ME4245E Robot Kinematics, Dynamics and Control

Quiz 1 (31 Aug 2010), 20:30-21:30

Name: ___________________________________ Matric number:_______________

Please make sure all your solutions are complete. For problems 1 and 2, you need not evaluate nor compute the final numeric value of your answers. For problem 3, please provide the numeric values of the angles.

1. (30 marks) Fig. 1 shows two frames A and B whose relative rotation and position are indicated in the figure; all coordinates are expressed in Frame A. Point C has coordinates (10, 20, 30) in Frame A. Find its coordinates in Frame B.

2. (40 marks) Fig. 2 shows Frames B and C attached to the same rigid body whose position and orientation are given by ${}^{A}T_{c}$. ${}^{B}T_{c}$ is known and Frame A is a fixed reference frame. The body undergoes the following sequence of motions:
   1^{st}> Rotation about the z axis of Frame C by 30 degrees
   2^{nd}> Rotation about the x axis of Frame A by 60 degrees
   3^{rd}> Translation along Frame B by (40, 50, 60)
   Determine the new position and orientation of Frame C in Frame A. Express this as a homogeneous transformation matrix.

3. (30 marks) The orientation of a rigid body B in A is known to be:
$$
{}^{A}R_{B} = \begin{pmatrix} 0.663 & 0.105 & 0.741 \\ 0.383 & 0.803 & -0.457 \\ -0.643 & 0.587 & 0.492 \end{pmatrix}
$$
   Find the complete solution for the roll (ϕ), pitch (θ) and yaw (φ) angles that describe this orientation. If there is more than 1 solution, please provide all the solutions sets. The roll, pitch and yaw angles are related to the rotation matrix as follows:
$$
{}^{A}R_{B} = \begin{pmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta\sin\varphi - \sin\phi\cos\varphi & \cos\phi\sin\theta\cos\varphi + \sin\phi\sin\varphi \\ \sin\phi\cos\theta & \sin\phi\sin\theta\sin\varphi + \cos\phi\cos\varphi & \sin\phi\sin\theta\cos\varphi - \cos\phi\sin\varphi \\ -\sin\theta & \cos\theta\sin\varphi & \cos\theta\cos\varphi \end{pmatrix}
$$

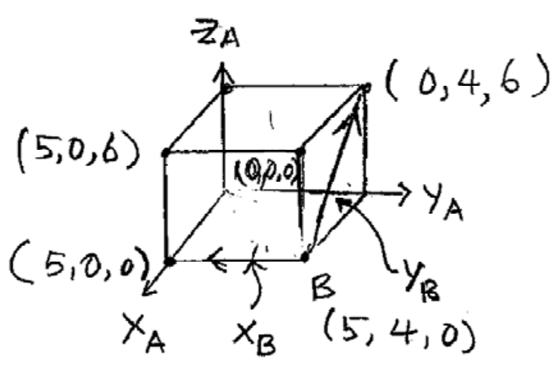

Fig. 1

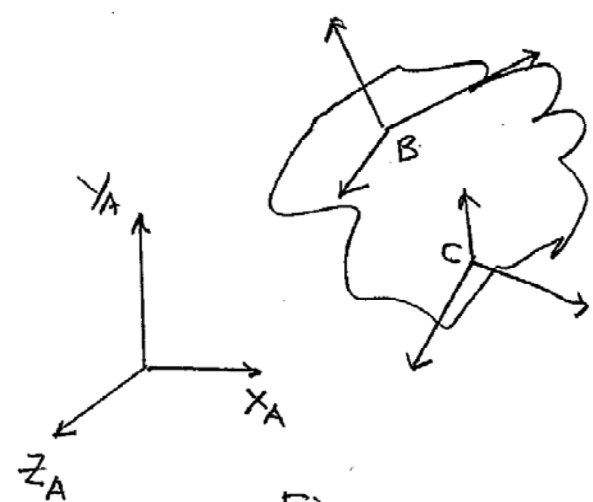

Fig. 2

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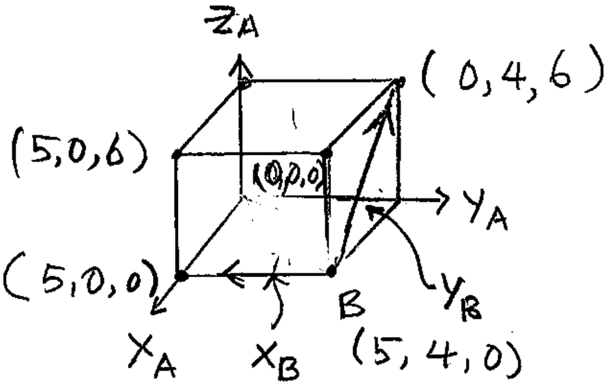

Fig. 1

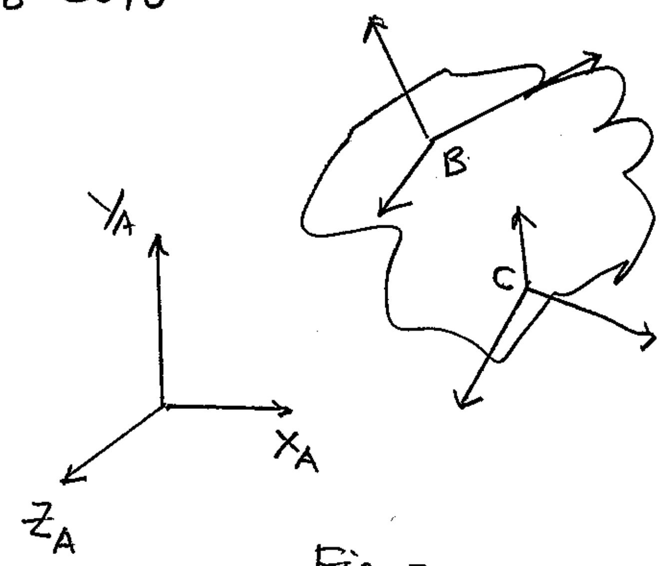

Fig. 2

1. $${}^{A}T_{B} = \begin{pmatrix} {}^{A}x_B & {}^{A}y_B & {}^{A}z_B & {}^{A}P_B \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{where } {}^{A}P_B = \begin{pmatrix} 5 \\ 4 \\ 0 \end{pmatrix} \quad {}^{A}z_B = {}^{A}x_B \times {}^{A}y_B$$

$${}^{A}x_B = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \qquad {}^{A}y_B = \begin{pmatrix} -5 \\ 0 \\ 6 \end{pmatrix} \frac{1}{\sqrt{5^2+6^2}}$$

$${}^{A}P_C = \begin{pmatrix} 10 \\ 20 \\ 30 \\ 1 \end{pmatrix} \qquad \underbrace{{}^{B}P_C}_{4\times 1} = {}^{B}T_A \, {}^{A}P_C = {}^{A}T_B^{-1} \, {}^{A}P_C$$

Final coords of C in B = (x, y, z)

where $${}^{B}P_C = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

2.

${}^{A}T_{C}$ ✓

${}^{B}T_{C}$ ✓

Let B_0, C_0 = before motion.

B_1, C_1 = after 1st motion, etc.

$${}^{A}T_{C_0} = {}^{A}T_{C}$$

$${}^{A}T_{C_1} = {}^{A}T_{C_0}\, Rot(z, 30^\circ) \qquad Rot(z,30^\circ) = \begin{pmatrix} \cos 30 & -\sin 30 & 0 & 0 \\ \sin 30 & \cos 30^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^{A}T_{C_2} = Rot(x, 60^\circ)\, {}^{A}T_{C_1}, \quad Rot(x, 60^\circ) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 60 & -\sin 60 & 0 \\ 0 & \sin 60 & \cos 60 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^{A}T_{B_2} = {}^{A}T_{C_2}\, {}^{C_2}T_{B_2} \qquad {}^{C_2}T_{B_2} = {}^{B}T_{C}^{-1}$$

$${}^{A}T_{B_3} = {}^{A}T_{B_2}\, Trans(40, 50, 60), \quad Trans(40, 50, 60^\circ) = \begin{pmatrix} 1 & 0 & 0 & 40 \\ 0 & 1 & 0 & 50 \\ 0 & 0 & 1 & 60 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^{A}T_{C_3} = {}^{A}T_{B_3}\, {}^{B_3}T_{C_3}, \quad {}^{B_3}T_{C_3} = {}^{B}T_{C}$$

3.

since $|n_z| \neq 1$ $\qquad n_z = -0.643$

$$\theta = ATAN2\left(\frac{-n_z}{\pm\sqrt{n_x^2+n_y^2}}\right) = ATAN2\left(\frac{+0.643}{\pm\sqrt{1-0.643^2}}\right)$$

$$ATAN2\left(\frac{+0.643}{\sqrt{1-0.643^2}}\right) = +40^\circ \qquad ATAN2\left(\frac{+0.643}{-\sqrt{1-0.643^2}}\right) = 140^\circ$$

Solution 1:

For $\theta = +40^\circ$, $\cos 40^\circ > 0$

$$\phi = ATAN2\left(\frac{n_y}{n_x}\right) = ATAN2\left(\frac{0.383}{0.663}\right) = 30^\circ$$

$$\psi = ATAN2\left(\frac{o_z}{a_z}\right) = ATAN2\left(\frac{0.587}{0.492}\right) = 50^\circ$$

Solution 2:

For $\theta = 140^\circ$, $\cos 140^\circ < 0$

$$\phi = ATAN2\left(\frac{-n_y}{-n_x}\right) = ATAN2\left(\frac{-0.383}{-0.663}\right) = -150^\circ$$

$$\psi = ATAN2\left(\frac{-o_z}{-a_z}\right) = ATAN2\left(\frac{-0.587}{-0.492}\right) = -130^\circ$$

Solution 1: $(30, 40, 50)$

Solution 2: $(-150, +140, -130^\circ)$