**ME4245 Quiz 2, 15 October 2013 – 16:00-17:15**

Chapters 2 to 4 (Robot Kinematics of Position, Velocity and Statics)
Answer all questions. Please provide complete answers with all expressions clearly indicated and derived from the known quantities. You need not evaluate nor simplify your answers.

1. Figure 1 shows a robot with three rotational joints whose positive directions of motion are indicated in the figure. The figures show the robot at its "zero configuration" (all the joint variables are zero), where all the links are along the YZ plane of Frame U. Frame U is fixed and attached to the base of the robot (e.g., table where robot is installed). Frame E is the end-effector frame. A tool is rigidly attached to the end-effector, with Frame T and the tool-frame. Assign frames according to the Denavit Hartenberg convention. Derive the complete expression of the position and orientation of Frame T with respect to U (${}^{U}T_{T}$), as a function of the joint coordinates ($q_1$, $q_2$, $q_3$).

2. Frame B is attached to an inclined plane (B) which carries a rectangular block (C) which is sliding down the plane, as shown in Fig. 2. Frame A is fixed to the ground. At the time instant shown in Fig 2, the inclined plane (B) is moving with a translational velocity of ${}^{A}u_{B}$ and angular velocity of ${}^{A}\omega_{B}$. Frame C is attached to the rectangular block and sliding down the plane at a speed of 1 m/sec. Determine the translational and angular velocities of Block C with respect to (and expressed in) Frame A.

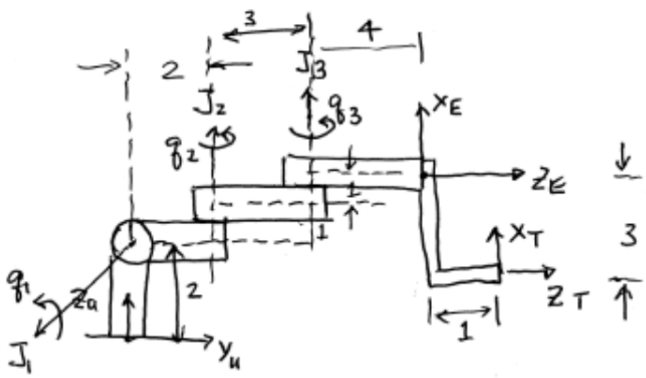
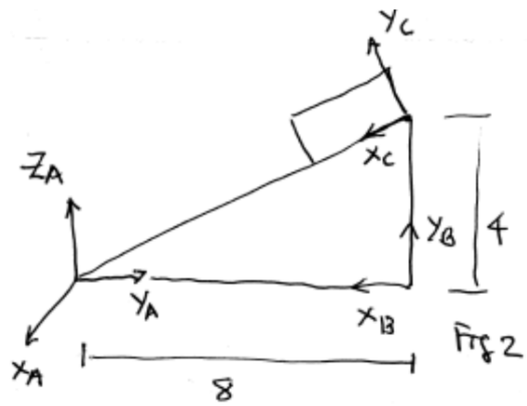



Fig 1




3. Fig 3 shows a robot with 2 joints (1 rotational and 1 translational).

a) Derive the expression for the full manipulator Jacobian that relates the 2 joint velocities with the translational and angular velocities of the last link (Frame E) with respect to Frame U.

b) Does the robot have singularities for the following tasks? If so, indicate the singular configurations.

   1> positioning task along xy plane of Frame U
   2> positioning and rotating along axes parallel to $X_U$ & $Z_U$, respectively.
   3> positioning and rotating along axes parallel to $Y_U$ & $Z_U$, respectively.

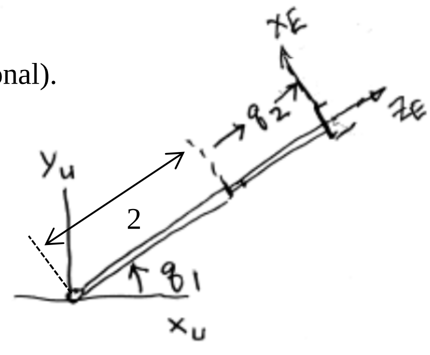



Fig 3

1.

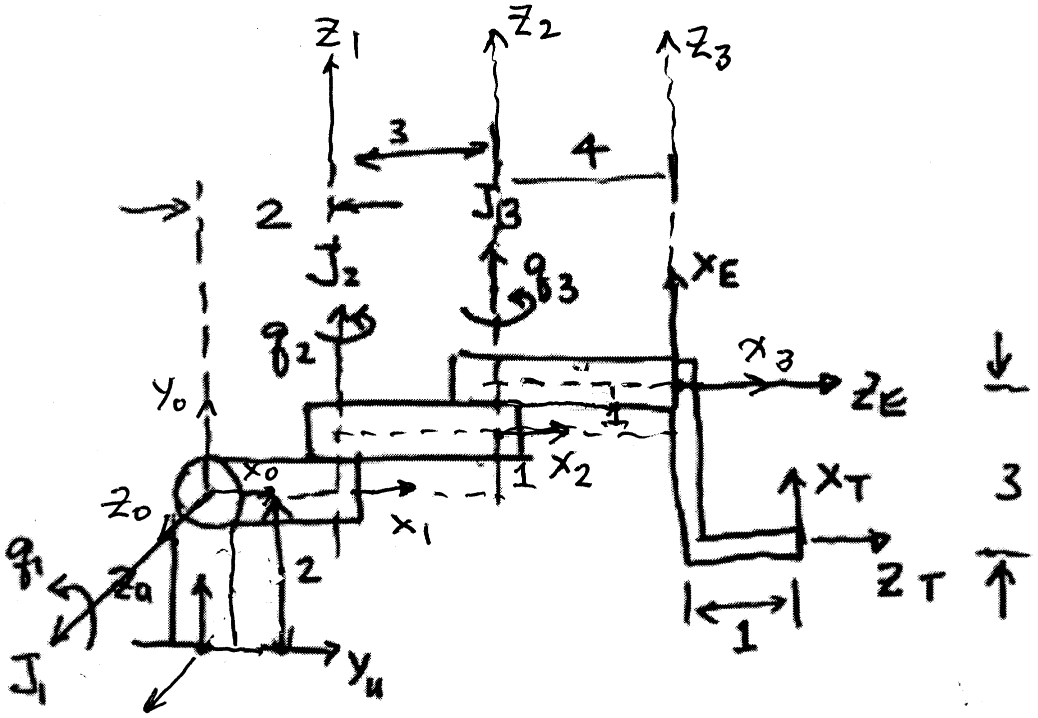
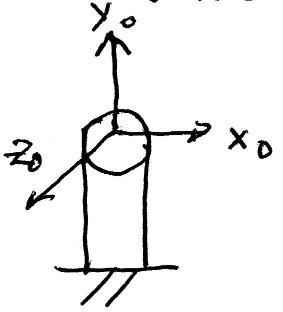
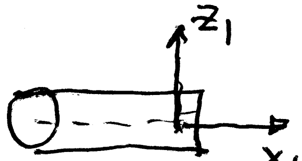


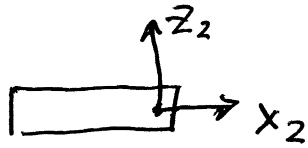

Fig 1



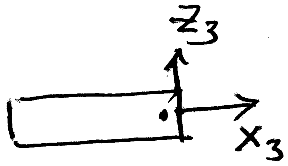

Link 0




Link 1




Link 2




Link 3

$$^3T_T = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^uT_0 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

| | $\theta$ | $r$ | $d$ | $\alpha$ |
|---|---|---|---|---|
| 1 | $q_1 = 0°$ | 0 | 2 | $-90°$ |
| 2 | $q_2 = 0°$ | 1 | 3 | $0°$ |
| 3 | $q_3 = 0°$ | 1 | 4 | $0°$ |

$$^iT_{i+1} = \begin{pmatrix} \cos q_i & -\cos(\alpha_i)\sin q_i & \sin\alpha_i \sin q_i & d_i \cos q_i \\ \sin q_i & \cos(\alpha_i)\cos q_i & -\sin\alpha_i \cos q_i & d_i \sin q_i \\ 0 & \sin(\alpha_i) & \cos\alpha_i & r_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$i = 0, 1, 2$

$$^uT_T = {}^uT_0\,{}^0T_1\,{}^1T_2\,{}^2T_3\,{}^3T_T$$

From ${}^AT_C = {}^AT_B\,{}^BT_C$

2. ${}^A\dot{T}_C = {}^AT_B\,{}^B\dot{T}_C + {}^A\dot{T}_B\,{}^BT_C$

$${}^A\dot{T}_B = \begin{pmatrix} {}^A\hat{\omega}_B\,{}^AR_B & {}^Au_B \\ 0 & 0 \end{pmatrix}$$

$${}^B\dot{T}_C = \begin{pmatrix} {}^B\hat{\omega}_C\,{}^BR_C & {}^Bu_C \\ 0 & 0 \end{pmatrix}$$

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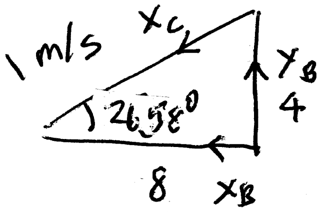
$${}^Au_C = {}^Au_B + {}^AR_B\,{}^Bu_C + {}^A\omega_B \times ({}^AR_B\,{}^BP_C)$$

$${}^A\omega_C = {}^A\omega_B + {}^AR_B\,{}^B\omega_C$$

$${}^AR_B = \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad {}^BP_C = \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}$$

$${}^Bu_C = 1\begin{pmatrix} +\cos 26.58^\circ \\ -\sin 26.58^\circ \\ 0 \end{pmatrix} = \begin{pmatrix} +0.8944 \\ -0.4472 \\ 0 \end{pmatrix}$$




$${}^B\omega_C = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$ since translaty only.

3. $x_E = (2+q_2)\cos q_1 \qquad \dot{x}_E = -(2+q_2)\sin q_1\,\dot{q}_1 + \dot{q}_2 \cos q_1$

$y_E = (2+q_2)\sin q_1 \qquad \dot{y}_E = (2+q_2)\cos q_1\,\dot{q}_1 + \dot{q}_2 \sin q_1$

$$J = \begin{pmatrix} -(2+q_2)\sin q_1 & \cos q_1 \\ (2+q_2)\cos q_1 & \sin q_1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}$$

- For x, y, task. singularities are:
  $\det(J_{2\times2(\text{upper})}) = -(2+q_2)(S_1^2 + C_1^2)$
  $= -(2+q_2) = 0$
- For $x, \omega_z$ task: singularities: $q_2 = -2$ //
  1st row + last row
  $-\cos q_1 = 0$, $q_1 = 90^\circ, 270^\circ$ //
- For $y, \omega_z$ task. singularities are.
  2nd row + last row.
  $-\sin q_1 = 0 \quad q_1 = 0^\circ$ or $180^\circ$
  //