Name:_____   Matric Number:_____
(as it appears in your NUS Student card)

Answer all the two questions in this quiz. You need not simplify your answers. But, please make sure all expressions are complete. Please note that the 2nd question is at the back of this page.

1. Fig.1 shows a rectangular block of dimensions 1 m x 2 m x 3 m with Frame B rigidly attached to it. The block is initially positioned in Frame A as shown in Fig. 1.
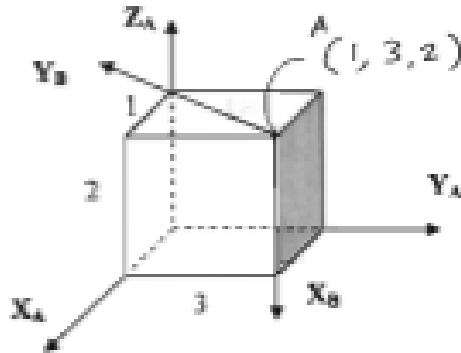

a. Determine the position of Frame B in A.
b. Determine the orientation of Frame B in A
c. Determine the position of Frame A in B.

$$^A\hat{X}_B = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \qquad ^A\hat{Y}_B = \begin{pmatrix} -1 \\ -3 \\ 0 \end{pmatrix} \cdot \frac{1}{\sqrt{10}}$$

Fig. 1

Ans:

$$^A\hat{Z}_B = {}^A\hat{X}_B \times {}^A\hat{Y}_B$$

$$^A\hat{Z}_B = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \times \begin{pmatrix} -1/\sqrt{10} \\ -3/\sqrt{10} \\ 0 \end{pmatrix} = \begin{pmatrix} -3/\sqrt{10} \\ 1/\sqrt{10} \\ 0 \end{pmatrix}$$

(a) $\quad ^AP_B = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$

(position of B in A)

(b) $^A R_B = \begin{pmatrix} 0 & -1/\sqrt{10} & -3/\sqrt{10} \\ 0 & -3/\sqrt{10} & 1/\sqrt{10} \\ -1 & 0 & 0 \end{pmatrix}$

(orientation of B in A)

(c). $^A T_B = \begin{pmatrix} ^A R_B & ^A P_B \\ 0 & 1 \end{pmatrix}$; $\quad ^B T_A = \begin{pmatrix} ^A R_B^T & -^A R_B^T \, ^A P_B \\ 0 & 1 \end{pmatrix}$

$$^B P_A = -^A R_B^T \, ^A P_B = -\begin{pmatrix} 0 & 0 & -1 \\ -\frac{1}{\sqrt{10}} & -3/\sqrt{10} & 0 \\ -3/\sqrt{10} & 1/\sqrt{10} & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

$$^B P_A = \begin{pmatrix} +2 \\ +10/\sqrt{10} \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ \sqrt{10} \\ 0 \end{pmatrix}$$

2.   Frames A and B are fixed to the ground with its relative
configuration (position and orientation) ^AT_B known. Frame C is attached to a moving object. The
object is initially at a configuration indicated by ^AT_C; with respect to Frame A. The object then
undergoes the following sequence of motions:
1> rotation about x axis of frame A by 30 degrees
2> rotation about its own y axis (Frame C) by 60 degrees
3> Translation along Frame B by (4,5,6) m

Determine the new configuration of Frame C. in Frame A.

Ans:

$$^AT_B = known.$$
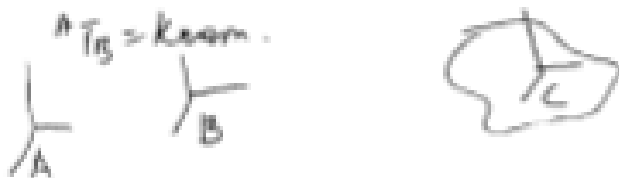
$$^AT_{C_0} = \; ^AT_C \; (given) = initial \; configuration \; of \; C$$

$$^AT_{C_1} = Rot(x, 30°) \; ^AT_{C_0} \quad (after \; motion \; 1))$$

$$where \; Rot(x, 30°) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & cos30° & -sin30° & 0 \\ 0 & sin30° & cos30° & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^AT_{C_2} = \; ^AT_{C_1} \; Rot(y, 60°) \quad (after \; motion \; 2))$$

$$where \; Rot(y, 60°) = \begin{pmatrix} cos60° & 0 & sin60° & 0 \\ 0 & 1 & 0 & 0 \\ -sin60° & 0 & cos60° & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^BT_{C_2} = \; ^BT_A \; ^AT_{C_2} = \; ^AT_B^{-1} \; ^AT_{C_2}$$

$$^BT_{C_3} = Trans(4,5,6) \; ^BT_{C_2} \quad (after \; motion \; 3))$$

$$^AT_{C_3} = \; ^AT_B \; ^BT_{C_3} = \; ^AT_B \; Trans(4,5,6) \; ^AT_B^{-1} \; Rot(x, 30°) \; ^AT_C \; Rot(y, 60°)$$