Name: ____ Matric No:______________________

ME4245 Quiz 1.2 (Chapters 2-3) 14 Sept 2016, 17:00-18:00

This is an open book/notes exam. But there is strictly no sharing of books and materials. Solve all problems. Unless otherwise specified, you don't need to simplify nor evaluate the expressions. But please make sure all expressions are complete, with all detailed steps leading to the final answer.

1. Fig. 1 shows a robot with 2 rotational joints and 1 translational joint. Frame A is attached to a table, where the robot is mounted. The 1st link (ADB) rotates about the Y axis of Frame A. The 2nd link is BC and translates along BC (and hence the distance between B and C varies as the 2nd joint moves). The third link CE rotates about an axis parallel to Y_A (vertical). Frame E is attached to the end-effector (last link) such that Z_E is along CE and X_E points down (parallel to Y_A). The coordinates shown are all expressed in Frame A. The robot's joint coordinates are β_1, β_2,β_3, whose positive directions are around Y_A, along BC (from B to C), and around an axis parallel to Y_A in C, respectively. At the configuration shown, $\beta_1 = \beta_3 = 0$ and β_2 = length of BC.

20 marks a) Assign Frames to the robot according to the D-H convention. (You can draw frames onto to the robot in Fig. 1 below.)

12 marks b) Derive the table of kinematic parameters and indicate which parameter is variable.

10 marks c) Derive an expression for the position and orientation of Frame E with respect to Frame A, ${}^{A}T_{E}$, as a function of the joint coordinates β_1, β_2,β_3.

12 marks d) Draw each link separately, showing the D-H frame attached to each link.

16 marks e) Derive the manipulator Jacobian matrix, at the configuration shown in Fig. 1 (i.e., the Jacobian evaluated at $\beta_1 = \beta_3 = 0$ and β_2 = length of BC). This Jacobian relates the generalized 6 x 1 velocity of Frame E in Frame A, with the joint velocities (time derivatives of β_1, β_2,β_3).

2. Fig. 2 shows a planar robot with a rotational joint followed by a translational joint, and the joint coordinates are θ and r, respectively. The position and orientation of the end-effector is described by the task space coordinates (x, y, θ). Determine the singularities of the robot with respect to the following tasks:

10 marks each (total=30)

a) moving along x
b) moving along y
c) planar rotation around the z-axis

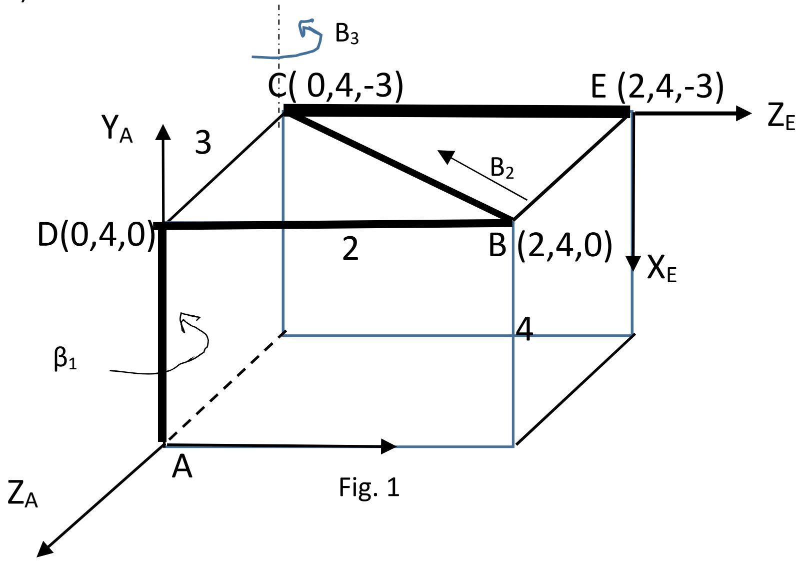


Fig. 1

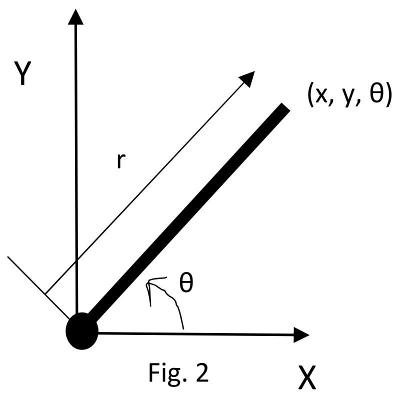
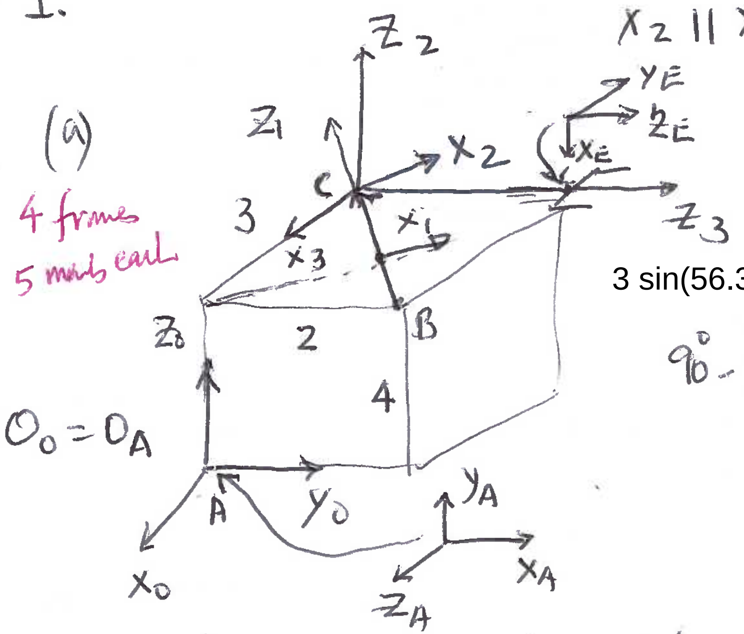


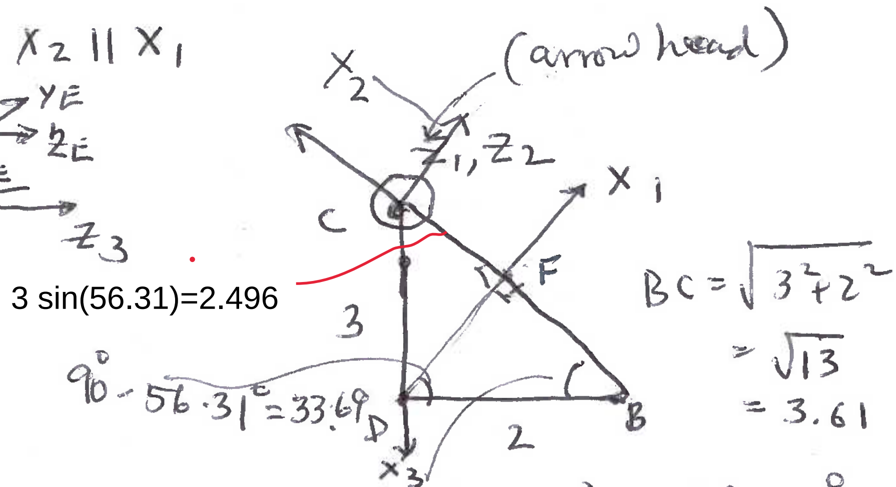
Fig. 2

1.

(a)

4 frames
5 marks each





$ATAN2(3,2) = 56.31^\circ$

$DF = 2 \sin 56.31^\circ = 1.66$

b)

1 mark × 12

i	θ_i	r_i	d_i	α_i
1	$q_1 = 90+33.69 = 123.69$	4	1.66	-90°
2	0°	q_2=2.496	0	$+90^\circ$
3	$q_3 = -(90+33.69) = -123.69$	0	0	-90°

$${}^{i-1}T_i = Rot(z,\theta_i)\, Trans(0,0,r_i)\, Trans(d_i,0,0)\, Rot(x,\alpha_i)$$

$i = 1, 2, 3$

c)

$${}^{A}T_0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ 3

$${}^{3}T_E = \begin{pmatrix} 0 & +1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ 3

1 $\quad {}^{A}T_E = {}^{A}T_0\, {}^{0}T_1\, {}^{1}T_2\, {}^{2}T_3\, {}^{3}T_E = f(q_1, q_2, q_3)$

1 $\quad \beta_1 = q_1 - 123.69^\circ$

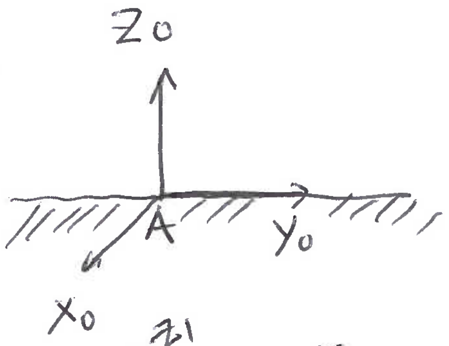
1 $\quad \beta_2 = q_2$

1 $\quad \beta_3 = q_3 + 123.69$

$${}^{A}T_E = f(\beta_1, \beta_2, \beta_3)$$

(d) there are 4 D-H frames, frames 0 to 3

link 0 = table

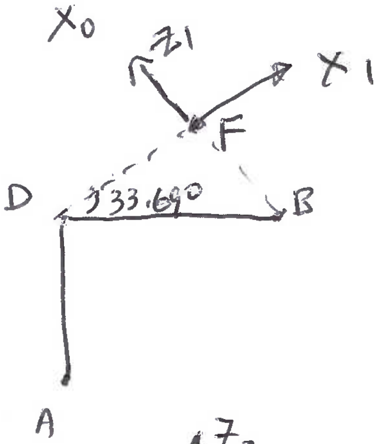


Frame 0 attached to the table

(Frame A, given, also attached to table)

4

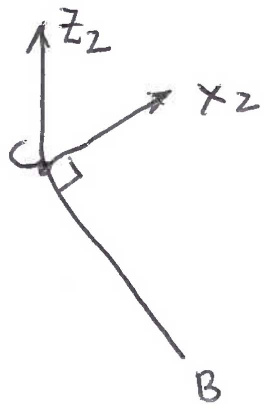
link 1



DF = 1.66

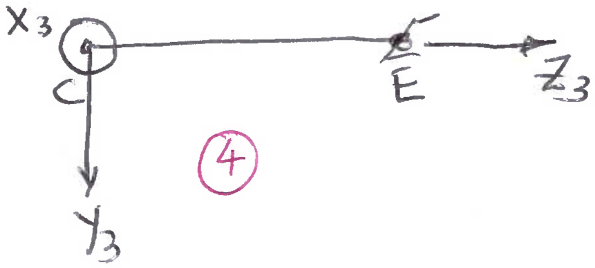
4

link 2



4

link 3



4

e) Jacobian at conf. shown

1 $${}^{A}J_{E} = \begin{pmatrix} J_1 & J_2 & J_3 \end{pmatrix}$$

5 $$J_1 = \begin{pmatrix} {}^{A}z_0 \times ({}^{A}p_E - {}^{A}p_A) \\ {}^{A}z_0 \end{pmatrix} \qquad {}^{A}z_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

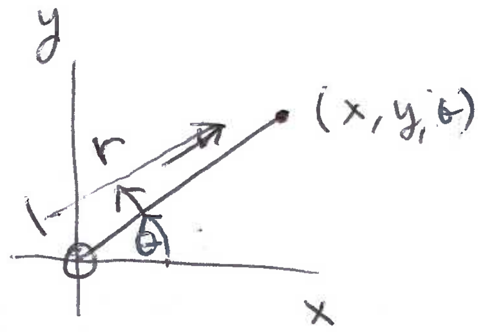
$${}^{A}p_E = \begin{pmatrix} 2 \\ 4 \\ -3 \end{pmatrix} \qquad {}^{A}p_A = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

5 $$J_2 = \begin{pmatrix} {}^{A}z_1 \\ 0 \end{pmatrix} \qquad {}^{A}z_1 = \begin{pmatrix} -2 \\ 0 \\ -3 \end{pmatrix} \times \frac{1}{\sqrt{(-2)^2 + (-3)^2}}$$

5 $$J_3 = \begin{pmatrix} {}^{A}z_2 \times ({}^{A}p_E - {}^{A}p_C) \\ {}^{A}z_2 \end{pmatrix}$$

$${}^{A}z_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad {}^{A}p_C = \begin{pmatrix} 0 \\ 4 \\ -3 \end{pmatrix}$$

2.



$u_x = \dot{x}$

$u_y = \dot{y}$

$w_z = \dot{\theta}$

$x = r\cos\theta$

$y = r\sin\theta$

$\dot{x} = -r\sin\theta\,\dot{\theta} + \dot{r}\cos\theta$

$\dot{y} = r\cos\theta\,\dot{\theta} + \dot{r}\sin\theta$

$$\begin{pmatrix} u_x \\ u_y \\ w_z \end{pmatrix} = \begin{pmatrix} -r\sin\theta & \cos\theta \\ r\cos\theta & \sin\theta \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{\theta} \\ \dot{r} \end{pmatrix}$$

10

If task is

u_x :

singularities at

$r = 0$ and $\theta = 90°$

10

If task is

u_y :

singularities at

$r = 0$, $\theta = 0°$

10

If task is

w_z :

no singularity

Note: singularities can also be obtained by inspection