NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR THE DEGREE OF B.ENG

Semester I 1996/1997

EE4304/ME4245 ROBOTICS

Oct/Nov 1996 Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains five (5) questions in 2 Sections, and comprises seven (7) pages.
2. Answer all questions (Q.1, 2 and 3) in Section A, and any 1 question (Q.4 or 5) in Section B.
3. All questions carry equal marks.
4. This is an open-book examination.

SECTION A: COMPULSORY
(Answer all the three questions in this section.)

Q.1 Figure 1 shows a schematic diagram of a robot with four joints (J_1, J_2, J_3, J_4). All joints are rotational except for the 2nd joint J_2, which is a translational joint. The axes of motion of all the joints are parallel to each other and to $\mathbf{y}_B$, as indicated in Figure 1. Frame B is fixed to the ground and $\mathbf{y}_B$ is aligned with the first joint axis. Frame E is attached to the end-effector (last link) in a manner indicated in Figure 1.

(a) Assign Cartesian coordinate frames to each link of the robot according to the Denavit-Hartenberg convention discussed in class. In assigning the frames, as far as possible, make all constant parameters zero.
(4 marks)

(b) Derive the table of kinematic parameters and indicate which parameter is the joint coordinate (i.e., which joint parameter is variable).
(3 marks)

(c) Derive an expression for the position and orientation of the end-effector expressed in Frame B, ${}^{B}T_{E}$.
(4 marks)

(d) Determine the values of the joint coordinates for the configuration shown in Figure 1.
(3 marks)

(e) Is it possible to find a decoupled system for the robot? If so, identify the decoupled system. If not, explain why not.
(3 marks)

(f) Derive the **complete** inverse kinematic solutions for this robot. That is, derive the expressions for q_1, q_2, q_3 and q_4 as functions of elements of ${}^{B}T_{E}$.
(8 marks)

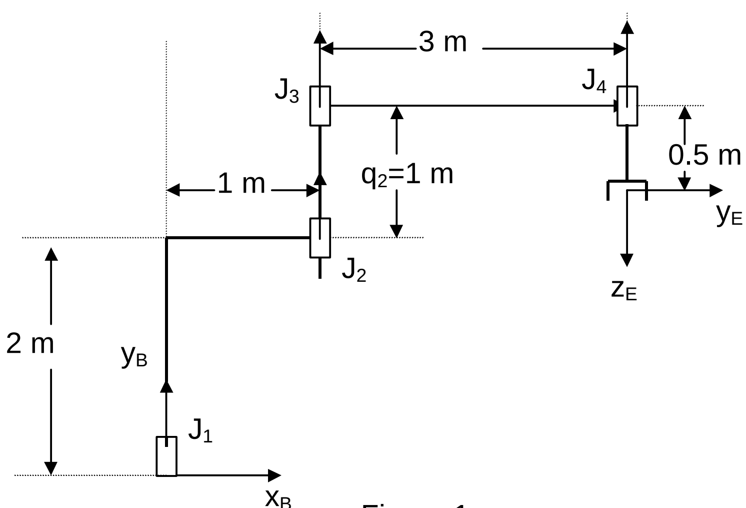

Figure 1

Q.2 (a) Frame B is attached to a rigid rectangular block as shown in Figure 2. The block is initially positioned in space with reference to Frames A and E which are fixed frames of reference. The block undergoes the following ordered sequence of motion:

1st Rotation about z_A by 30°.
2nd Rotation about y_A by 45°.
3rd Rotation about x_B by 90°.
4th Rotation about x_E by 60°.

(i) Express the initial position and orientation of Frame B in A using a homogeneous transformation matrix, ${}^{A}T_B$. (3 marks)

(ii) Determine an expression for the new position and orientation of Frame B in A after the four motions. (Express this in terms of known quantities; you do not need to numerically evaluate this.) (8 marks)

(iii) Determine an expression for the new coordinates of the vertex point C in Frame A after the four motions. (Express the position in terms of known quantities.) (4 marks)

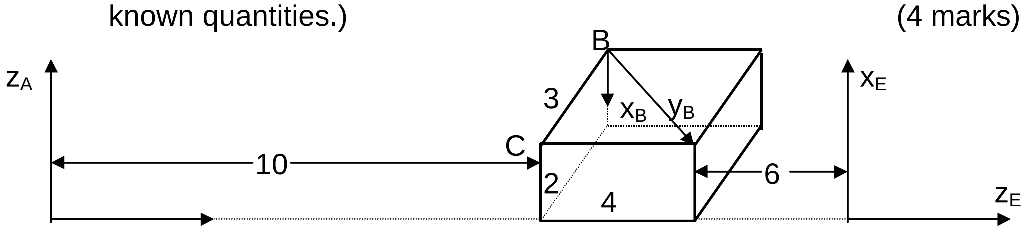

Figure 2

(b) Figure 3 shows a 2-link robot attached to a rigid block B at Joint A, which is a rotational joint whose axis of motion is parallel to z_B. Link L_1 is parallel to the xy plane of Frame B, has length 2 m, and is rotating at a constant speed of 3 rad/s. Link L_2 has length 1 m and is rotating at a constant speed of 2 rad/s. The axis of motion of L_2 is along Link L_1 in the direction shown in Figure 3. The block B is sliding up the inclined plane at a constant rate of 1 m/s. At a certain instant of time t_1, the links are parallel to the xy plane of Frame B (configuration shown in Figure 3.)

(i) Determine the translational velocity of D and angular velocity of link L_2 at this instant of time t_1. (6 marks)

(ii) Assume that the point mass load with a weight of 10 N is attached to to point D. (The direction of gravity is along the negative y_B axis.) Determine the torques at joints A and C to support the load. (4 marks)

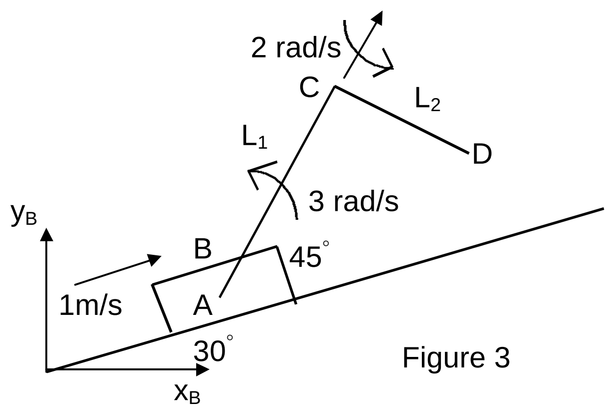

Figure 3

Q.3 Assume that the robot shown in Figure 4 is in the vertical plane, the mass of each link is concentrated at a point in the indicated mass centres with m_1 and m_2 being the equivalent masses of links 1 and 2 respectively, L_{1c} is the length from the centre of mass m_1 to joint 1, L_1 and L_2 are the lengths of links 1 and 2, t_1 and t_2 are the output torques of motors 1 and 2 which are located at the joints.

(a) Give two different sets of generalised coordinates for the robot manipulator. Draw two separate figures of the manipulator indicating the generalised coordinates that you choose. Indicate the type of joint for the each of the two joints as well.

(3 marks)

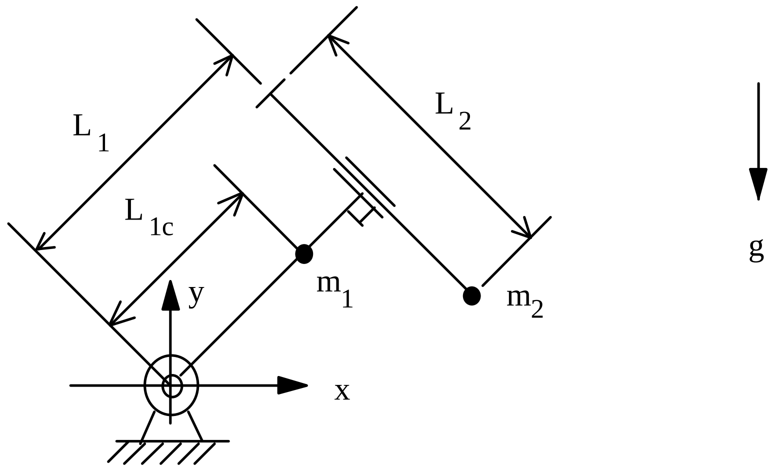

Figure 4

(b) For any one set of generalized coordinates from (a), find

(i) the positions and velocities of m_2 and m_3 with respect to the base coordinate system $x_o y_o z_o$; (4 marks)

(ii) the expression for matrix *D(q)* from

$$K = \frac{1}{2}\dot{q}^T D(q)\dot{q};$$ (2 marks)

(iii) the potential energy *V(q)* of the robot; (2 marks)

(iv) the Lagrange-Euler equations

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = t$$

with $C(q,\dot{q})$ being defined by the christoffel symbols, and *G(q)* being derived from the potential energy. (6 marks)

(c) List at least three general properties about the dynamical equations in Part (b). (3 marks)

(d) Design a computed torque controller for this robot. Discuss its advantages and disadvantages. (5 marks)

SECTION B
(Answer Only One out of the Two Questions in this Section)

Q.4 (a) For the system shown in Figure 5, m_1 and m_2 are the masses of blocks 1 and 2, *k* is the spring constant of the linear spring connecting the two blocks, d_1 and d_2 are the displacements as indicated, and f_1 and f_2 are external forces on blocks 1 and 2.

Derive the equations of motion for the system using any method you feel comfortable with, and express the obtained dynamics in state space form if the states are chosen as: $x_1 = d_1$, $x_2 = d_2$, $x_3 = \dot{d}_1$ and $x_3 = \dot{d}_2$.

(9 marks)

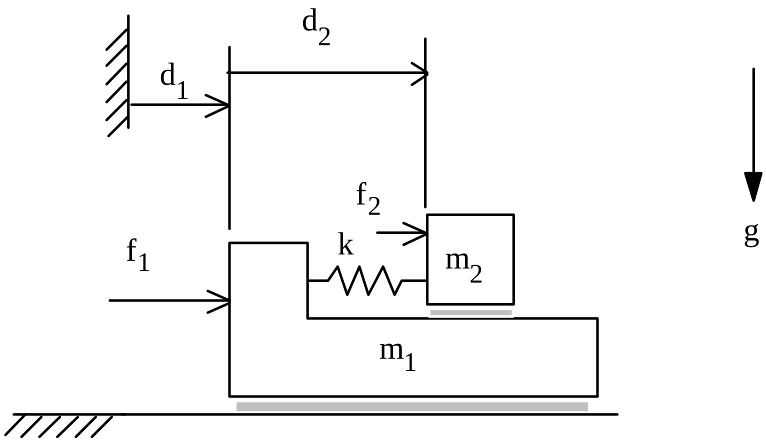

Figure 5

(b) If the generalized coordinates, d_1 and d_2, are chosen as shown in Figure 6 and all other parameters are the same as for the robot shown in Figure 5, derive the equations of motion for the system, and state the difference between the two resulting dynamic equations.

(6 marks)

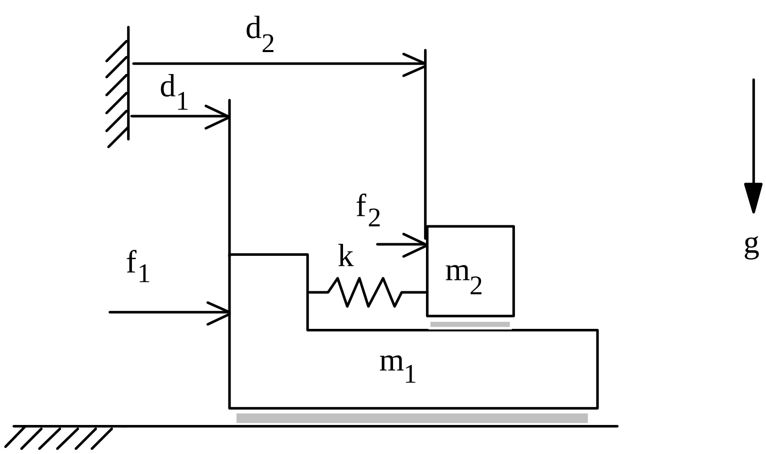

Figure 6

Question Q.4 is continued on Page 6.

(c) Consider the dynamical equations of a robot which are given as:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau$$

Let q_d be the desired trajectory, and $\dot{q}_d$ and $\ddot{q}_d$ be the first and second derivatives of the desired trajectory. Define tracking errors $e = q_d - q$, and $r = \dot{e} + \lambda e$. Further, define $\dot{q}_r = \dot{q}_d + \lambda e$, and let the control law be:

$$\tau = D(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + G(q) + Kr$$

where $K = K^T > 0$.

(i) Show that the closed-loop system is stable by choosing an appropriate Lyapunov function.

(5 marks)

(ii) Briefly discuss the difference between this controller and that from the computed torque method. In implementing the above controller, exact knowledge is required. Give an adaptive control scheme to overcome this problem, and show that the closed-loop system is stable.

(5 marks)

Q.5 (a) For the three degrees-of-freedom robot shown in Figure 7, joint 1 is rotational while joints 2 and 3 are translational. Here, I_1 is the moment of inertia of joint 1, and m_2 and m_3 are equivalent masses of links 2 and 3. Further, q_1 is the angular displacement of joint 1, and q_2 and q_3 are the displacements of links 2 and 3. Let τ_1 be the output torque of motor 1, and f_2 and f_3 be the output forces of motors 2 and 3.

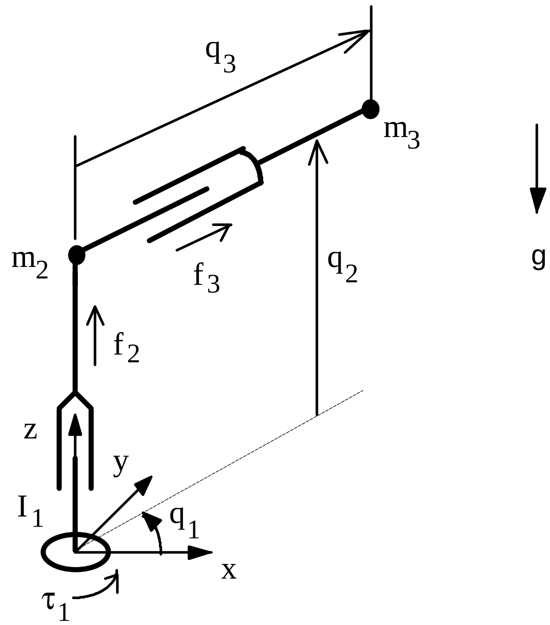

Figure 7

Derive the equations of motion for the three degrees-of-freedom robot which should be written in the form:

$$D(q)\ddot{q} + H(q,\dot{q}) + G(q) = \tau$$

where $q = [q_1\ q_2\ q_3]^T$, $\tau = [\tau_1\ f_2\ f_3]^T$, $D(q)$ is the inertia matrix, $H(q,\dot{q})$ is the coriolis and centrifugal force vector, and $G(q)$ is the gravitational force vector.

(13 marks)

(b) A SCARA robot is employed in a robotic workcell to assemble hard disk drives. The robot picks up the disk media and places them into a hub. Describe how the following topics in robotics are employed in order for the robot to operate and accomplish its task:

i) forward kinematics of position;
ii) inverse kinematics of position;
iii) force/torque transformations;
iv) Jacobian of the robot;
v) velocity transformations.

(12 marks)

END of PAPER