}S_2 = S_3
$$

$$
\theta_3 = 0,\ 180^\circ
$$

Task (u_x, u_y)

singularities when $\theta_2 = 0, \theta_3 = 0$ (by inspection)

or $\theta_2 = 0, \theta_3 = 180^\circ$

$\theta_2 = 180^\circ, \theta_3 = 0^\circ$

$\theta_2 = 180^\circ, \theta_3 = 180^\circ$

or

$$\begin{matrix} 1 & 0 & 0 \\ 0 & -S_2 - S_{23} & -S_{23} \end{matrix}$$

1 2 3

all possible determinant $(2\times 2) = 0$ if $S_2 = 0, S_{23} = 0$
(3 of them)

Task (u_x, w_x)

no singularities: by inspection

or

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$\Downarrow$

$\neq 0$ determinant

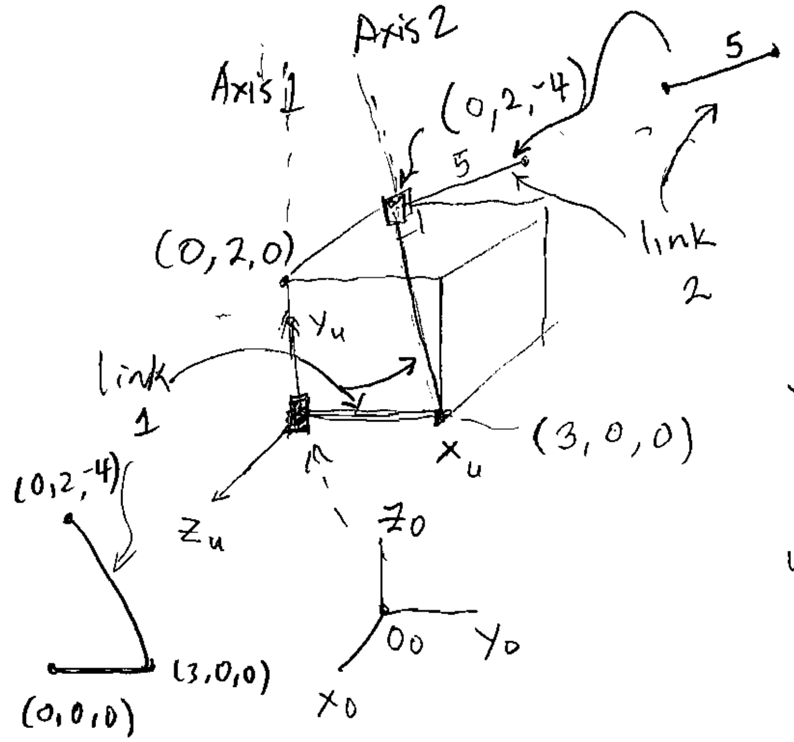
one 2×2 matrix is always non-zero

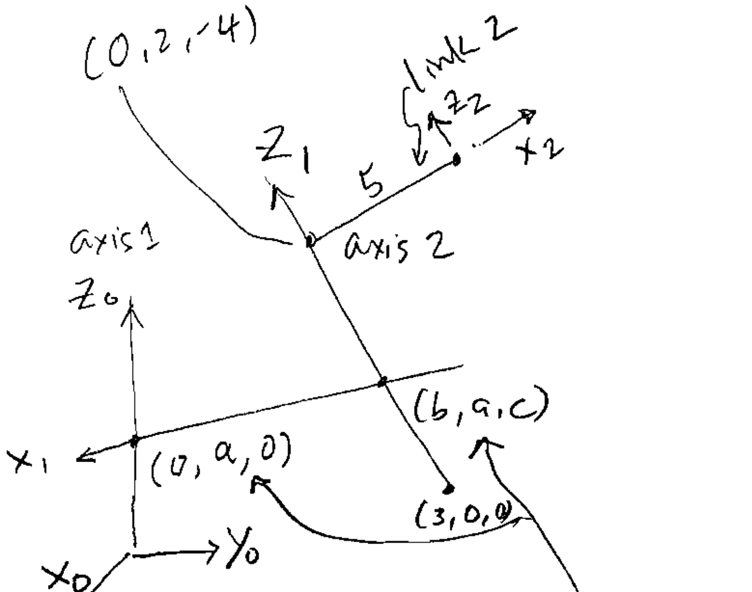
Task (u_x, w_z)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

↳ always singular.

#3





$${}^{u}z_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad {}^{u}z_1 = \begin{pmatrix} -3 \\ 2 \\ -4 \end{pmatrix} \times \frac{1}{\sqrt{}}$$

$${}^{u}x_1 = \text{along} = {}^{u}z_0 \times {}^{u}z_1 = \begin{pmatrix} -4 \\ 0 \\ 3 \end{pmatrix}$$

Since ${}^{u}x_1$ has 0 in its y component, this means ${}^{u}x_1$ is parallel to xz plane of Frame u (see figure, "a" appears in both points)

Solve for a, b, c (Intersection pts of common normal)

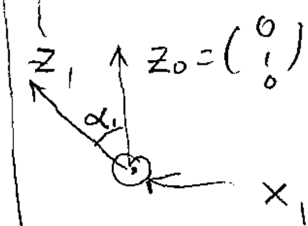
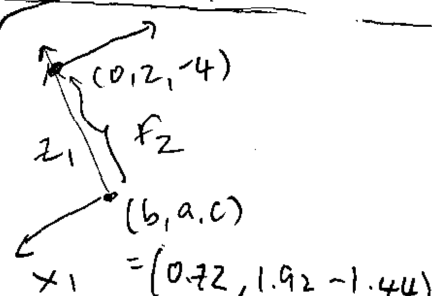
$${}^{u}x_1 : \frac{-b}{-c} = \frac{-4}{3} \qquad {}^{u}z_1 : \underbrace{\frac{b-3}{-3} = \frac{a}{2}}_{(2)} \underbrace{= \frac{c}{-4}}_{(3)}$$

$$-3b = 4c \quad (1)$$

Solving (1), (2) & (3) simultaneously yields

$$a = \frac{18}{25} = 0.72 \qquad b = \frac{48}{25} = 1.92 \qquad c = \frac{-36}{25} = -1.44$$

Observe to confirm that a, b, c make sense.

Link	θ	r	d	α
1	$q_1 =$ $x_0 \cdot x_1$ $\begin{pmatrix}0\\0\\1\end{pmatrix}\cdot\begin{pmatrix}-4\\0\\3\end{pmatrix} = 3$ $= 1 \cdot \sqrt{-4^2+3^2}\cos\theta_1$ $\theta_1 = 53.13^\circ$ but by right hand rule $q_1 = -53.13^\circ$	0.72	$\sqrt{b^2+c^2} =$ $-\sqrt{(1.92)^2+(-1.44)^2} =$ -2.4	$\begin{pmatrix}-3\\2\\-4\end{pmatrix}$  $z_1 \cdot z_0 =$ $\begin{pmatrix}-3\\2\\-4\end{pmatrix}\cdot\begin{pmatrix}0\\1\\0\end{pmatrix} =$ $= \lvert z_1\rvert\lvert z_0\rvert\cos\alpha_1$ $2 = \sqrt{(-3)^2+2^2+(-4)^2}\cos\alpha_1$ $\alpha_1 = 68.20^\circ$
2	q_2 angle with respect to x_1 about z_1 $q_2 = 0$ when $\vec{x}_1$ is parallel to $\vec{x}_2$ + facing same direction.	 $r_2 = \sqrt{0.72^2+(2-1.92)^2+(-4-1.44)^2}$ $= 5.49$ $r_2 = +5.49$ Calong positive z_1	5	0°

$$ {}^{u}T_{0} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$