National University of Singapore — Department of Mechanical Engineering

ME4245 — Quiz 1 — 6 Sept 2006, 16:00-16:45

1. Frame C is attached to a corner of the rectangular block as shown in Fig. 1. The y axis of Frame C is directed from corner C to D, while the x axis is directed from C to E. The coordinates of the corners are expressed in Frame A. Determine the homogeneous transformation matrix (4 x 4) that describes the position and orientation of Frame C in A.

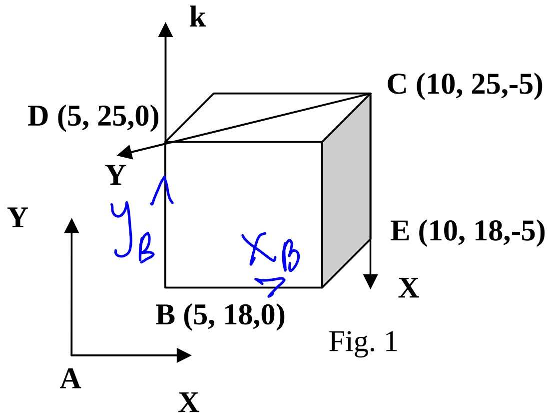

Fig. 1

$$^A x_C = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

$$^A y_C = \begin{pmatrix} 5-10=-5 \\ 0 \\ 0+5=5 \end{pmatrix} \times \frac{1}{\sqrt{5^2+5^2}}$$

$$^A z_C = {}^A x_C \times {}^A y_C$$

$$^A p_C = \begin{pmatrix} 10 \\ 25 \\ -5 \end{pmatrix} \quad \therefore \; {}^A T_C = \begin{pmatrix} ^A x_C & ^A y_C & ^A z_C & ^A p_C \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2. The rectangular block is initially at a position and orientation shown in Fig 1. The k axis and Frame C are attached to the block. The block then undergoes the following sequence of motion:
   1> rotation about x axis of frame A by 30 degrees
   2> Translation along Frame C by (5,10,15) m
   3> rotation about k axis (directed from B to D) by 90 degree
   Determine the new position and orientation of Frame C. in Frame A.

$$^A T_{C_1} = Rot(x, 30^\circ)\, {}^A T_{C_0} \quad \text{where } {}^A T_{C_0} = {}^A T_C, \; Rot(x, 30^\circ) = \ldots$$

$$^A T_{C_2} = {}^A T_{C_1}\, Trans(5,10,15) \quad \text{Where } Trans(5,10,15) = \begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 15 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{Let } \vec{BD} = {}^A y_B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad {}^A x_B = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad {}^A z_B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$^A T_{B_2} = {}^A T_{C_2}\, {}^{C_2} T_{B_2}; \quad {}^{C_2} T_{B_2} = {}^{C_2} T_{A_2}\, {}^{A_2} T_{B_2} \quad \text{or} \quad \underbrace{{}^C T_B = {}^C T_A\, {}^A T_B}_{\text{at same time instant}}$$

$$^{A_2} T_{B_2} = {}^A T_B = \begin{pmatrix} ^A x_B & ^A y_B & ^A z_B & ^A p_B \\ 0 & 0 & 0 & \end{pmatrix} \qquad {}^A p_B = \begin{pmatrix} 5 \\ 18 \\ 0 \\ 1 \end{pmatrix}$$

$$^A T_{B_3} = {}^A T_{B_2}\, Rot(y, 90^\circ) \quad \text{where } Rot(y, 90^\circ) = \ldots$$

$$^A T_{C_3} = {}^A T_{B_3}\, {}^B T_C \quad \text{where } {}^B T_C = {}^C T_B^{-1}$$

3. Fig 2 shows a planar robot with its first joint rotational followed by a translational joint. The 2nd link is at 90 degrees with respect to the 1st link. The robot is connected to a table (Frame A). Frame E is attached to the end effector (last link).
   a) Assign frames to the robot according to the Denavit Hartenberg (DH) convention discussed in class.
   b) Determine the 4 kinematic parameters that describe the spatial relationship between adjacent links. (You need to provide 8 parameters.) Indicate which parameters are the joint coordinates/variables.
   c) Determine the spatial relationship (4x4 homogeneous transformation matrix) between the DH frame attached to the table/ground (Frame 0) and Frame A.
   d) Determine the spatial relationship (4x4 homogeneous transformation matrix) between the frame attached to the last link (Frame 2) and Frame E.

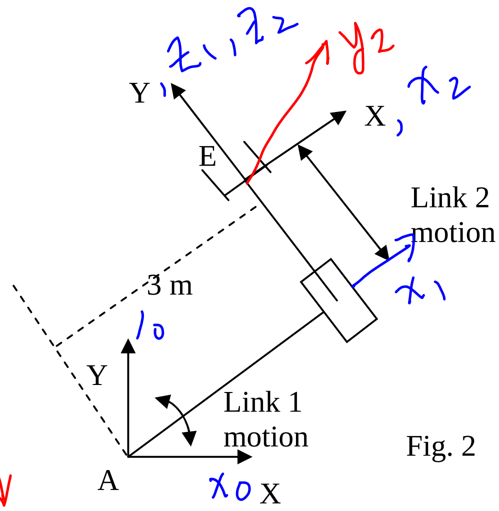

Fig. 2

	θ	r	d	α
1	q_1	0	3	$-90°$
2	0	q_2	0	0

Why?

$${}^{A}T_E = {}^{A}T_0 \, {}^{0}T_1 \, {}^{1}T_2 \, {}^{2}T_E$$

$${}^{A}T_0 = I$$

$$
{}^{2}T_E = \begin{array}{c} \begin{array}{cccc} {}^{2}x_E & {}^{2}y_E & {}^{2}z_E & \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}
$$