ME4245E Robot Kinematics, Dynamics and Control
Quiz 1.2 19 Sept 2012, 20:00 – 21:00

Please make sure all your solutions are complete. You need not evaluate nor compute the final numeric value of your answers.

The figure below shows a robot with 1st 2 joints rotational, and the third joint translational. At the configuration shown in the figure, the directions of the joint axes are:

1st Joint: Rotational, along Y_U
2nd Joint: Rotational, along an axis parallel to Z_U (BC)
3rd Joint: Translational, along axis parallel to Y_U (DE)

1. Assign frames to the robot according to the Denavit Hartenberg (DH) convention.

2. Draw the individual rigid bodies and show how the frames are attached to each individual body. (*Hint: There should be four rigid bodies, one fixed, and three moving.*)

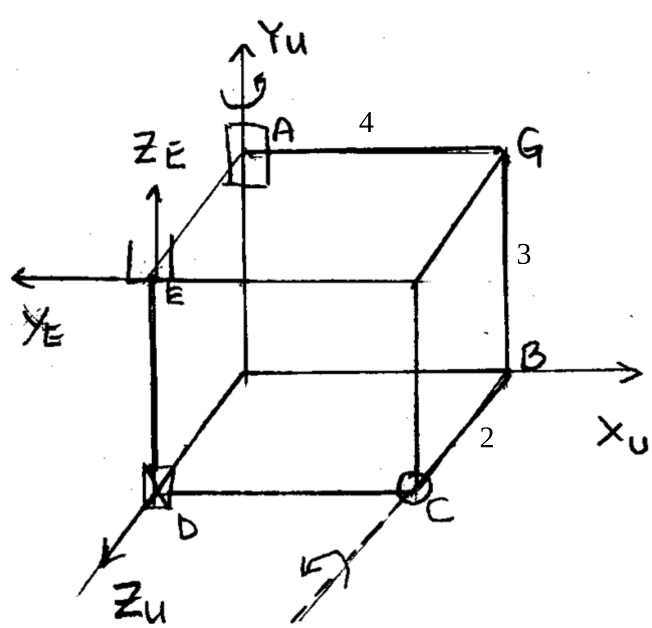
3. What are the kinematic (DH) parameters of the robot? Indicate which parameters are the joint variables.

4. Determine the expression for the position and orientation of Frame E in Frame U (${}^{U}T_{E}$) as a function of the joint variables.

5. Describe the orientation of Z_E when the robot joints are at their zero positions.

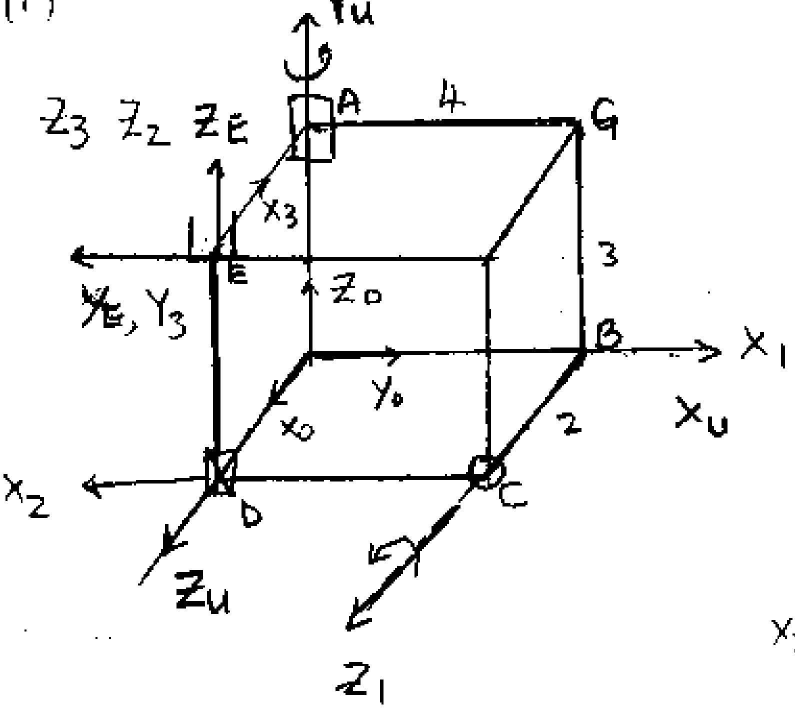
6. Which element(s) in ${}^{U}T_{E}$ is (are) constant(s)? What is (are) the value(s) of the constant(s)?

7. The robot's end-effector is carrying a tool F according to the relative position and orientation defined by ${}^{E}T_{F}$. Frame F is attached to the tool and the origin of Frame F is at the tool tip. Determine the expression for the position of F, ${}^{U}P_{F}$ as a function of the 3 joint variables (q_1, q_2, q_3) you have defined. (Added question after the quiz: Determine also the translational velocity ${}^{U}U_{F}$ as a function of the joint coordinates and velocities.)

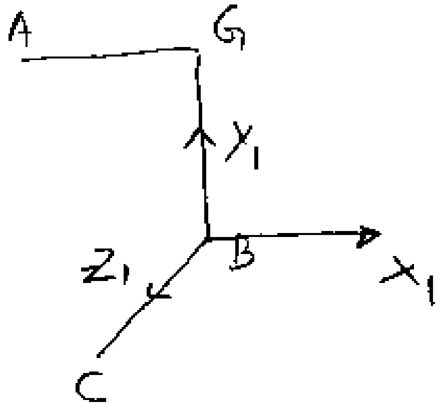


1st link: AGBC (rotating)
2nd link: CD (rotating)
3rd link: ED (translating)

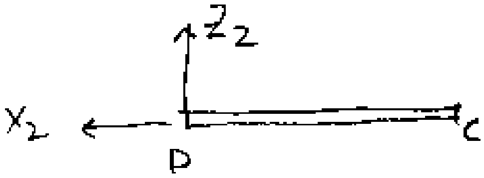
(1)



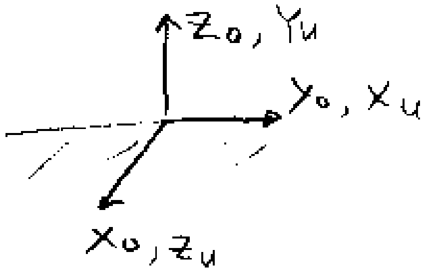
(2)



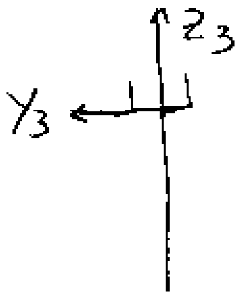
Link 1



Link 2



Base (table)
Frame ϕ



Link 3

(3)

i	θ	r	d	α
i = 1	$q_1 = 90^\circ$	0	4	90°
2	$q_2 = 180^\circ$	2	4	90°
3	-90°	$q_3 = 3$	0	0°

(4)

$$ {}^{u}T_0 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{i-1}T_i = \begin{pmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & d_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & d_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & r_i \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

for $i = 1, 2, 3$

$\theta_i, r_i, d_i, \alpha_i$ from DH Table in #(3)

$$ {}^{3}T_E = I_{4\times4} $$

$$ {}^{u}T_E = {}^{u}T_0 \, {}^{0}T_1 \, {}^{1}T_2 \, {}^{2}T_3 \, {}^{3}T_E $$

(5) ${}^{u}Z_E$ will be point down (along $-Y_u$ direction)

(6) $$ {}^{u}T_E = \begin{pmatrix} X & X & X & X \\ 0 & X & X & X \\ X & X & X & X \\ 0 & 0 & 0 & 1 \end{pmatrix} $$ Element (2, 1) = 0

row column

${}^{u}X_E$ will always be parallel to XZ Plane of Frame U

(By inspection)

(7)

$$J = \begin{pmatrix} J_1 & J_2 & J_3 \end{pmatrix}$$

$$J_1 = {}^{u}Z_0 \times ({}^{u}P_F - {}^{u}P_0)$$

$$J_2 = {}^{u}Z_1 \times ({}^{u}P_F - {}^{u}P_1)$$

$$J_3 = {}^{u}Z_2$$

$${}^{u}T_F = {}^{u}T_E \, {}^{E}T_F \quad \text{, where}$$

$$\begin{pmatrix} {}^{u}P_F \\ 1 \end{pmatrix} = {}^{u}T_F \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} {}^{u}Z_0 \\ 0 \end{pmatrix} = {}^{u}T_0 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} ; \begin{pmatrix} {}^{u}P_0 \\ 1 \end{pmatrix} = {}^{u}T_0 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} {}^{u}Z_1 \\ 0 \end{pmatrix} = {}^{u}T_1 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} , \quad {}^{u}T_1 = {}^{u}T_0 \, {}^{0}T_1$$

$$\begin{pmatrix} {}^{u}Z_2 \\ 0 \end{pmatrix} = {}^{u}T_2 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} , \quad \begin{pmatrix} {}^{u}P_1 \\ 1 \end{pmatrix} = {}^{u}T_1 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} {}^{u}P_2 \\ 1 \end{pmatrix} = {}^{u}T_2 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$${}^{u}U_F = J_1 \dot{q}_1 + J_2 \dot{q}_2 + J_3 \dot{q}_3 \quad //$$