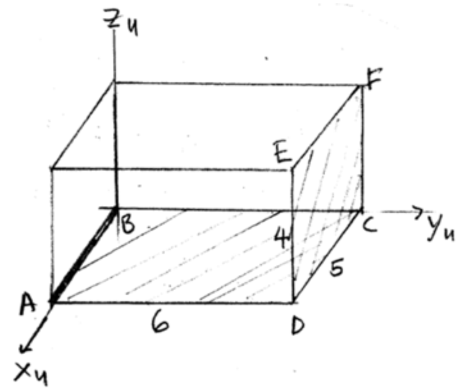
ME4245 Quiz 1 **1 hr – Open Book and Notes** **9 Sept 2015**

Name:__________________________________ **Matric:________________**

Answer the 2 Questions below. Give your answers in this examination script. You can use additional sheets if needed, but be sure to write your name and matric number in each sheet. You need not simplify your answers. You can leave your answers in expression form, but with quantities clearly indicated.

1. (40 marks) Fig 1 (below) shows a rigid body ('L" shaped plate ABCDEF) that is initially located at Frame U. The body undergoes the following sequence of motions:



1^{st}> Rotation about Xu = AB by 30 degrees
2^{nd}> Rotation about Yu = BC by 60 degrees
3^{rd}> Rotation about EF by 80 degrees
4^{th}> Translation along AF by 2 units

Find the new coordinates of F in U.

Solution:

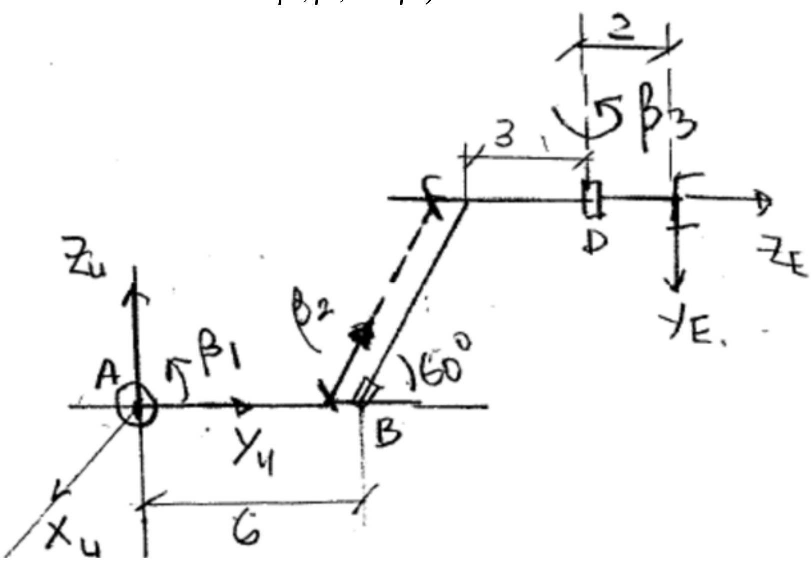
2. (60 marks) Fig 2 (below) shows a robot with 3 joints: 1st link (AB) rotates (around X_U) with the joint variable β_1; 2nd link (BCD) translates (along BC) with joint variable β_2; and the 3rd link (DE, end-effector) rotates with joint variable β_3. The zero positions and positive directions of each joint variable are shown in the figure. In the configuration shown, all the 3 links of the robot are in the YZ plane of Frame U and the 3rd link rotates about an axis parallel to Z_U at D. Frame U is attached to the base of the robot (table robot is mounted on). Frame E is attached to the last link (link 3 = end-effector) of the robot.

(a) Assign frames to each of the 3 links according to the Denavit Hartenberg convention given in class. (You can draw the frames using the figure below)

(b) Derive the table of kinematic parameters.

(c) Draw each of the links together with the frame attached to the link. Indicate the location of the frame relative to the link.

(d) Derive an expression for the 4x4 homogenous transformation matrix describing the position and orientation of the end-effector (Frame E) with respect to Frame U, as a function of β_1, β_2, and β_3).



1

- Let Frame F be attached to Rigid Body as shown: $x_F = \overrightarrow{EF}$ $y_F = \overrightarrow{FC}$

$O_F = F$ (origin of F)

$$ {}^{u}T_F = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 6 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix} = {}^{u}T_{F_0} $$

$$ {}^{u}T_{F_1} = Rot(x, 30°)\ {}^{u}T_{F_0} $$

$$ {}^{u}T_{F_2} = Rot(y, 60°)\ {}^{u}T_{F_1} $$

$$ {}^{u}T_{F_3} = {}^{u}T_{F_2} \cdot Rot(x, 80°) $$

Let Frame A be attached to the rigid body as shown:

$x_A = \overrightarrow{AD}$ $y_A = \overrightarrow{AB}$

$$ {}^{u}T_{A_3} = {}^{u}T_{F_3}\ {}^{F}T_A \quad \text{where } {}^{A}T_F = \begin{pmatrix} 0 & 0 & -1 & 6 \\ 1 & 0 & 0 & 5 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

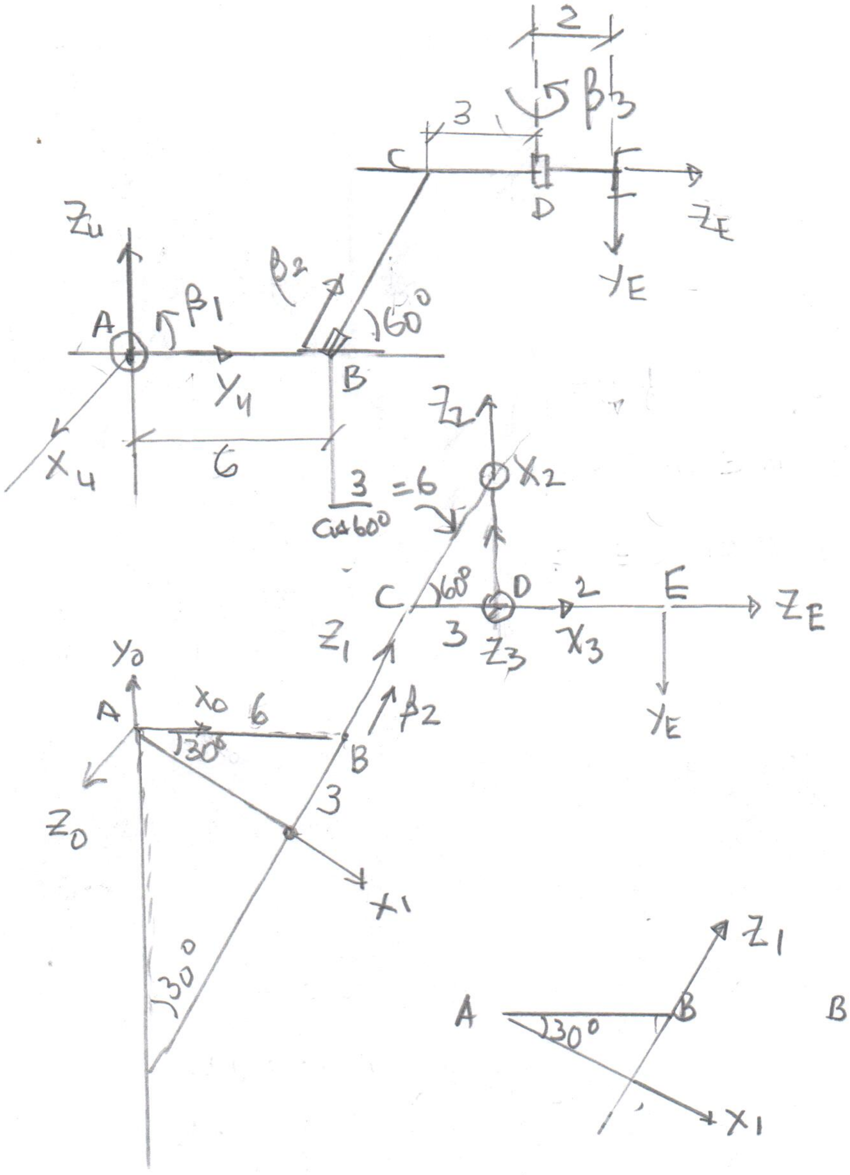
$$ {}^{u}T_{A_4} = {}^{u}T_{A_3}\ Trans\ \frac{(6, 5, 4) \cdot 2}{\sqrt{6^2+5^2+4^2}} \qquad {}^{F}T_A = {}^{A}T_F^{-1} $$

$$ {}^{u}T_{F_4} = {}^{u}T_{A_4}\ {}^{A}T_F $$

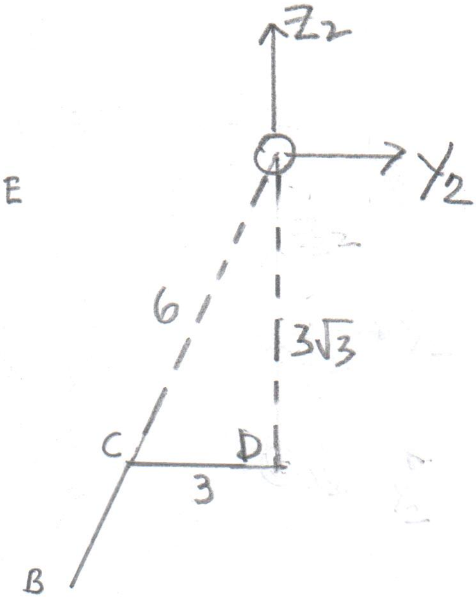
Coords of F in U is:

$$ {}^{u}P_{F_4} = {}^{u}T_{F_4} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad \text{(4th column)} $$

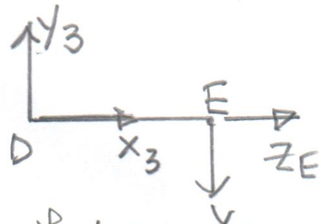
2



Link 1



Link 2



Link 3

	θ	r	d	α	
1	$-30^\circ+\beta_1$	0	$6\cos 30^\circ$	-90°	${}^0T_1$
2	-90°	$3+\beta_2+6$	0	$+30^\circ$	${}^1T_2$
3	$90^\circ+\beta_3$	$-3\sqrt{3}$	0	90°	${}^2T_3$

$${}^{i-1}T_i = \begin{pmatrix} \cos\theta_i & -\cos\alpha_i\sin\theta_i & \sin\alpha_i\sin\theta_i & d_i\cos\theta_i \\ \sin\theta_i & \cos\alpha_i\cos\theta_i & -\sin\alpha_i\cos\theta_i & d_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & r_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^uT_0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad {}^3T_E = \begin{pmatrix} 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^uT_E = {}^uT_0\ \underset{\downarrow\ f(\beta_1)}{{}^0T_1}\ \underset{\downarrow\ f(\beta_2)}{{}^1T_2}\ \underset{\downarrow\ f(\beta_3)}{{}^2T_3}\ {}^3T_E$$