ME4245 Robot Kinematics, Dynamics and Control
Quiz 1.2 Sept 2010 (1 hr)

Name: ___________________________________ Matric number:_______________

The figure below shows a robot with three joints. The first 2 are rotational and the last joint is translational. The axes of motion are indicated by the joint variables j_1, j_2, and j_3. The first link is UBC which always lies on the XY plane of Frame U (which is fixed); UB and BC segments of the 1st link has lengths of 2 and 3 respectively. Figure 1 shows the robot at its zero position ($j_1 = j_2 = j_3 = 0$). At the zero position, CE is on the XY plane of Frame U, and UB is aligned with Y_U.

Please make sure all your solutions are complete.

1. (40 marks) Assign Frames according to the DH convention discussed in class

2. (40 marks) Provide the table of kinematic (DH) parameters and indicate which parameters are variables

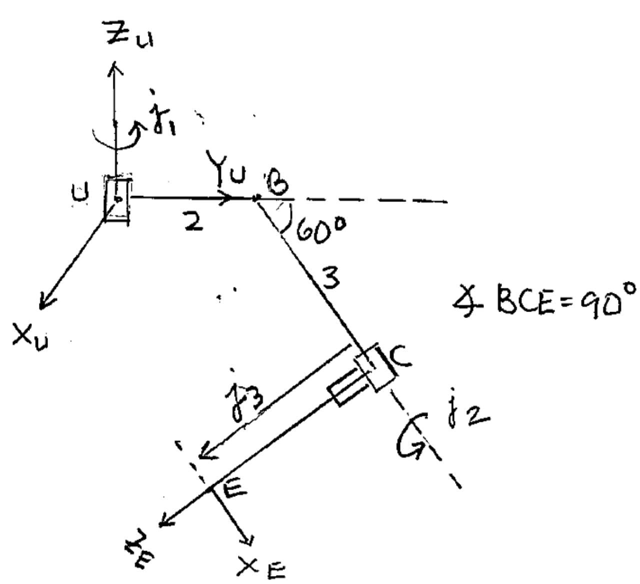
3. (5 marks) Derive the expressions that relate the joint variables j_1, j_2, and j_3 with the kinematic parameters that are variables.

4. (15 marks) The position and orientation of the end-effector (Frame E) in Frame U is:

$$
{}^{U}T_E = \begin{pmatrix} \sin j_1 & \cos j_1 \cos j_2 & -\cos j_1 \sin j_2 & \sqrt{3}\cos j_1 + 4\sin j_1 - j_3 \cos j_1 \sin j_2 \\ -\cos j_1 & \cos j_2 \sin j_1 & -\sin j_1 \sin j_2 & -4\cos j_1 + \sqrt{3}\sin j_1 - j_3 \sin j_1 \sin j_2 \\ 0 & \sin j_2 & \cos j_2 & j_3 \cos j_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

   a. How many degrees of freedom does the robot have in terms of orientation capability of the end-effector?
   b. How many degrees of freedom does the robot have in terms of positioning capability of the end-effector?
   c. Given the desired orientation (as a 3x3 rotation matrix) of the end-effector, solve for the joint variables to achieve the desired orientation.

5. (Bonus question: 10 marks) Given the desired position of the end-effector, solve for the joint variables to achieve the desired position.



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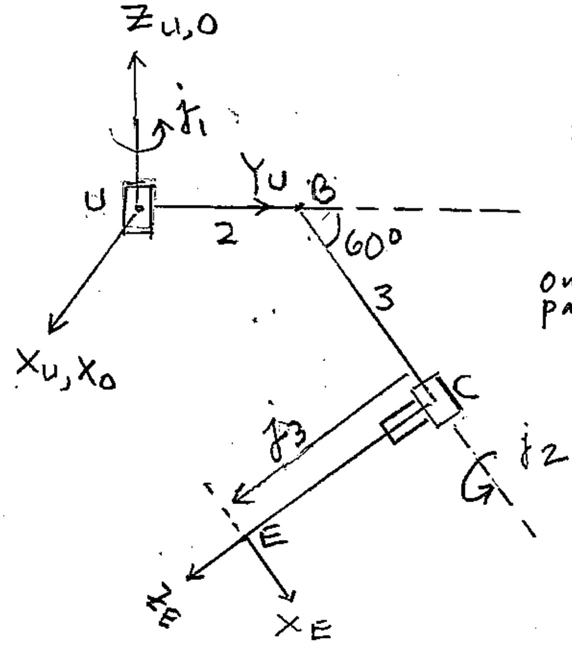

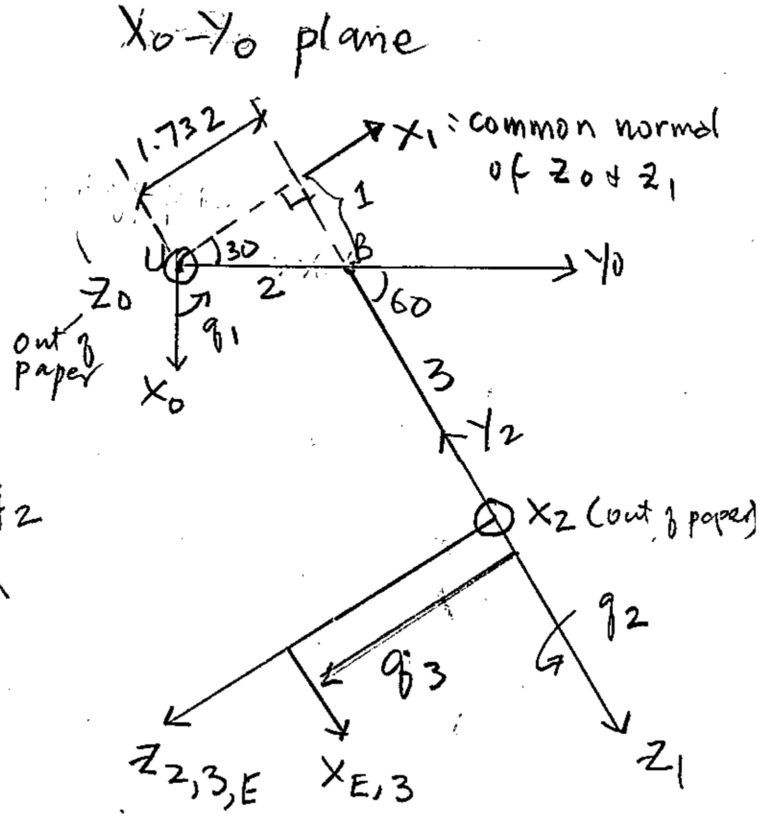

Link	θ	r	d	α
1	$q_1 = 120°$	0	1.732	90°
2	$q_2 = 90°$	4	0	$-90°$
3	$-90°$	q_3	0	0

$j_1 = 0 \Leftrightarrow q_1 = 120°$, j_1 & q_1 have the same pos. directions of motion

$q_1 = j_1 + 120°$

$j_2 = 0° \Leftrightarrow q_2 = 90°$, j_2 & q_2 have the same positive directions of motion

$q_2 = j_2 + 90°$

$q_3 = j_3$

$$^{u}T_0 = I_{4\times4} \quad \text{(4x4 Identity Matrix)}$$

$$^{3}T_E = I_{4\times4}$$

$$^{u}T_E = {}^{u}T_0 \; {}^{0}T_1 \; {}^{1}T_2 \; {}^{2}T_3 \; {}^{3}T_E$$

$$^{i-1}T_i = Rot(z, \theta_i)\, Trans(r_i, 0, 0)\, Trans(0, 0, d_i)\, Rot(x, \alpha_i)$$

Where $\theta_i, r_i, d_i, \alpha_i$ from DH Table

$$^{u}T_E = f(j_1, j_2, j_3)$$

4. (c)

$$R = \begin{pmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{pmatrix} = \begin{pmatrix} \sin j_1 & x & x \\ -\cos j_1 & x & x \\ 0 & \sin j_2 & \cos j_2 \end{pmatrix}$$

$$j_1 = \mathrm{ATAN2}\left(\frac{n_x}{-n_y} \right)$$

$$j_2 = \mathrm{ATAN2}\left({}^{o_z}/_{a_z} \right)$$

4. (a) : 2 DOF (orientational)

(b) 3 DOF (Positional)

$$q_1 = \dot{j}_1, \quad q_2 = \dot{j}_2, \quad q_3 = \dot{j}_3$$

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In[27]:= A1 = amat[q1, 0, 1.732, 90 * Pi / 180]

Out[27]= {{Cos[q1], 0, Sin[q1], 1.732 Cos[q1]},
          {Sin[q1], 0, -Cos[q1], 1.732 Sin[q1]}, {0, 1, 0, 0}, {0, 0, 0, 1}}

In[28]:= A2 = amat[q2, 4, 0, -90 * Pi / 180]

Out[28]= {{Cos[q2], 0, -Sin[q2], 0}, {Sin[q2], 0, Cos[q2], 0}, {0, -1, 0, 4}, {0, 0, 0, 1}}

In[29]:= A3 = amat[-90 * Pi / 180, q3, 0, 0]

Out[29]= {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 1, q3}, {0, 0, 0, 1}}

In[30]:= T3 = A1 . A2 . A3

Out[30]= {{Sin[q1], Cos[q1] Cos[q2], -Cos[q1] Sin[q2], 1.732 Cos[q1] + 4 Sin[q1] - q3 Cos[q1] Sin[q2]},
          {-Cos[q1], Cos[q2] Sin[q1], -Sin[q1] Sin[q2],
           -4 Cos[q1] + 1.732 Sin[q1] - q3 Sin[q1] Sin[q2]},
          {0, Sin[q2], Cos[q2], q3 Cos[q2]}, {0, 0, 0, 1}}

In[31]:= MatrixForm[T3]

Out[31]//MatrixForm=
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$$\begin{pmatrix} \mathrm{Sin}[q1] & \mathrm{Cos}[q1]\,\mathrm{Cos}[q2] & -\mathrm{Cos}[q1]\,\mathrm{Sin}[q2] & 1.732\,\mathrm{Cos}[q1] + 4\,\mathrm{Sin}[q1] - q3\,\mathrm{Cos}[q1]\,\mathrm{Sin}[q2] \\ -\mathrm{Cos}[q1] & \mathrm{Cos}[q2]\,\mathrm{Sin}[q1] & -\mathrm{Sin}[q1]\,\mathrm{Sin}[q2] & -4\,\mathrm{Cos}[q1] + 1.732\,\mathrm{Sin}[q1] - q3\,\mathrm{Sin}[q1]\,\mathrm{Sin}[q2] \\ 0 & \mathrm{Sin}[q2] & \mathrm{Cos}[q2] & q3\,\mathrm{Cos}[q2] \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(1) $P_x = 1.732 C_1 + 4 S_1 - q_3 C_1 S_2$

(2) $P_y = -4 C_1 + 1.732 S_1 - q_3 S_1 S_2$

(3) $P_z = q_3 C_2$

Case 6

$$a \sin\theta + b \cos\theta = d$$
$$a \cos\theta - b \sin\theta = c$$

$$\theta = \mathrm{ATAN2}\left(\frac{ad - bc}{ac + bd}\right)$$

$$\underset{d}{P_x} = \overset{a}{4} S_1 + \overbrace{(1.732 - q_3 S_2)}^{b} C_1$$

$$\underbrace{-P_y}_{c} = 4 C_1 - (1.732 - q_3 S_2) S_1$$

$$\theta_1 = \mathrm{ATAN2}\left(\frac{4P_x + (1.732 - q_3 S_2) P_y}{-4P_y + (1.732 - q_3 S_2) P_x}\right) \quad \text{unique}$$

once q_3 & S_2 are known.

$$b = 1.732 - q_3 S_2$$

$$P_x S_1 = 4S_1^2 + b S_1 C_1$$

$$+ \quad -P_y C_1 = 4C_1^2 - b C_1 S_1$$

$$\underbrace{P_x}_{b} S_1 - \overbrace{P_y}^{a} C_1 = \overbrace{4}^{c} \qquad \text{Case 4}$$

θ_1

$$\boxed{\theta_1 = \mathrm{ATAN2}\left(\frac{P_x}{-P_y}\right) + \mathrm{ATAN2}\left(\frac{I_1\sqrt{P_x^2 + P_y^2 - 16}}{16}\right) \qquad I_1 = \pm 1 \quad (2 \text{ sol'ns})}$$

From (1)

$$q_3 S_2 = \frac{1.732 C_1 + 4S_1 - P_x}{C_1}$$

$C_1 \neq 0$, and $q_3 \neq 0$

or

From (2) $$q_3 S_2 = \frac{1.732 S_1 - 4C_1 - P_y}{S_1}$$

if $S_1 \neq 0$, and $q_3 \neq 0$

From (3)

$$q_3 C_2 = P_z$$

Let $q_3 S_2 = A$

$q_3 C_2 = P_z$

$q_3^2 = A^2 + P_z^2$

$$\boxed{\begin{array}{ll} q_3 = I_3 \sqrt{P_z^2 + A^2} & S_1 \neq 0 \text{ or } C_1 \neq 0 \\ I_3 = \pm 1 \quad (2 \text{ sol'ns}) & \end{array}}$$

$$\boxed{\theta_2 = \mathrm{ATAN2}\left(\frac{A/q_3}{P_z/q_3}\right) \quad 1 \text{ sol'n.}}$$

$\therefore$ total 4 sol'ns. $I_1 = \pm 1$, $I_3 = \pm 1$