National University of Singapore
Faculty of Engineering

Drill Problem Set 1: ME4245/EE4304: Robotics by: Marcelo Ang Jr.

1. Frame B is initially coincident to frame A in Figure1(a). Frame B is then rotated 30 degrees about the vector described by the directed line segment from P to Q (following the right-hand rule). Determine the position and orientation of the new frame B with respect to frame A. Express your answer in the form of a homogeneous transformation matrix.

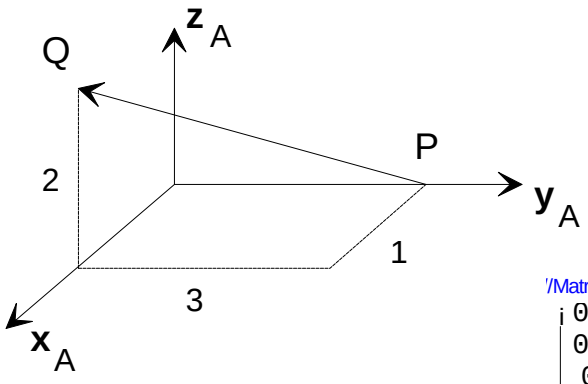


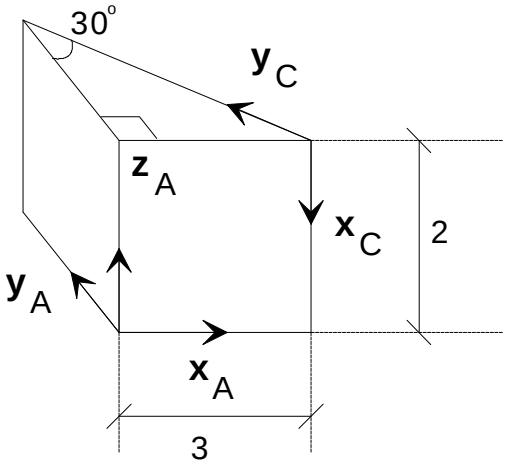
Figure 1 (a)

Ans:

'/MatrixForm=

$$\begin{pmatrix} 0.876 & -0.296 & -0.382 & 0.888 \\ 0.239 & 0.952 & -0.191 & 0.144 \\ 0.42 & 0.0762 & 0.904 & -0.229 \\ 0 & 0 & 0 & 1. \end{pmatrix}$$

(b) Referring to Figure 1(b), determine the homegeneous transformation matrix that describes frame C in frame A. Also determine the homegeneous transformation matrix that describes frame A in frame C.



Ans:

$$^{A}T_{C} = \begin{pmatrix} 0 & -0.5 & 0.866 & 3 \\ 0 & 0.866 & 0.5 & 0 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^{C}T_{A} = \begin{pmatrix} 0 & 0 & -1 & 2 \\ -0.5 & 0.866 & 0 & 1.5 \\ 0.866 & 0.5 & 0 & -2.598 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

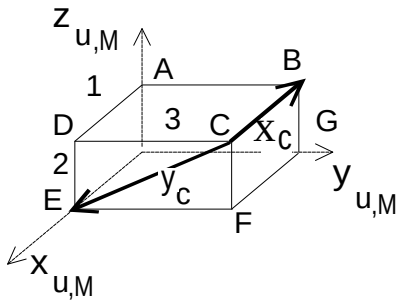
Figure 1(b)

2. Frames M and C are attached rigidly to a cuboid as shown in Fig. 3. Frame U is fixed and serves as the universe frame of reference. The cube undergoes the following motion in the indicated sequence:

1> Rotation about the z axis of Frame C by 30°, then
2> Translation of (1, 2, 3) along Frame C, then
3> Rotation about the x axis of Frame M by 45°, and then
4> Rotation about the y axis of Frame U by 60°.

Let ${}^{U}T_{C_i}$ and ${}^{U}T_{M_i}$ be the 4 × 4 homogeneous transformation matrices that describes the position and orientation of Frames *C* and M, respectively, in *U* after motion *i*.

Find
i. ${}^{U}T_{C_1}$
ii. ${}^{U}T_{C_2}$
iii. ${}^{U}T_{C_3}$
iv. ${}^{U}T_{C_4}$
v. ${}^{U}T_{M_4}$



line segment lengths:
AD=1
DC=3
DE=2

Figure 3

Ans:

$${}^{U}T_{C_1} = \begin{pmatrix} -0.866 & 0.5 & 0 & 1 \\ -0.416 & -0.721 & -0.555 & 3 \\ -0.277 & -0.48 & 0.832 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad {}^{U}T_{C_2} = \begin{pmatrix} -0.866 & 0.5 & 0 & 1.134 \\ -0.416 & -0.721 & -0.555 & -0.521 \\ -0.277 & -0.48 & 0.832 & 3.258 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

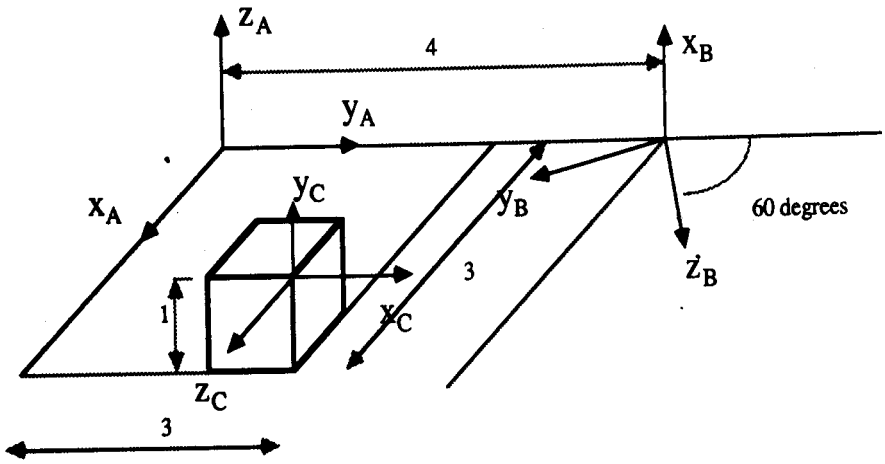
$${}^{U}T_{C_3} = \begin{pmatrix} -0.866 & 0.354 & 0.354 & 1.662 \\ -0.416 & -0.117 & -0.902 & -2.696 \\ -0.277 & -0.928 & 0.249 & 4.872 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^{U}T_{C_4} = \begin{pmatrix} -0.6732 & -0.627 & 0.392 & 5.05 \\ -0.416 & -0.117 & -0.902 & -2.696 \\ 0.611 & -0.77 & -0.182 & 0.997 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^{U}T_{M_4} = \begin{pmatrix} 0.6732 & 0.304 & 0.674 & 2.117 \\ 0.416 & 0.598 & -0.685 & -3.535 \\ -0.611 & 0.742 & 0.276 & -1.169 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3 Frame C is firmly attached to a corner of the rigid cube with z_C parallel x_A and y_C parallel to z_A, as shown in the figure below. Frame B is located at a fixed position and orientation with respect to Frame A with x_B parallel to z_A and the angle 60 degrees represents a rotation about x_B. The following ordered sequence of motions is applied to the cube

I) rotation about y_B by 45 degrees, followed by
II) rotation about x_C by 30 degrees.

Find the new position and orientation of Frame C expressed in Frame A.



Ans:

$$ {}^{A}T_{C} = \begin{pmatrix} -0.127 & -0.140 & 0.982 & 1.855 \\ 0.927 & -0.370 & 0.067 & 2.339 \\ 0.354 & 0.918 & 0.177 & 2.190 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$