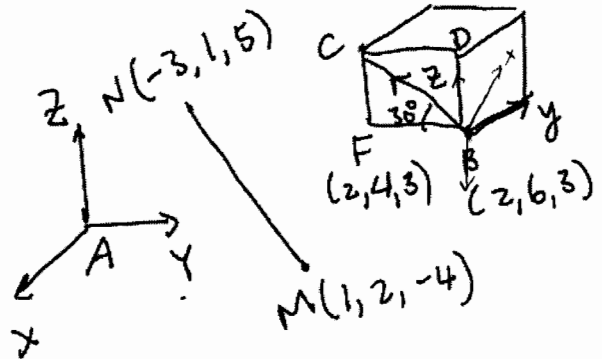


Name (as in Matric Card): _____ Matric # _____

Please answer all questions. You don't need to simplify your answers; but please make sure your answers are complete. (For example, you can leave your answer as ${}^A T_B = {}^A T_C {}^C T_B$ without multiplying the two matrices; but you have to give complete expressions for ${}^A T_C$ and ${}^C T_B$.) Please write your solutions on this sheet itself. If you need to use separate sheets, please write your name and matric number clearly on your answer sheets. This is an "open-book/notes" examination. But no sharing of books or notes are allowed.

The figure shows two frames, A and B. B is attached to a rectangular block at one of its corners whose coordinates are (2,6,3) in Frame A. The Z axis of Frame B is directed along line segment from B to C and is on the plane defined by BDCF, which is parallel to ZY plane of Frame A. Z axis of Frame B makes an angle of 30° with respect to line segment BF. M and N are fixed to Frame A at the indicated coordinates.



The block undergoes the following motion in the indicated sequence:

- 1st) Rotation about Z axis of Frame A by 30 degrees,
- 2nd) Rotation of 60 degrees about an axis indicated by line segment from M to N. $\rightarrow$ let this be Z axis of Frame Q
- 3rd) Rotation of 90 degrees about its own y axis (y axis of Frame B)

Determine new position of the corner F of the block in Frame A.

Solution: $Rot(z, \theta) = \begin{pmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ $Rot(y, \theta) = \begin{pmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

${}^A T_{B_0} \text{ (before motion)} = \begin{pmatrix} {}^A x_B & 0 & 0 & 2 \\ 0 & -\cos 30^\circ & 0 & 6 \\ 0 & \sin 30^\circ & 0 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ ${}^A x_B = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -\cos 30^\circ \\ \sin 30^\circ \end{pmatrix}$

${}^A T_{B_1} = Rot(z, 30^\circ) {}^A T_{B_0}$

${}^Q T_{B_2} = Rot(z, 60^\circ) {}^Q T_{B_1}$ where ${}^Q T_{B_1} = {}^Q T_A {}^A T_{B_1}$ ${}^Q T_A = {}^A T_Q^{-1}$

Let M = origin of Frame Q, $\vec{MN} = \text{Z axis of Frame Q} = \frac{1}{\sqrt{9^2 + 1^2 + 4^2}} \begin{pmatrix} -4 \\ -1 \\ 9 \end{pmatrix} = {}^A z_Q$

${}^A T_Q = \begin{pmatrix} {}^A x_Q & {}^A y_Q & {}^A z_Q & -\frac{1}{4} \\ 0 & 0 & 0 & 1 \end{pmatrix}$ let ${}^A y_Q = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

pick any a, b, c such that ${}^A y_Q \cdot {}^A z_Q = 0$ (many possible are).

Then ${}^A x_Q = {}^A y_Q \times {}^A z_Q$ (Note many possible ${}^A p_Q$'s also)

${}^Q T_{B_3} = {}^Q T_{B_2} Rot(y, 90^\circ)$

${}^A T_{B_3} = {}^A T_Q {}^Q T_{B_3}$

${}^A p_{F_3} = {}^A T_{B_3} {}^B p_{F_3}$

where ${}^B p_{F_3} = Rot(y, -30^\circ) \begin{pmatrix} 0 \\ 0 \\ 6-4 \\ 1 \end{pmatrix}$