**ME4245 Quiz 1.2 Robot Kinematics & Statics 20 Sep 2017, 16:15-17:30**

Open book and notes. But strictly no sharing of books and notes. Answer all questions completely, but you do no need to simplify your answers. Make sure all expressions are complete.

1. (40 marks) Figure 1 shows a robot with 2 rotational joints (at A and C). The first and 2nd rotational joint axes are along $X_A$ and BC, respectively. The 1st moving link is ABC where the angle 30 degrees is constant. The 2nd moving link is CE where Frame E is attached to the 2nd link. Note that CE makes a constant angle of 90 degrees with CB. Frame A is attached to the ground as shown. AB = 6, BC = 6, and CE = 2.

a) Assign Frames to the robot according to the D-H convention given in class. Draw each link separately, and include the frame attached to the respective link.

b) Determine the table of kinematic parameters, and indicate which ones are the joint variables $q_1$ and $q_2$.

c) Indicate the values of the joint variables at the configuration shown in Fig 1.

d) Derive the complete expression for the position and orientation of Frame E in A, ${}^{A}T_{E}$, as a function of $q_1$ and $q_2$. You don't need to simplify the expression. But make sure it's complete and only function of $q_1$ and $q_2$. Be sure to give expressions of each symbol you have used for ${}^{A}T_{E}$.

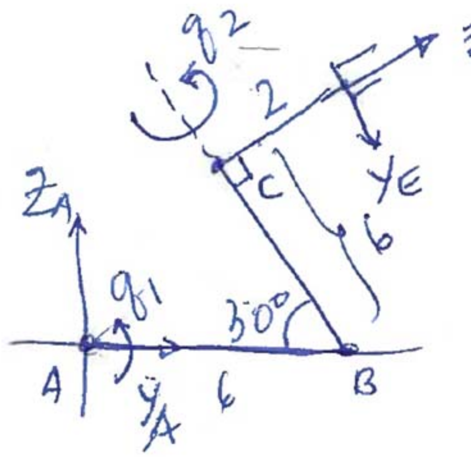


Figure 1

2. (40 marks) Figure 2 shows a robot with 3 joints: 1st 2 joints provide planar rotation along the XY plane of Frame 0 (their axes of rotation are parallel to $Z_0$, XY Plane is plane of the paper). Frame 0 is attached to the robot base/ground. Frame E is attached to the third link, which translates along an axis parallel to $Z_0$ (out of the plane of the paper). The zero positions and positive directions of motions for the three joint variables q1, q2, and q3 are shown in Figure 2.

a) Determine the expression for the complete manipulator Jacobian that relates the translational and angular velocities of Frame E in 0 as a function of the three joint velocities.

b) For the task consisting of x-, y- and z- translational velocities of point E with respect to Frame 0 ($u_x$, $u_y$ and $u_z$) determine the singular configurations of the robot, if any.

c) For the task consisting of translational velocities of point E along the x and y axis of Frame 0 and rotational velocity of Frame E around an axis parallel to $Z_0$ ($u_x$, $u_y$ and $\omega_z$), determine the singular configurations of the robot, if any.

d) For the task ($u_y$ and $u_z$), determine the singular configurations of the robot, if any

e) For the task ($u_y$ and $\omega_z$), determine the singular configurations of the robot, if any

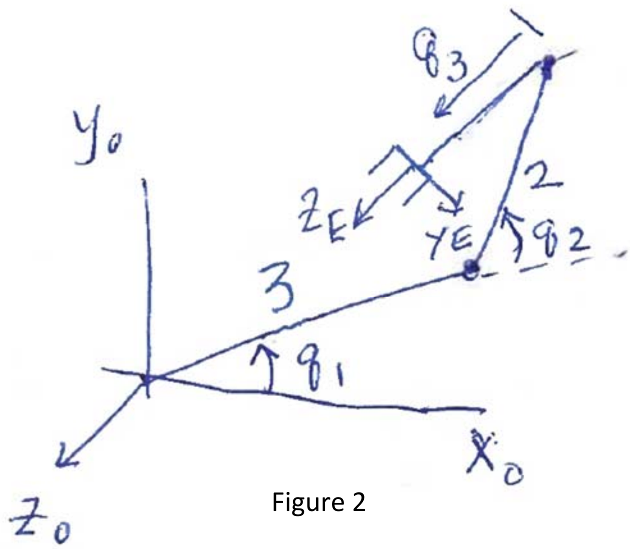


Figure 2

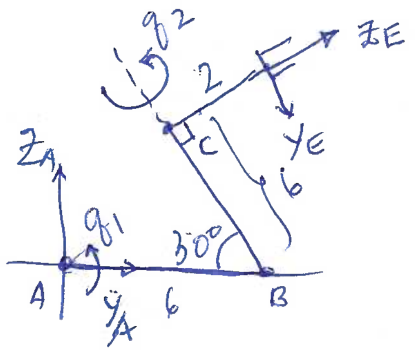
3. (20 marks) The robot with N joints is carrying a payload (M = 10 kg) in its end-effector (Frame E). The 3 D position of the centre of gravity of M with respect to E is given by ${}^{E}P_{M}$. The robot is standing on the ground, with Frame A attached to its base (ground). The direction of the acceleration due to gravity (9.8 m/s$^2$) is in the negative Z direction of Frame A. The payload is held at a static position and orientation given by ${}^{A}T_{E}$. At this configuration, the Jacobian of E with respect to A is known, ${}^{A}J_{E}$ and all the joint positions of the robot, q1, … qN are known.

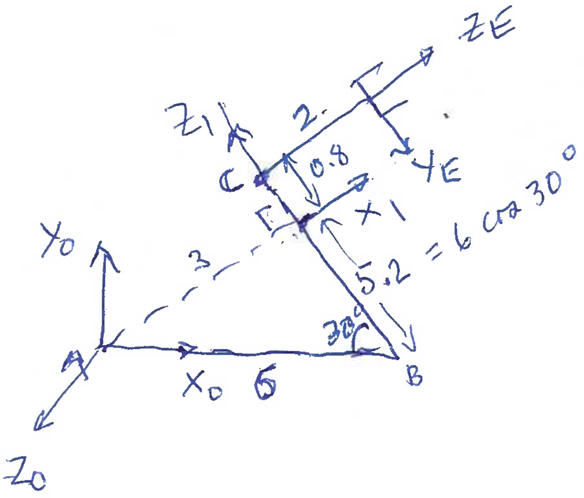
a) Determine the expression for the joint actuator forces/torques to maintain the payload at this static position. Express this expression as a function of the given known quantities.

b) A force/torque sensor is installed between the base of the robot and the ground, such that ${}^{S}T_{A}$ is known. Frame S is the frame attached to the sensor. The sensor gives 3 x 1 force sensor ${}^{S}f_{S}$ and 3x1 moment ${}^{S}n_{S}$ readings. Determine the expression for the force and moment readings of the sensor as a function of the known quantities.

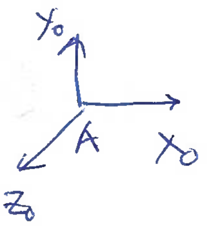
1.

a)

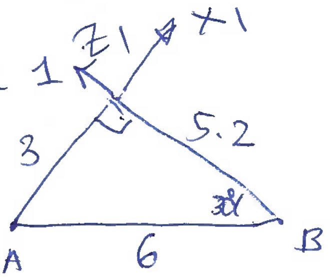


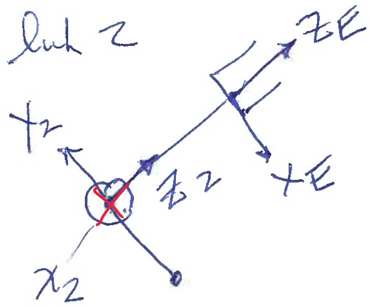

link 0




link 1




link 2




b)

c)

| | $\theta$ | $r$ | $d$ | $\alpha$ |
|---|---|---|---|---|
| 1 | $q_1 = 60^\circ$ | 0 | 3 | $-90^\circ$ |
| 2 | $q_2 = +90^\circ$ | 0.8 | 0 | $+90^\circ$ |

$$^{2}T_E = \begin{pmatrix} 0 & +1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

d)

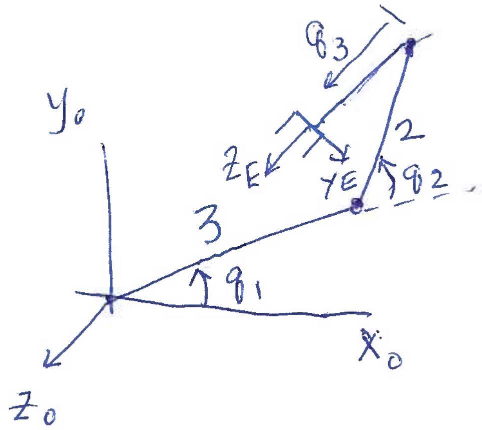
$$^{A}T_0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^{A}T_E = {}^{A}T_0 \, {}^{0}T_1 \, {}^{1}T_2 \, {}^{2}T_E$$

$$^{0}T_1 = \begin{pmatrix} \cos q_1 & -\cos(-90^\circ)\sin q_1 & \sin(-90^\circ)\sin q_1 & 3\cos q_1 \\ \sin q_1 & \cos(-90^\circ)\cos q_1 & -\sin(-90^\circ)\cos q_1 & 3\sin q_1 \\ 0 & \sin(-90^\circ) & \cos(-90^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^{1}T_2 = \begin{pmatrix} \cos q_2 & -\cos(-90^\circ)\sin q_2 & \sin(-90^\circ)\sin q_2 & 0 \\ \sin q_2 & \cos(-90^\circ)\cos q_2 & -\sin(-90^\circ)\cos q_2 & 0 \\ 0 & \sin(-90^\circ) & \cos(-90^\circ) & 0.8 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2.




$$^0V_E = \underbrace{\left(^0J_E\right)}_{6\times 3}\begin{pmatrix}\dot q_1\\ \dot q_2\\ \dot q_3\end{pmatrix} = \begin{matrix}^0u_E\\ ^0\omega_E\end{matrix}$$

By inspection:

$$^0P_E = \begin{matrix} x = 3C_1 + 2C_{12} \rightsquigarrow \dot x = -3S_1\dot q_1 - 2S_{12}(\dot q_1+\dot q_2) \\ y = 3S_1 + 2S_{12} \rightsquigarrow \dot y = +3C_1\dot q_1 + 2C_{12}(\dot q_1+\dot q_2) \\ z = q_3 \qquad \dot z = \dot q_3 \end{matrix}$$

a) $\therefore$

$$^0J_E = \begin{pmatrix} -3S_1-2S_{12} & -2S_{12} & 0 \\ 3C_1+2C_{12} & 2C_{12} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

b) For Task: $u_x, u_y$ & $u_z$

$$\det\begin{pmatrix} -3S_1-2S_{12} & -2S_{12} & 0 \\ 3C_1+2C_{12} & 2C_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= -6S_1C_{12} - 4S_{12}C_{12} - \left[-6C_1S_{12} - 4C_{12}S_{12}\right] = 0$$

$$= 6(S_{12}C_1 - C_{12}S_1) = 6S_2 \quad \therefore q_2 = 0° \text{ or } 180°$$

c) For Task $u_x$, $u_y$ + $w_z$, always singular

because
$$J = \begin{pmatrix} -3S_1 - 2S_{12} & -2S_{12} & 0 \\ 3C_1 + 2C_{12} & 2C_{12} & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

$\det(J)$ = always zero

d) For Task $U_y$, + $U_z$

$$J = \begin{array}{cccc} & (1) & (2) & (3) \\ & \multicolumn{3}{c}{\begin{pmatrix} 3C_1 + 2C_{12} & 2C_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}} \end{array}$$

$\det(J_{12}) = 0$ $\quad \det(J_{13}) = 3C_1 + 2C_{12} = 0$

$\theta_1 = 90°, \ \theta_2 = 0°$

$\det(J_{23}) = 0$ $\quad \det(J_{12}) = C_{12} = 0$

Singularities when all det $= 0$

$C_{12} = 0$ and $C_1 = 0$

$(\theta_1 + \theta_2) = \pm 90°$ $\quad \theta_1 = \pm 90°$

$(\theta_1, \theta_2) = (90°, 0°)$, or $(90°, -180°)$, or

$= (-90°, 0°)$ or $(-90°, 180°)$,

e) For taking $v_y$ & $\omega_z$

$$J = \begin{pmatrix} 3c_1 + 2c_{12} & 2c_{12} & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

$$\det\begin{pmatrix} 3c_1 + 2c_{12} & 2c_{12} \\ 1 & 1 \end{pmatrix} = 3c_1 + 2c_{12} - 2c_{12} = 0$$

$$3c_1 = 0$$

$$\theta_1 = \pm 90^\circ \;//$$

3.

a) Given: ${}^{E}P_M$, ${}^{A}T_E$, ${}^{A}J_E$, $q$, ${}^{A}g = \begin{matrix} 0 \\ 0 \\ -9.8 \end{matrix}$

Joint actuator Forces: $\tau$

$$\tau = {}^{A}J_E^{T} \, {}^{A}F_E$$

$${}^{A}F_E = \begin{pmatrix} {}^{A}f_E \\ {}^{A}n_E \end{pmatrix} \qquad {}^{A}f_E = \begin{pmatrix} 0 \\ 0 \\ 9.8 \end{pmatrix}$$

$${}^{A}n_E = {}^{A}R_E \, {}^{E}P_M \times {}^{A}f_E$$

${}^{A}R_E$ = 3x3 part of ${}^{A}T_E$:

$${}^{A}T_E = \begin{bmatrix} {}^{A}R_E & {}^{A}P_E \\ 0 & 1 \end{bmatrix}$$

b)

6 axis reading

$${}^{S}F_S = \begin{pmatrix} {}^{S}f_S \\ {}^{S}n_S \end{pmatrix}; \quad {}^{S}f_S = \left({}^{S}R_A \, {}^{A}f_E\right) \times (-1)$$

$${}^{S}n_S = \left({}^{S}R_A \, {}^{A}P_M\right) \times \left({}^{S}R_A \, {}^{A}f_E\right) \times (-1)$$

$${}^{A}P_M = {}^{A}T_E \, {}^{E}P_M$$

3x1 part