National University of Singapore
Department of Mechanical Engineering
ME4245 Quiz 1 25 Aug 2008, 15:00-15:40

Name:______________________________ Matric Number:______________________
(as it appears in your NUS Student card)

Answer all the two questions in this quiz. You need not simplify your answers. But, please make sure all expressions are complete. ~~Please note that the 2nd question is at the back of this page~~.

1. Fig.1 shows a cube (Body C) with a length of 2 m on each side. Frame C is attached to the cube at Corner C with its Z axis along directional line from C to A, and Y axis along directional line from C to F. The cube is currently at the indicated coordinates which are all expressed in Frame U.

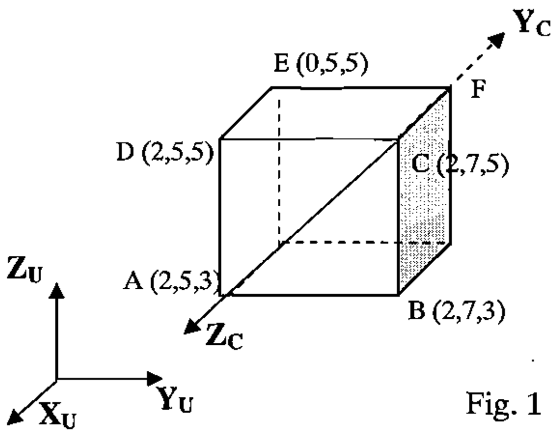

Fig. 1

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a. Determine the position and orientation of Frame C with respect to Frame U. Express the position and orientation as a 4 x 4 homogeneous transformation matrix ${}^{U}T_{C}$

b. A Point "G" has coordinates (1,2,3) m in Frame U. What are its coordinates in Frame C.

Ans:

2. The Body C undergoes the following sequence of motions starting from an initial configuration shown in Fig. 1

1st> Rotation about the Z axis of Frame U by 30 degrees
2nd> Translation along itself (Frame C) by (4, 5, 6) m
3rd> Rotation about the X axis of Frame U by 60 degrees

Determine the 4x4 homogenous transformation matrix ${}^{U}T_{C}$ representing the final position and orientation of Body (Frame) C in Frame U.

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1. a) ${}^{u}P_c = \begin{pmatrix} 2 \\ 7 \\ 5 \\ 1 \end{pmatrix}$ ${}^{u}R_c = \begin{pmatrix} {}^{u}x_c & {}^{u}y_c & {}^{u}z_c \end{pmatrix}$

${}^{u}y_c = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

${}^{u}z_c = \begin{pmatrix} 2-2 \\ 5-7 \\ 3-5 \\ 0 \end{pmatrix} \times \frac{1}{\sqrt{\cdot}} = \begin{pmatrix} 0 \\ -2 \\ -2 \\ 0 \end{pmatrix} \frac{1}{\sqrt{(-2)^2+(-2)^2}}$

${}^{u}x_c = \underbrace{{}^{u}y_c}_{3\times1} \times \underbrace{{}^{u}z_c}_{3\times1}$ + add $\emptyset$

${}^{u}T_c = \begin{pmatrix} {}^{u}x_c & {}^{u}y_c & {}^{u}z_c & {}^{u}P_c \end{pmatrix}$

b. ${}^{u}P_G = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix}$

${}^{c}P_G = {}^{c}T_u \, {}^{u}P_G$

$\downarrow$

${}^{u}T_c^{-1} =$

${}^{u}T_c^{-1} =$

$\left[\begin{array}{c|c} {}^{u}R_c^{T} & -{}^{u}R_c^{T}\,{}^{u}P_c \\ \hline 0 & 1 \end{array}\right]$

${}^{u}R_c = 3\times3$ (upper left)
${}^{u}P_c = 3\times1$ (upper right)
coords is 3×1 part of ${}^{u}P_c$

2. Let ${}^{u}T_{c_0} = {}^{u}T_c$ (from #1)

${}^{u}T_{c_1} = Rot(z, 30^\circ)\, {}^{u}T_c$ When $Rot(z, 30^\circ) =$

$$\begin{pmatrix} \cos 30^\circ & -\sin 30^\circ & 0 & 0 \\ \sin 30^\circ & \cos 30^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

${}^{u}T_{c_2} = {}^{u}T_{c_1} \underbrace{Trans(4,5,6)}$

$$\begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

${}^{u}T_{c_3} =$

$= \underbrace{Rot(x, 60^\circ)}\, {}^{u}T_{c_2}$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 60^\circ & \sin 60^\circ & 0 \\ 0 & \sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$