NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR THE DEGREE OF B.TECH

Mechanical and Manufacturing Engineering

Semester II 1998/1999

TM4245 ROBOTICS

April/May 1999 Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains four (4) questions and comprises six (6) pages.
2. Answer all four questions.
3. All questions carry equal marks.
4. This is an open-book examination.

Q.1. (a) Complete the frame assignment for Frame 1 (in Fig. 1a) according to the Denavit-Hartenberg convention given in class. Identify the four Denavit-Hartenberg kinematic paramters that relate the relative position and orientation of Frames 1 and 0.

(7 marks)

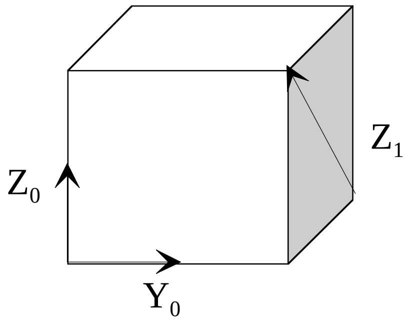


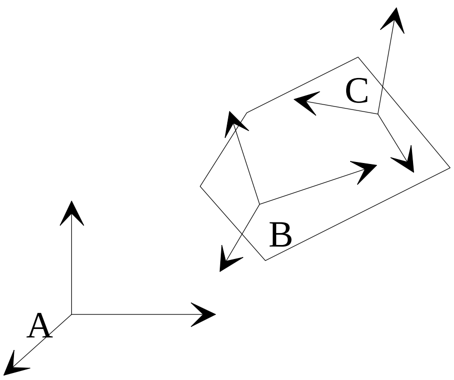
Fig. 1a

(b) Frames B and C (in Fig. 1b) are attached to the same moving rigid body with ${}^{B}T_{C}$ as known. At a certain instant of time when ${}^{A}T_{B}$ is at the given configuration, Frame B has translational and angular velocities of $(1, 2, 3)^T$ m/s and $(4, 5, 6)^T$ rad/s, respectively, with respect to Frame A. (8 marks)

Determine:

(i) the angular velocity of Frame C with respect to Frame A.

(ii) the translational velocity of Frame C with respect to Frame A.



$$ {}^{B}T_{C} = \begin{bmatrix} 0.866 & 0 & 0.5 & 7. \\ 0.433 & 0.5 & -0.75 & 8. \\ -0.25 & 0.866 & 0.433 & 9. \\ 0 & 0 & 0 & 1. \end{bmatrix} $$

$$ {}^{A}T_{B} = \begin{bmatrix} 1. & 0 & 0 & 0 \\ 0 & 0.5 & -0.866 & 0 \\ 0 & 0.866 & 0.5 & 0 \\ 0 & 0 & 0 & 1. \end{bmatrix} $$

Fig. 1b

Q.1 is continued on Page 3

(c) Frame C is attached to a rigid body and D is a reference point on the rigid body. Frames A and E are fixed to the world. The following are known quantities:

${}^{C}P_{D}$: position of point D in Frame C

${}^{A}T_{E}$: position and orientation of Frame E in Frame A

${}^{A}T_{C}$: initial position of Frame B in Frame A.

The rigid body undergoes the following motion in the indicated sequence:

1) rotation about z_A by 20 degrees,

2) rotation about x_C by 30 degrees,

3) rotation about y_E by 40 degrees.

Derive the expression for the new position of point D in Frame A (${}^{A}P_{D}$) as a function of the known quantities.

(10 marks)

Q.2. Fig. 2 shows a three-DOF robot that operates in the x-y plane which is vertical. The gravitational force is acting in the negative y_0 direction. The first joint is translational and along x_0. The second and third joints are rotational, and their joint axes of motion are parallel to z_0. The 2nd and 3rd links have lengths of 3 and 2 respectively. The three joint coordinates (q_1, q_2, q_3) have zero positions and positive motion directions as indicated in the figure.

(a) Find an expression for ${}^{0}T_{E}$, that describes the position and orientation of Frame E in Frame 0, as a function of the three joint coordinates.

(7 marks)

(b) Given: ${}^{0}T_{E} = \begin{bmatrix} nx & ox & ax & px \\ ny & oy & ay & py \\ nz & oz & az & pz \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Derive an expression for the joint coordinates (q_1, q_2, q_3) as a function of the elements of ${}^{0}T_{E}$.

(9 marks)

Q.2 is continued on Page 4

(b) The robot carries a tool at Frame E. The mass of the tool is 5 kg and it is assumed that the mass of the tool is concentrated at the origin of Frame E. When the robot is at a configuration (q_1, q_2, q_3) = (1, 30°, 60°), the robot uses the tool to exert a force of 10 N on the environment in a direction parallel to the positive x_0. Determine the joint force required at joint 1, and the joint torques required at joints 2 and 3 to exert this force.

(9 marks)

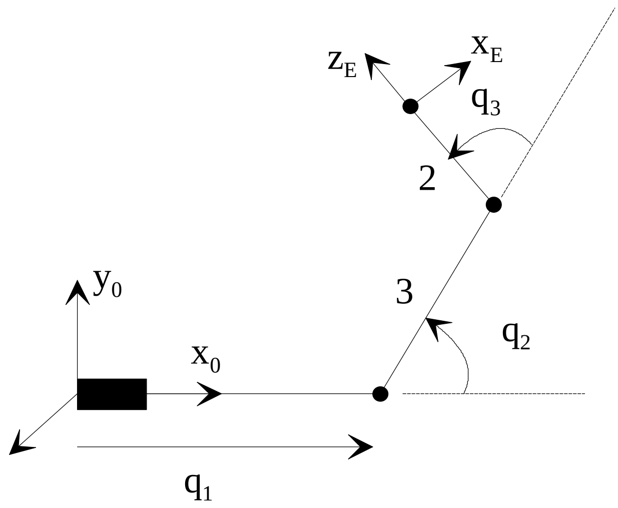


Fig. 2

Q.3 For the system shown in Figure 3, m_1 and m_2 are the masses of blocks 1 and 2, k is the spring constant of the linear spring connecting the two blocks, x_1 and x_2 are the displacements as indicated, and f is the external forces on block 1. Assume that the system is in the vertical plane under the influence of gravity.

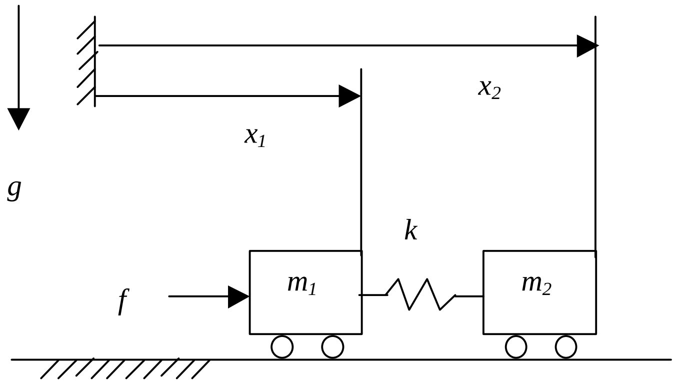


Figure 3

(a) Find the kinetic and potential energies of the system and derive the equations of motion using Lagrange-Euler formulation.

(15 marks)

(b) Find the equations of motion using Newton-Euler formulation. Are they the same as that obtained in Q.3 (a)?

(5 marks)

(c) Are the generalized coordinate unique? If no, draw an additional figure of the system indicating the different generalized coordinates that you choose. If yes, explain.

(5 marks)

Q.4 (a) For a single-link robot with a rotary joint, assume that the robot is motionless at $\theta = -10^o$. It is desired to move the joint in a smooth manner to $\theta = 60^o$ in 5 seconds. Find the coefficients of a cubic, which accomplishes this motion and brings the arm to rest at the goal. Sketch profiles of position, velocity and acceleration.

(5 marks)

(b) Consider a system described by the following dynamic equations:

$$(m_1 l_1^2 + m_2 d_2^2)\ddot{\theta}_1 + 2m_2 d_2 \dot{\theta}_1 \dot{d}_2 + 10(m_1 l_1 + m_2 d_2)cos\theta_1 = \tau_1$$

$$m_2 \ddot{d}_2 - m_2 d_2 \dot{\theta}_1^2 + 10 m_2 sin\theta_1 = f_2$$

where τ_1 and f_2 are the generalized torque and force for the first and the second degrees of freedom.

(i) Determine the generalized coordinates, the inertia matrix, the Coriolis force and centrifugal force vector, and the gravitational force vector.

(5 marks)

(ii) List at least three structural properties about the dynamic equation of the system.

(5 marks)

(iii) Design a computed torque controller for this robot such that the resulting closed-loop system is decoupled, critically damped and with natural frequency $\omega = 5$. Explain the advantages and disadvantages of such a design method.

(10 marks)

END OF PAPER