NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR THE DEGREE OF B.ENG

Semester I 1995/1996

ME4245/EE4304 ROBOTICS

Oct/Nov 1995 Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains five (5) questions in 2 Sections, and comprises six (6) pages.
2. Answer all questions (Q.1, 2 and 3) in Section A, and any 1 question (Q.4 or 5) in Section B.
3. All questions carry equal marks.
4. This is an open-book examination.

SECTION A: COMPULSORY
(Answer all the three questions in this section.)

Q.1 A 3-dof planar robot is shown in Figure 1. All joints are rotational and all the joint axes are parallel. The robot is designed to position the end effector (Frame E) at a point (p_x, p_y) in the x_0y_0 plane and to orient the end-effector such that z_E makes an angle Φ with respect to the horizontal (x_0) axis. The lengths of each link are L_1, L_2, and L_3, respectively, starting from the base. Assume that $L_1 > L_2 > L_3$. The zero position of each joint is indicated in the figure together with the direction for positive motion.

(a) Derive the homogenous tranformation matrix ${}^0\mathbf{T}_E$ that describes the position and orientation of the end-effector (Frame E) in the base frame (Frame 0) as a function of the three joint coordinates q_1, q_2 and q_3.

(*Hint*: *You may be able to determine the orientation part of* ${}^0\mathbf{T}_E$ *by inspection.)*

(8 marks)

(b) Derive the **complete** inverse kinematic solutions for this robot. That is, derive the expressions for q_1, q_2 and q_3 as functions of p_x, p_y and Φ.

(8 marks)

(c) Derive an expression that describes the position and orientation workspace of this robot. Assume that there are no joint limits.

(9 marks)

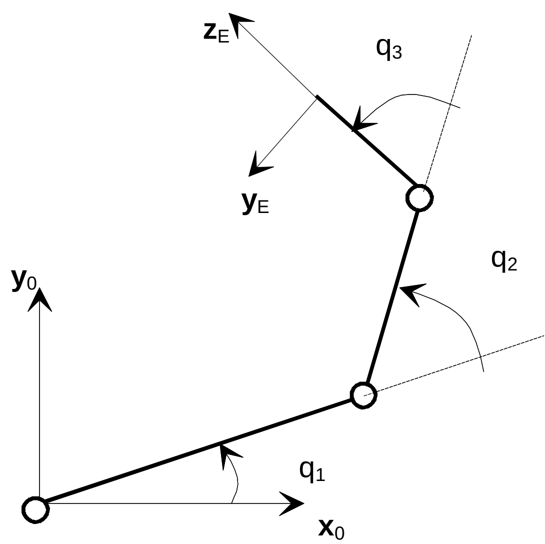

Figure 1

Q.2 (a) The 3-DOF manipulator in Fig. 2 has one translational joint (q_1) followed by two rotational joints (q_2 and q_3). The axes of motion are as shown in the figure. At the configuration shown, we have q_1 = 100 mm, $q_2 = 0°$ and $q_3 = 45°$. Frame U serves as the fixed frame of reference. Frame E is attached rigidly to the last link (end-effector) of the robot.

(i) Assign frames to each link according to the Denavit-Hartenberg convention discussed in the class. Make sure that the frame assignments are consistent with the positive directions of motion and zero positions indicated in Fig. 2

(6 marks)

(ii) Derive the table of kinematic parameters consistent with your frame definitions in (i). That is, identify the four kinematic parameters for each of the three links of the robot. Indicate which of the four kinematic parameters for each link is variable.

(6 marks)

(iii) Derive expressions for ${}^{U}\mathbf{T}_0$ and ${}^{3}\mathbf{T}_E$ that describe the position and orientation relationships between the Denavit-Hartenberg frames attached to the base and the last link, and the given frames (U and E).

(3 marks)

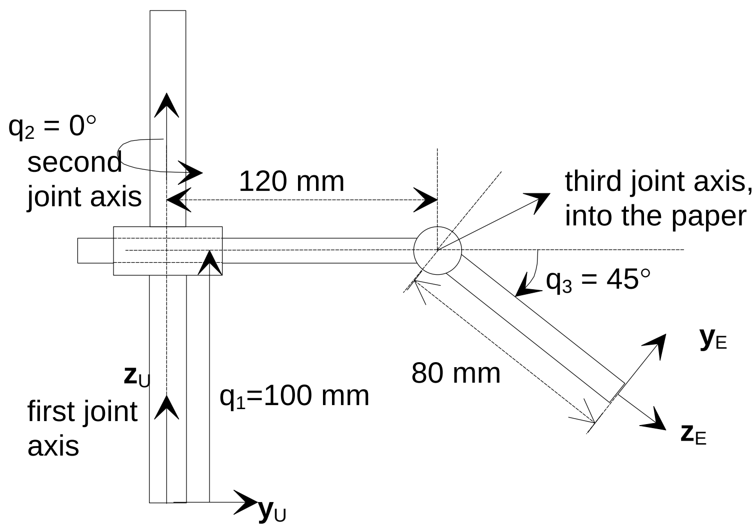

Figure 2

(b) Frame A is fixed and Frames B and C are attached to the same rigid body that is moving. At a certain time instant, t = t_1, the position and orientation of Frames C and A are known to be ${}^{A}\mathbf{T}_B(t_1)$ and ${}^{A}\mathbf{T}_C(t_1)$, respectively. Then, the rigid body undergoes the following ordered sequence of motion: .

<1> Rotation about the y axis of Frame A by θ_1, followed by
<2> Translation along Frame B by (x,y,z), followed by
<3> Rotation about the x axis of Frame C by θ_2.

Derive an expression for the new position and orientation of Frame B in Frame A, ${}^{A}\mathbf{T}_B$. (You do not need to simplify your expression. Express your answer in terms of matrix products.)

(10 marks)

Q.3 (a) In Figure 3, m is the mass of the payload, m_{1c} and m_{2c} are the masses of links 1 and 2, and L_{1c} and L_{2c} are the lengths from the centres of masses m_{1c} and m_{2c} to joint 1 and joint 2, respectively. Let t_1 and t_2 be the output torques of motors 1 and 2 which are located at the joints. Show that the equations of motion for the two degrees-of-freedom robot in the horizontal plane can be written as

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} = t$$

where $q = [q_1 \quad q_2]^T$, $\tau = [\tau_1 \quad \tau_2]^T$, $D(q)$ is the inertia matrix, and $C(q,\dot{q})$ is the matrix defined by the so-called Christoffel Symbols by determining the expressions for $D(q)$ and $C(q,\dot{q})$. Assume that the mass of each link is concentrated at a point in the indicated mass centres.

(12 marks)

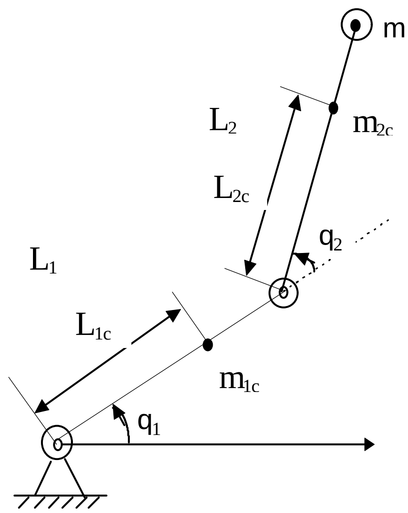

Figure 3: Two link rotary robot in the horizontal plane

(b) Find an inverse dynamics control law τ to transform this system into a double integrator system:

$$\ddot{q} = v$$

where v is the new control to be designed.

(6 marks)

(c) Using the control law

$$v = r + K_V\dot{q} + K_p q$$

where r is the reference signal, select values of K_v and K_p so that the resulting system is decoupled, critically damped, and with natural frequency $\omega = 5$ rad/s.

(7 marks)

SECTION B
(Answer Only One out of the Two Questions in this Section)

Q.4 (a) For the pole-cart system shown in Figure 4, f_1 is the external force on the cart, t_2 is the torque to drive the pole, m_1 and m_2 are the lumped equivalent masses of the cart and the pole, x and q are the displacements of the cart and the pole, and L is the length of the pole. Assume that the system is in the vertical plane and there is no friction. Derive the dynamic equations using any method you feel comfortable with.

(10 marks)

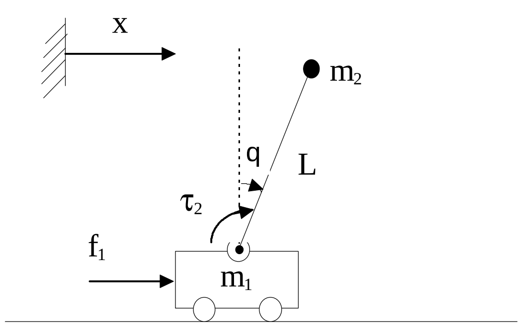

Figure 4: Pole-cart system with two inputs

(b) Let q_d be the desired trajectory, and $\dot{q}_d$ and $\ddot{q}_d$ be the first and second derivatives of the desired trajectory. Define tracking errors e = q_d - q, and r = $\dot{e} + \lambda$ e. Further, define $\dot{q}_r = \dot{q}_d + \lambda$ e, and let the control law be:

$$t = D(q)\,\ddot{q}_r + C(q,\dot{q})\dot{q}_r + G(q) + Kr$$

where $K = K^T > 0$.

(i) Draw a block diagram of the control scheme.

(5 marks)

(ii) Show that the closed-loop system is stable by choosing an appropriate Lyapunov function.

(5 marks)

(iii) Discuss briefly the advantages and disadvantages in implementing the above control scheme.

(5 marks)

Q.5 (a) Given a system described by the following dynamic equations:

$$(m_1 l_1^2 + I_1 + I_2 + m_2 q_2^2)\ddot{q}_1 + 2m_2 q_2 \dot{q}_1 \dot{q}_2 + (m_1 l_1 + m_2 q_2) g \cos q_1 = \tau_1$$

$$m_2 \ddot{q}_2 - m_2 q_2 \dot{q}_1^2 + m_2 g \sin q_1 = \tau_2$$

where m_i, I_i and l_i are constants, the gravitational acceleration g is 9.8m/s^2, and q_i and τ_i are the generalised coordinates and generalised forces/torques for i = 1,2.

(i) Determine the inertia matrix, the coriolis force and centrifugal force vectors, and the gravitational force vector.

(4 marks)

(ii) Determine the state equations using the state $x_1 = q_1$, $x_2 = q_2$, $x_3 = \dot{q}_1$ and $x_4 = \dot{q}_2$.

(5 marks)

(iii) Briefly discuss the basic idea of the independent joint control scheme and the advantages and disadvantages in implementing this scheme.

(4 marks)

(b) Figure 5 shows a six-axis force torque sensor mounted between the end of a robotic arm and a rigid tool consisting of straight segments of lengths 2 cm, 3 cm and 4 cm. Frame S is the sensor frame and all sensor readings are expressed in this frame. The straight segments of the tool of lengths 2 cm, 3 cm and 4 cm, are parallel to the z, y and x axes of Frame S, respectively. A 10 N downward force is exerted on the end of the tool (at point E) in the direction parallel to the z axis of Frame S. Determine all the six readings of the force sensor.

(12 marks)

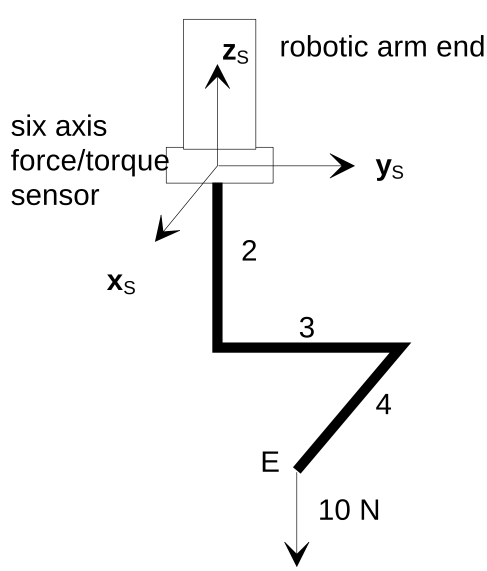

Figure 5

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