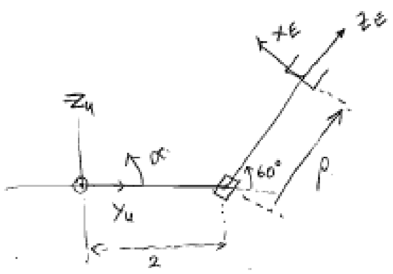
National University of Singapore Department of Mechanical Engineering
ME4245E Quiz 2 15 Sept 2008, 19:30-21:45

Name:______________________________ Matric Number:______________________
(as it appears in your NUS Student card)

Answer all the 3 questions in this quiz. You need not simplify your answers. But, please make sure all expressions are complete. Please write your answers here and please feel free use the back pages and/or add additional pages.

1. (60 marks) The figure below shows a planar robot with two joints. Frame U is fixed to the ground. The first link rotates about the X-axis of Frame U. The second link, i.e., the end-effector, translates along an axis which is 60 degrees rotated counterclockwise from the 1st link. The angle of 60 degrees does not change as the robot moves. The robot joint variables (or coordinates) are defined by α and ρ, with zero positions and positive directions indicated by the tails and heads of the arrows, respectively. Frame E is attached to the end-effector as shown.



a. Assign frames to the robot according to the D-H convention discussed in class.

b. Determine all the kinematic parameters (i.e., fill in the D-H table of kinematic parameters) and identify which of the parameters are the joint variables. Please note that the D-H joint variables q_1 and q_2 are not necessarily the same as α and ρ. Identify the values of the D-H joint variables q_1 and q_2 when the robot is at the configuration shown in the figure above.

c. Draw the robot when the D-H joint variables q_1 and q_2 are both equal to zero.

d. Determine the expression for ${}^{U}T_{E}$ (4 x 4 homogeneous transformation matrix describing the position and orientation of Frame E with respect to Frame U) as a function of α and ρ.

2. *(15 marks)* Frame U is a frame fixed to the ground. A body "A" (with Frame A attached to it) is in motion while carrying another body "B" (with Frame B attached to it). Body B is also in motion with respect to A. At some instant of time, the following quantities are known:

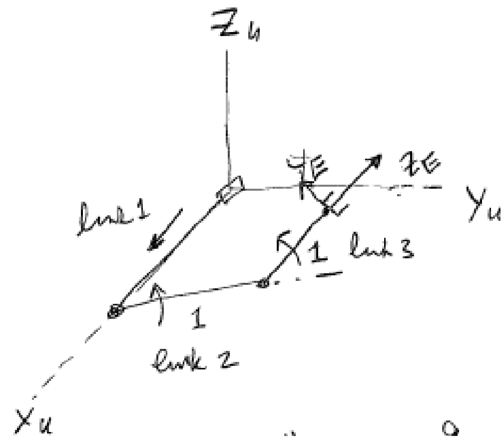
   - Motion of A with respect to U: translational velocity of ${}^{U}u_A \in R^{3x1}$ and angular velocity of ${}^{U}\omega_A \in R^{3x1}$
   - Position (${}^{U}p_A \in R^{3x1}$) and Orientation (of ${}^{U}R_A \in R^{3x3}$) of A with respect to U.
   - Position (${}^{A}p_B \in R^{3x1}$) and Orientation (of ${}^{A}R_B \in R^{3x3}$) of B with respect to A.
   - Body B is translating along the Z axis of body A at 10 m/s

   Determine the expressions for the following velocities in terms of the known quantities above:

   a. translational velocity of B with respect to U.

   b. angular velocity of B with respect to U.

3. *(25 marks)* The figure on the right shows a robot with three joints. The first link translates along the X axis of Frame U which is fixed to the ground. The second and third links rotate about two axes that are parallel to the X-axis of Frame U. Links 2 and 3 always move in a plane parallel the YZ plane of Frame U. Frame E is attached to the end-effector (link 3) such that its Y and Z axes are always parallel to the YZ plane of Frame U.



a. Determine the full manipulator Jacobian that relates the 6 x 1 generalized (translational (u_x, u_y, u_z) and angular (ω_x, ω_y, ω_z)) velocity of the end-effector (Frame E) with the joint velocities.

b. If the task of interest is (u_x, u_y, u_z), determine the singular configurations, if any, of the robot. If applicable, describe the lost degrees of freedom for each singular configuration of the robot.

c. If the task of interest is (u_x, u_y), determine the singular configurations, if any, of the robot. If applicable, describe the lost degrees of freedom for each singular configuration of the robot.

d. If the task of interest is (u_x, ω_x), determine the singular configurations, if any, of the robot. If applicable, describe the lost degrees of freedom for each singular configuration of the robot.

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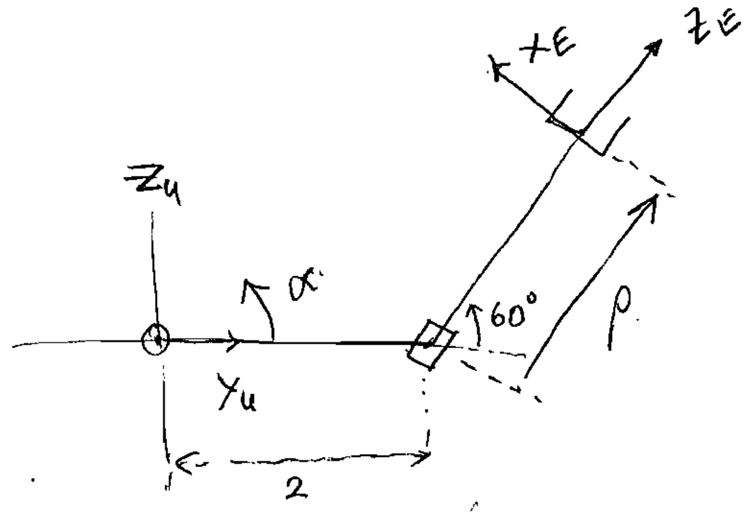
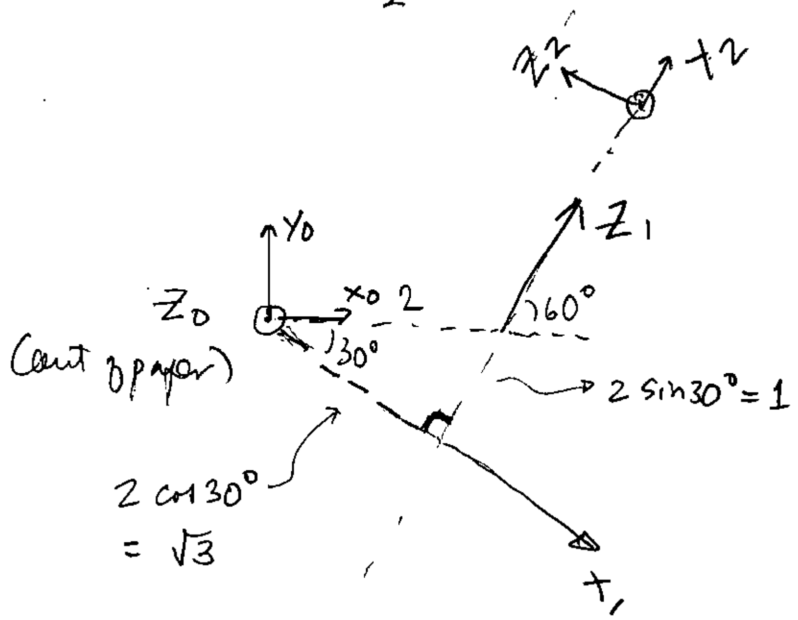
#1



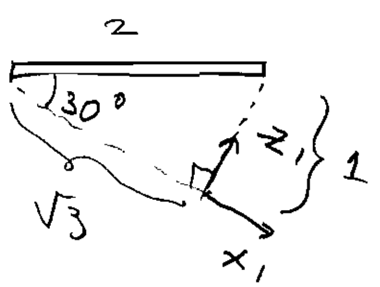


Link	θ	r	d	α
1	$q_1 = -30^\circ$	0	$\sqrt{3}$	-90°
2	-90°	$q_2 = 1 + \rho$	0	90°

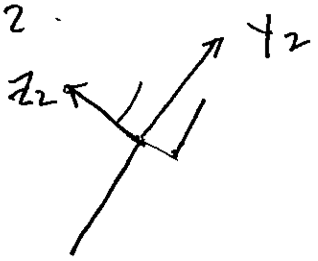
$$q_1 = -30^\circ + \alpha$$

$$q_2 = 1 + \rho$$

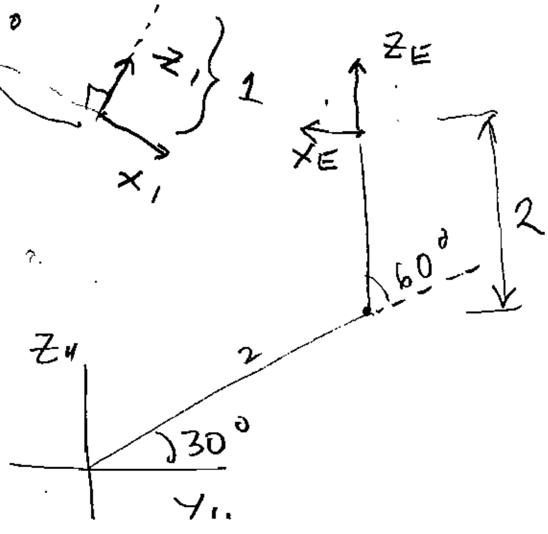
Link 1:



Link 2



when $q_1 = 0$
$q_2 = 3$



$$ {}^{u}T_0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$ By inspection.

$$ {}^{2}T_E = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$ By inspection

$$ {}^{0}T_1 = \begin{pmatrix} \cos q_1 & -\cos(-90^\circ)\sin q_1 & \sin(-90^\circ)\sin q_1 & \sqrt{3}\cos q_1 \\ \sin q_1 & \cos(-90^\circ)\cos q_1 & -\sin(-90^\circ)\cos q_1 & \sqrt{3}\sin q_1 \\ 0 & \sin(-90^\circ) & \cos(-90^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{1}T_2 = \begin{pmatrix} \cos(-90^\circ) & -\cos(90^\circ)\sin(-90^\circ) & \sin 90^\circ \sin(-90^\circ) & 0 \\ \sin(-90^\circ) & \cos(90^\circ)\cos(-90^\circ) & -\sin 90^\circ \cos(-90^\circ) & 0 \\ 0 & \sin(90^\circ) & \cos 90^\circ & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ q_1 = -30^\circ + \alpha, \quad q_2 = 1 + \rho $$

$$ {}^{u}T_E = {}^{u}T_0 \; {}^{0}T_1 \; {}^{1}T_2 \; {}^{2}T_E $$

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#2 QUIZ 2, ME4245

$$^{u}T_B = {}^{u}T_A \; {}^{A}T_B$$

$$^{u}\dot{T}_B = {}^{u}T_A \; {}^{A}\dot{T}_B + {}^{u}\dot{T}_A \; {}^{A}T_B$$

translation only

$$\begin{pmatrix} {}^{u}\hat{\omega}_B \, {}^{u}R_B & {}^{u}u_B \\ \hline 0 & 0 \end{pmatrix} = \begin{pmatrix} {}^{u}R_A & {}^{u}p_B \\ \hline 0 & 1 \end{pmatrix} \begin{pmatrix} \cancel{{}^{A}\hat{\omega}_B \, {}^{A}R_B} & {}^{A}u_B \\ \hline 0 & 0 \end{pmatrix} + \begin{pmatrix} {}^{u}\hat{\omega}_A \, {}^{u}R_A & {}^{u}u_A \\ \hline 0 & 0 \end{pmatrix}$$

$$^{A}u_B = \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix} \qquad \begin{pmatrix} {}^{A}R_B & {}^{A}p_B \\ \hline 0 & 1 \end{pmatrix}$$

$$^{u}u_B = {}^{u}R_A \, {}^{A}u_B + \underbrace{{}^{u}\hat{\omega}_A \, {}^{u}R_A \, {}^{A}p_B}_{\downarrow} + {}^{u}u_A \quad //$$

$$^{u}\omega_A \times ({}^{u}p_B - {}^{u}p_A)$$

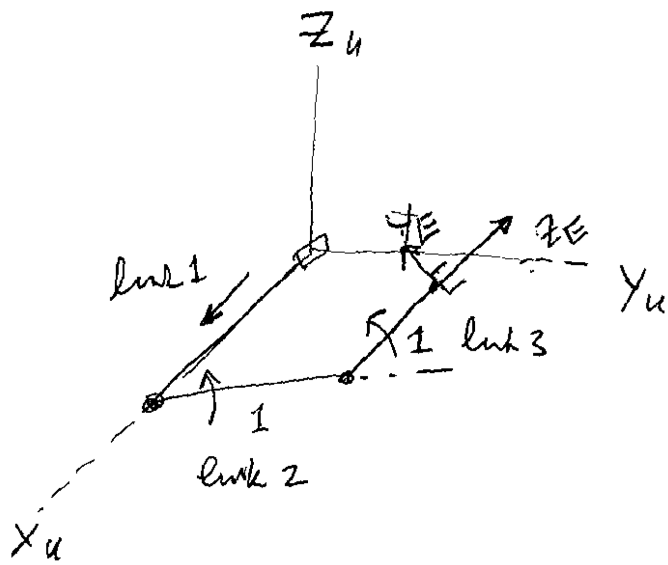
$$^{u}\hat{\omega}_B \, {}^{u}R_B = \cancel{{}^{u}R_A (\phi)} + {}^{u}\hat{\omega}_A \, {}^{u}R_A \, {}^{A}R_B$$

ϕ

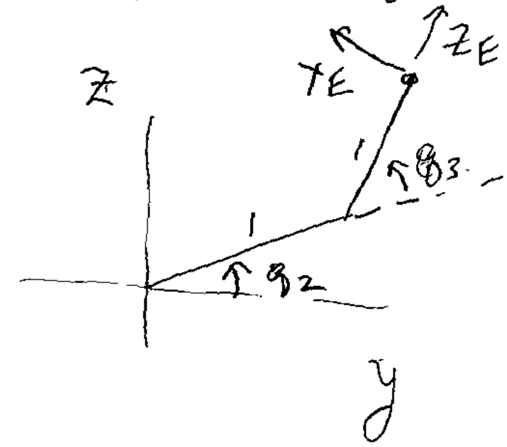
$$^{u}\hat{\omega}_B = {}^{u}\hat{\omega}_A$$

$$^{u}\omega_B = {}^{u}\omega_A \quad //$$

#3



On a plane parallel to YZ plane of Frame U



$$^{u}P_E = \begin{matrix} q_1 \\ \cos q_2 + \cos(q_2+q_3) \\ \sin q_2 + \sin(q_2+q_3) \end{matrix} = \begin{pmatrix} q_1 \\ C_2 + C_{23} \\ S_2 + S_{23} \end{pmatrix}$$

$$\frac{d}{dt}\,{}^{u}P_E = \begin{matrix} \dot q_1 \\ -S_2\dot q_2 - S_{23}(\dot q_2 + \dot q_3) \\ C_2 \dot q_2 + C_{23}(\dot q_2 + \dot q_3) \end{matrix} = \begin{matrix} \dot q_1 \\ (-S_2 - S_{23})\dot q_2 - S_{23}\dot q_3 \\ (C_2 + C_{23})\dot q_2 + C_{23}\dot q_3 \end{matrix}$$

$$^{u}V_E = {}^{u}J_E\,\dot q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -S_2-S_{23} & -S_{23} \\ 0 & C_2+C_{23} & C_{23} \\ 0 & \cancel{0}\ 1 & \cancel{0}\ 1 \\ 0 & 0 & \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \dot q_1 \\ \dot q_2 \\ \dot q_3 \end{pmatrix}$$

$$= \begin{matrix} u_x \\ u_y \\ u_z \\ \omega_x \\ \omega_y \\ \omega_z \end{matrix}$$

Task (u_x, u_y, u_z)

$$\det\begin{pmatrix} 1 & 0 & 0 \\ 0 & -S_2-S_{23} & -S_{23} \\ 0 & C_2+C_{23} & C_{23} \end{pmatrix} = (-S_2 - S_{23})C_{23} + (C_2 + C_{23})S_{23}$$

$$S_{23}C_2 - C_{23}S_2 = S_3$$

$$\theta_3 = 0,\ 180^\circ$$

Task (u_x, u_y)

singularities when $\theta_2 = 0, \theta_3 = 0$ (by inspection)

$\theta_2 = 0, \theta_3 = 180°$

$\theta_2 = 180°, \theta_3 = 0°$

$\theta_2 = 180°, \theta_3 = 180°$

or

$$\begin{matrix} 1 & 0 & 0 \\ 0 & -S_2 - S_{23} & -S_{23} \end{matrix}$$

②

①

③

all possible determinant $(2 \times 2) = 0$ if $S_2 = 0, S_{23} = 0$
(3 of them)

Task (u_x, ω_z)

no singularities: by inspection

or

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$\Downarrow$

$\neq 0$ determinant

One 2×2 matrix is always non-zero