NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR THE DEGREE OF B.ENG

Semester I   1998/1999

EE4304/ME4245 ROBOTICS

Nov 1998   Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains five (5) questions in 2 Sections, and comprises seven (7) pages.

2. Answer all questions (Q.1, 2 and 3) in Section A, and any 1 question (Q.4 or 5) in Section B.

3. All questions carry equal marks.

4. This is an open-book examination.

SECTION A: COMPULSORY
(Answer all the three questions in this section.)

Q.1.   In Figure 1, the cuboids have faces parallel to the coordinate planes of Frame U. The coordinates of the labeled corners of the cubes are with respect to Frame U. Frame E is positioned and oriented with respect to Frame U according to $^{U}T_{E}$, as given in Figure 1.

$$^{U}T_{E} = \begin{bmatrix} 0.35 & -0.61 & 0.71 & 1.3 \\ 0.93 & 0.13 & -0.35 & 0.12 \\ 0.13 & 0.78 & 0.61 & 3.5 \\ 0 & 0 & 0 & 1. \end{bmatrix}$$
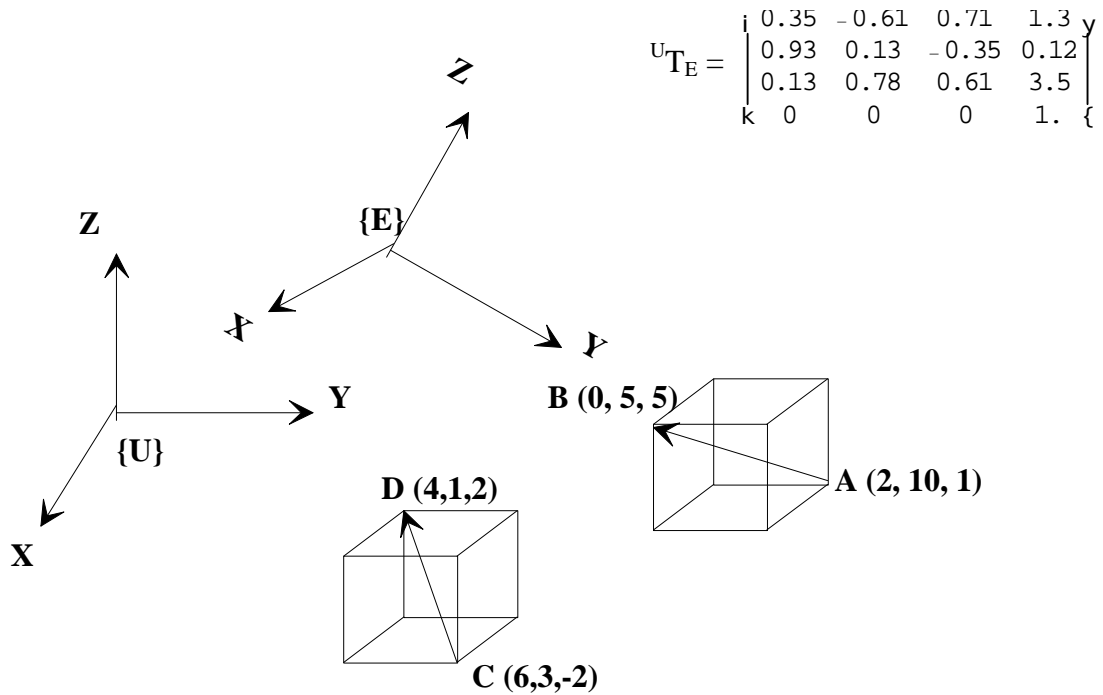

Fig. 1

(a)   Frame E undergoes the following motion in the following sequence:
First: a rotation about an axis from A to B by 30 degrees, followed by
Second: a translation along axis from C to D by 5 units.
Determine the new position and orientation of Frame E with respect to Frame U.

(15 marks)

(b)   At the instant shown in Figure 1, Frame E is translating along axis AB at 2 units/sec, while axis AB is rotating about axis CD at 30 deg/sec. Determine the translational velocity of Frame E with respect to Frame U.

(10 marks)

Q.2. (a) Figure 2 shows a manipulator with two rotational joints at locations A and D. The first moving link is link ABCD. ABC is on a plane and CD is normal to this plane. The first joint axis is along the z axis of Frame U. The second moving link is link DEFG (DEFG is on the same plane). The second joint axis is along link segment CD. When the two joints are moving, CD is always parallel to the xy plane of Frame U (horizontal plane) while planes ABC and DEF are always in a plane parallel to the z axis of Frame U (vertical plane). Frame G is attached to the second link as shown in Figure 2, with its yz plane always aligned to the plane defined by DEFG.

   (i) Assign coordinate frames to each link according to the Denavit-Hartenberg (DH) convention.

   (ii) Derive the 4 DH parameters for each of the 2 links. Indicate which parameter changes when the robot moves.

   (iii) Derive a closed-form expression for the position of the end-effector (Frame G) with respect to Frame U as a function of the 2 joint coordinates.

   (iv) Derive a closed-form expression for the orientation of the end-effector (Frame G) with respect to Frame U as a function of the 2 joint coordinates.
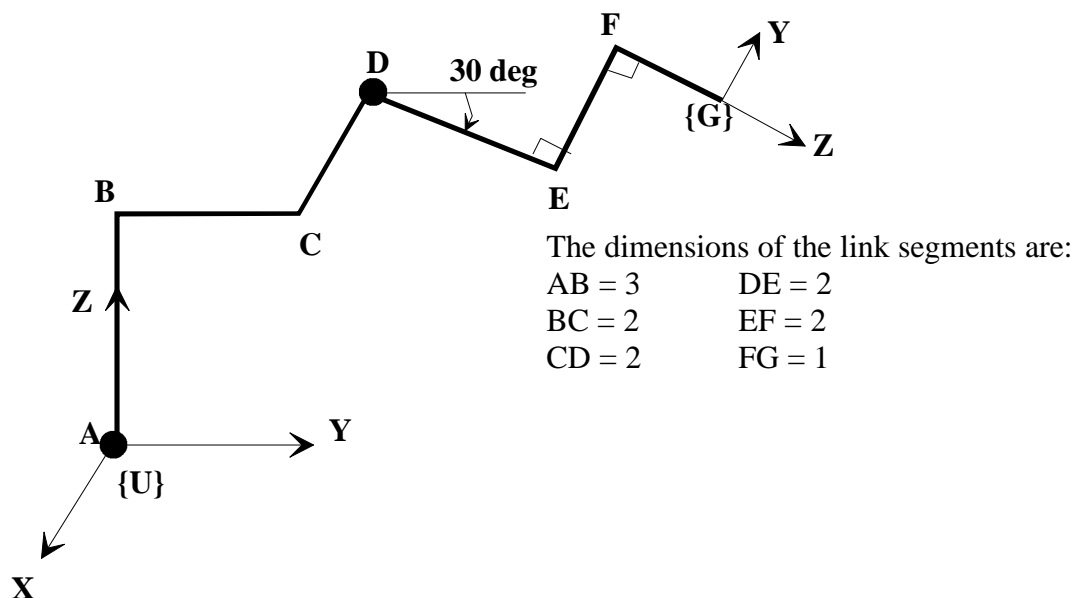
(15 marks)

The dimensions of the link segments are:
AB = 3   DE = 2
BC = 2   EF = 2
CD = 2   FG = 1

Figure 2

Q.2. (b) Figure 3 shows a planar manipulator with three rotational joints whose joint axes of motion are parallel to the z axis of Frame U.
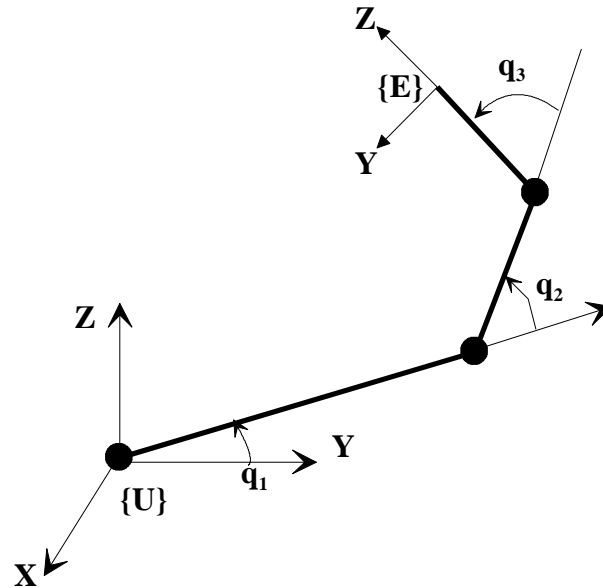

Figure 3

The position of the end-effector in Frame U is given by.

$P_x = 0$
$P_y = L_1 \cos q_1 + L_2 \cos (q_1 + q_2) + L_3 \cos (q_1 + q_2 + q_3)$
$P_z = L_1 \sin q_1 + L_2 \sin (q_1 + q_2) + L_3 \sin (q_1 + q_2 + q_3)$

(i) Derive the complete (6 x 3) manipulator Jacobian that relates the generalized (6 x 1) velocity of Frame E in Frame U and the vector of joint velocities.

(ii) What joint torques are needed to maintain a 10 kg mass at joint angles $q_1 = q_2 = q_3 = 30^\circ$? The link lengths are $L_1 = 1$ m, $L_2 = 0.5$ m, and $L_3 = 0.25$ m.

(10 marks)

Q.3 Assume that the two degree-of-freedom robot shown in Figure 4 is in the vertical plane under the influence of gravity. For this robot, let I_1 be the moment of inertia of the first link, and m_2 be the lumped equivalent mass of the second link, which is located at the place as indicated in the figure.

(a) Give two different sets of generalized coordinates for the robotic manipulator. Draw two separate figures of the manipulator indicating the generalized coordinates that you choose. Indicate the type of joints.
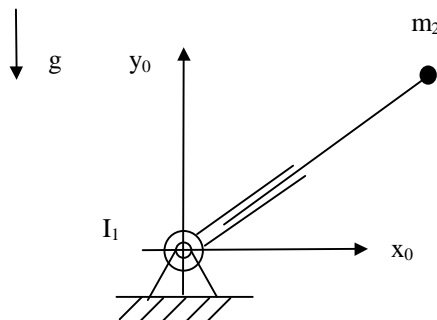
(3 marks)

Figure 4

(b) For any one set of generalized coordinates from (a), find

(1) the kinetic energy of the robot; (9 marks)

(2) the potential energy $V(q)$ of the robot; (3 marks)

(3) the Lagrange-Euler equations of motion which should be written in the form:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = t$$

where $D(q)$ is the inertia matrix, $C(q,\dot{q})$ is the Coriolis matrix defined by the so-called Christoffel Symbols, and $G(q)$ is the gravitational force vector.

(12 marks)

SECTION B
(Answer Only One out of the Two Questions in this Section)

Q.4 The dynamic equation of a robot in contact with environment is described by

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + J^T(q)\lambda = \tau$$

where $q \in R^n$ are the generalised coordinates, $\tau \in R^n$ are the independent controls, $D(q)$ is the inertia matrix, and $C(q,\dot{q}) \in R^{n \times n}$ is the matrix defined by the so-called Christoffel Symbols, $G(q) \in R^n$ denote the gravitational forces, $J(q)$ is the Jacobian matrix and λ is the Lagrange multipliers associated with the constrains.

(a) List at least three structural properties about the dynamic equation of the system.

(5 marks)

(b) Design a computed torque controller for this robot such that the resulting closed-loop system is decoupled, critically damped and with natural frequency $w_n = 5$. Are the design specifications reasonable?

(15 marks)

(c) Prove that the following control law

$$\tau = K_p e - K_d \dot{q} + G(q) + J^T(q)\lambda$$

where K_p and K_d are diagonal matrices, and $e = q_d - q$ with $q_d \equiv 0$, yields an asymptotically stable closed-loop system.

(5 marks)

Q.5 (a) Consider a single link prismatic robot with mass $m = 20$ mg. The robot is motionless at $q = -1$ m, and is required to move to $q = 1$ m in 2 seconds and stop there. Assume the joint motion is performed with constant accelerations as shown in Figure 5.
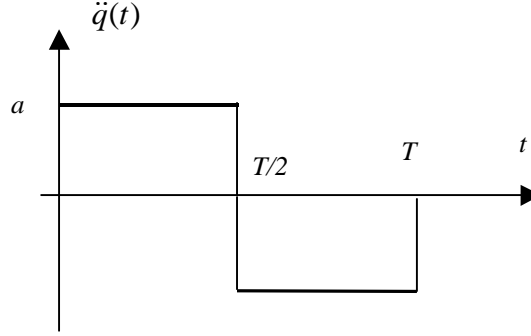

Figure 5

Determine the magnitude a of the joint acceleration, sketch the trajectories of $q(t)$ $\dot{q}(t)$, and the force required for this motion $F(t)$.

(7 marks)

(b) The dynamics of a robot are described by the following two equations

$$(m_1 l_{c1}^2 + m_3 l_{c3}^2 + m_4 l_1^2 + I_1 + I_3)\ddot{q}_1 + (m_3 l_2 l_{c3} - m_4 l_1 l_{c4})\cos(q_2 - q_1)\ddot{q}_2$$
$$- (m_3 l_2 l_{c3} - m_4 l_1 l_{c4})\sin(q_2 - q_1)\dot{q}_2^2$$
$$+ (m_1 l_{c1} + m_3 l_{c3} + m_4 l_1)g\cos q_1 = t_1$$

$$(m_3 l_2 l_{c3} - m_4 l_1 l_{c4})\cos(q_2 - q_1)\ddot{q}_1 + (m_2 l_{c2}^2 + m_3 l_3^2 + m_4 l_{c1}^2 + I_2 + I_4)\ddot{q}_2$$
$$+ (m_3 l_2 l_{c3} - m_4 l_1 l_{c4})\sin(q_2 - q_1)\dot{q}_1^2$$
$$+ (m_2 l_{c2} + m_3 l_2 + m_4 l_{c4})g\cos q_2 = t_2$$

Express the above equations in the linear-in-the-parameters form. Determine the inertia matrix, the Coriolis and Centrifugal force vector, and the gravitational force vector.

(5 marks)

(c) The PUMA 560 manipulator is to be used for polishing turbine blades. The end-effector holds a polishing tool that weighs 2 kg, and is required to exert at most a 10 N normal force to the turbine blade. Given the specifications of the PUMA 560 robot, explain the steps would you would take to determine if the PUMA 560 robot can be used for the polishing task.

(13 marks)

END OF PAPER