NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR THE DEGREE OF B.ENG.

(Semester I: 1999/2000)

EE4304/ME4245 - ROBOTICS

October/November 1999 - Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains FIVE (5) questions in 2 Sections and comprises SIX (6) pages.
2. Answer all questions (Q. 1, 2 and 3) in Section A, and any 1 question (Q. 4 or 5) in Section B.
3. All questions carry equal marks.
4. This is an open-book examination.

SECTION A: COMPULSORY
(Answer all three questions in this section.)

Q.1(a) Figure 1 shows a robot with three rotational joints. The axis motion of the first joint (Joint A) is along the x-axis of Frame U. The axis of motion for the 2nd joint (Joint B) is along the link AB. The 3rd joint (Joint C) rotates about an axis defined by BC × CD (where × is the vector cross product), i.e., the axis is normal to the plane formed by BCD. The three joint variables ϕ_1, ϕ_2, ϕ_3 have the following zero positions: ϕ_1 is zero when AB is parallel to the y-axis of Frame U, ϕ_2 is zero when ABCD is in the yz plane of Frame U, ϕ_3 is zero when CD is aligned with BC. The length of each link is 1 m. AB is always perpendicular to BC.

(i) Assign Frames to the robot according to the Denavit Hartenberg (DH) Convention discussed in class.

(4 Marks)

(ii) Determine the 4 DH parameters for each of the three links, i.e., complete the table of kinematic (DH) parameters. Indicate which parameter is the joint coordinate, q_i.

(4 Marks)

(iii) Determine the relationships between q_i and ϕ_i.

(2 Marks)

(iv) Frame D is attached to the last link such that its origin is at D and it's z axis is along the link CD. Derive the expressions describing the position ${}^{U}\mathbf{p}_D$ and the orientation vector ${}^{U}\mathbf{z}_D$ as functions of the three joint variables ϕ_1, ϕ_2, ϕ_3. You do not need to simplify the expressions. (You can leave the expression in matrix-vector and/or matrix-matrix product terms.)

(5 Marks)

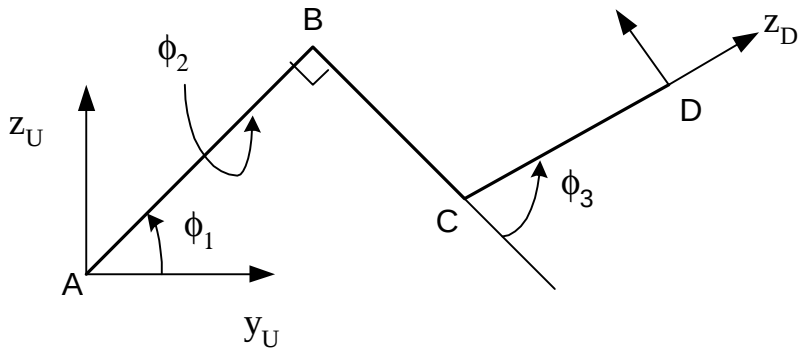

Figure 1

Q.1(b) Frames B and C are attached to the same rigid body with ${}^{B}T_{C}$ given. Frames A and D are fixed to the universe with ${}^{A}T_{D}$ given. The body is initially at ${}^{A}T_{B}$. The body undergoes the following ordered sequence of motions:

1> Rotation about z-axis of Frame A by 30 degrees
2> Rotation about x-axis of Frame C by 40 degrees
3> Rotation about y-axis of Frame D by 50 degrees

Determine the final position and orientation of Frame B with respect to Frame A, ${}^{A}T_{B}$. You do not need to simplify your answer. Express your answer in terms of a matrix product expression.

(10 Marks)

Q 2. The orientation of a rigid body can be described by the three angles α, β, γ where

$$R = \text{Rot}(x, \alpha)\,\text{Rot}(y, \beta)\,\text{Rot}(x, \gamma) =$$

$$\begin{pmatrix} \mathbf{Cos}[\beta] & \mathbf{Sin}[\beta]\,\mathbf{Sin}[\gamma] & \mathbf{Cos}[\gamma]\,\mathbf{Sin}[\beta] \\ \mathbf{Sin}[\alpha]\,\mathbf{Sin}[\beta] & \mathbf{Cos}[\alpha]\,\mathbf{Cos}[\gamma] - \mathbf{Cos}[\beta]\,\mathbf{Sin}[\alpha]\,\mathbf{Sin}[\gamma] & -\mathbf{Cos}[\beta]\,\mathbf{Cos}[\gamma]\,\mathbf{Sin}[\alpha] - \mathbf{Cos}[\alpha]\,\mathbf{Sin}[\gamma] \\ -\mathbf{Cos}[\alpha]\,\mathbf{Sin}[\beta] & \mathbf{Cos}[\gamma]\,\mathbf{Sin}[\alpha] + \mathbf{Cos}[\alpha]\,\mathbf{Cos}[\beta]\,\mathbf{Sin}[\gamma] & \mathbf{Cos}[\alpha]\,\mathbf{Cos}[\beta]\,\mathbf{Cos}[\gamma] - \mathbf{Sin}[\alpha]\,\mathbf{Sin}[\gamma] \end{pmatrix}$$

(a) Derive the complete inverse kinematic equations that compute α, β, γ given

$$R = \begin{pmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{pmatrix}$$

(15 Marks)

(b) Frames E and F are attached to the same rigid body with ${}^{E}T_{F}$ given. The rigid body is in motion and at a certain instant of time, the body is at given orientation (α, β, γ) such that ${}^{A}R_{E}$ = Rot (x, α) Rot (y, β) Rot (x, γ), and the position coordinates of E in Frame A is given as (a,b,c). At this same instant, the angles (α, β, γ) are changing at the given rates $(\alpha', \beta', \gamma')$ and the translational velocity of E is given as ${}^{A}u_{E} = (a', b', c')^{T}$.

(i) Determine the angular velocity of Frame F with respect to Frame A, ${}^{A}\omega_{F}$.
(ii) Determine the translational velocity of Frame F with respect to Frame A, ${}^{A}u_{F}$.

(Express your answers in terms of the given quantities. You do not need to simplify.)

(10 Marks)

Q.3 Assume that the robot shown in Figure 2 is in the vertical plane, the mass of each link is concentrated at a point in the indicated mass centers with m_1 and m_2 being the equivalent masses of links 1 and 2 respectively, L_{1c} is the length from the center of mass m_1 to joint 1, L_1 and L_2 are the lengths of links 1 and 2, τ_1 and τ_2 are the output torques of motors 1 and 2 which are located at the joints.

(a) Give two different sets of generalized coordinates for the robot manipulator. Draw two separate figures of the manipulator indicating the generalized coordinates that you choose. Indicate the type of joint for the each of the two joints as well.

(3 Marks)

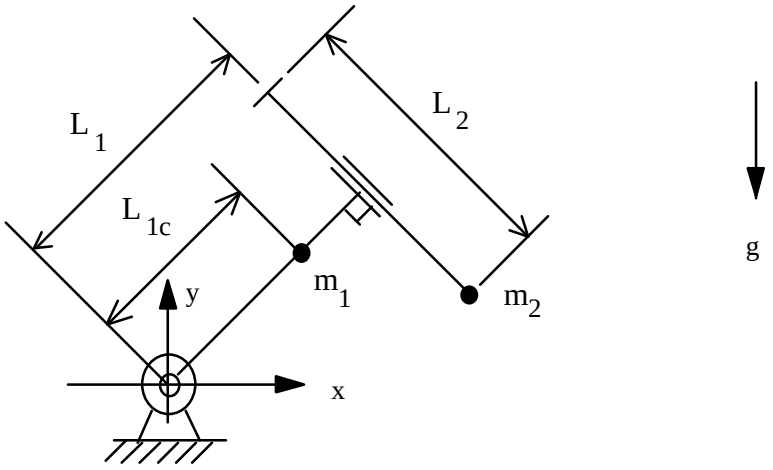

Figure 2

(b) Derive the Lagrange-Euler equations of the form

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau$$

where, $q = [q_1, q2]^T$, $\tau = [\tau_1, \tau_2]^T$, D(q) is the inertia matrix, $C(q,\dot{q})$ is the matrix defined by the so called Christoffel Symbols, and G(q) represent the gravitational forces.

(14 marks)

(c) Design a computed torque controller for this robot such that the resulting closed-loop system is decoupled, critically damped, and with a natural frequency $\omega = 4$ rad/s.

(8 Marks)

SECTION B
(Answer Only One out of the Two Questions in this Section)

Q.4 (a) The equations of motion of the two-link robot arm in Fig. 3 can be written in a compact matrix-vector form as:

$$\begin{bmatrix} d_{11}(q_2) & d_{12}(q_2) \\ d_{12}(q_2) & d_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1(t) \\ \ddot{q}_2(t) \end{bmatrix} + \begin{bmatrix} c_{12}(q_2)\dot{q}_2^2 + 2c_{12}(q_2)\dot{q}_1\dot{q}_2 \\ -c_{12}(q_2)\dot{q}_1^2 \end{bmatrix} + \begin{bmatrix} g_1(q_1,q_2) \\ g_2(q_1,q_2) \end{bmatrix} = \begin{bmatrix} t_1(t) \\ t_2(t) \end{bmatrix}$$

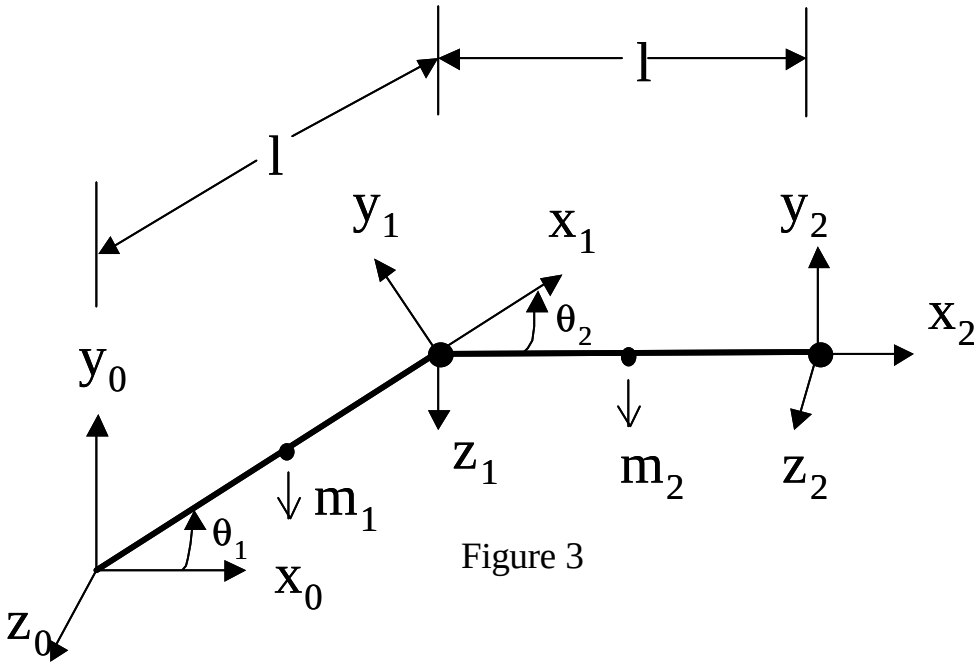

Figure 3

i. Choose an appropriate state variable vector x(t) and a control vector u(t) for this dynamic system.

(5 Marks)

ii. Express the equations of motion of this robot arm explicitly in terms of d_{ij}'s, c_{ij}'s, and g_i's in a state space representation with the chosen state-variable vector and control vector.

(10 Marks)

(b) Briefly discuss the basic idea of the independent joint control scheme and, the advantages and disadvantages in implementing such a scheme.

(6 Marks)

(c) Briefly discuss the advantages and disadvantages of the computed torque method.

(4 Marks)

Q.5 (a) A single-link rotary robot is required to move from $\theta(0) = 30°$ to $\theta(2) = 100°$ in 2 s. The joint velocity and acceleration are both zero at the initial and final positions. The trajectory may be composed of one or more polynomial segments whose orders are all the same.

i. What is the lowest degree polynomial that can be used to accomplish the motion?

(3 Marks)

ii. Determine the coefficients of a quadric polynomial that accomplishes the motion. You may split the joint trajectory in to several trajectory segments. (You can leave the expression in matrix-vector and/or matrix-matrix product terms.)

(10 Marks)

(b) Figure 4 shows a planar robot with 7 rotational joints. The axis of motion of each joint is parallel to the z axis of Frame U.

(3 Marks each)

i. How many degrees of freedom does this robot have in terms of its positioning and orienting capability?

ii. It is known that robots with redundant joints can also have singularities. At a singular configuration, what is the rank of the Jacobian matrix (that relates the joint and end-effector velocities)?

iii. Draw the robot at a singular configuration and indicate the rank of the Jacobian at the drawn singular configuration.

iv. What is the minimum number of joints for this planar robot to be able to have the same positioning and orienting capability as the 7 joints? (The positioning and orienting capabilities are independent.)

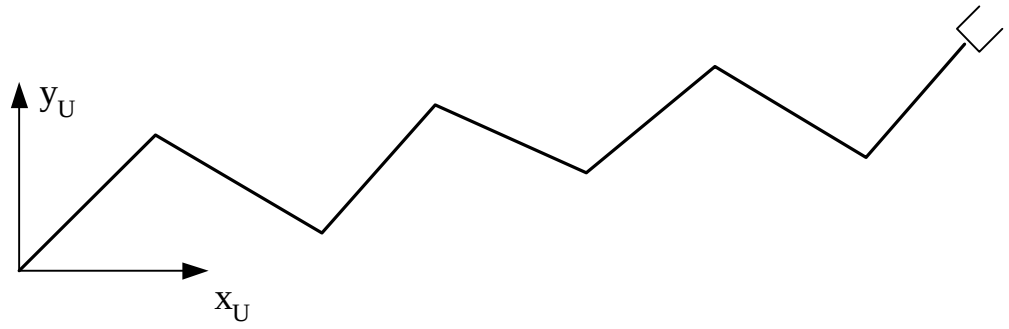

Figure 4

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