

National University of Singapore
Faculty of Engineering

Quiz 1

ME4245/EE4304: Robotics

5 September 1997

1. ABCD is a rectangle whose surface is at an angle of 30° with respect to the yz plane of Frame A (Fig.1). Frame E is attached to the rectangle as shown with the origin of Frame E at the geometric center of the rectangle. The x axis of Frame E is directed towards Corner A of the rectangle and the y axis is on the plane of the rectangle as shown in the figure. The rectangle has a width (AB) of 2 m and a height (BC) of 1.5 m.

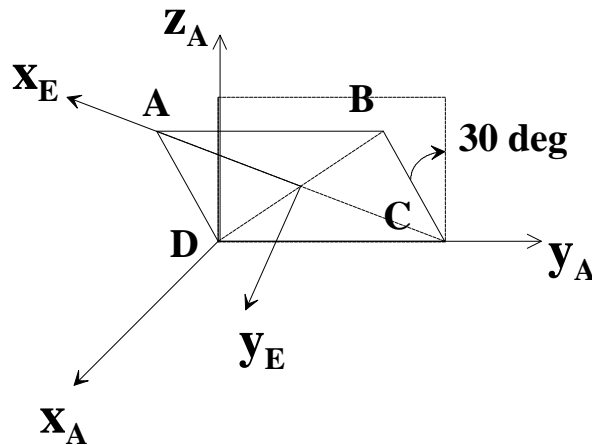


Fig. 1

- (a) Determine the position and orientation of Frame E in Frame A.

Ans:
$$\begin{bmatrix} i & 0.3 & -0.4 & 0.866025 & 0.375 \\ & -0.8 & -0.6 & 0. & 1. \\ k & 0.519615 & -0.69282 & -0.5 & 0.649519 \\ & 0. & 0. & 0. & 1. \end{bmatrix} \begin{matrix} y \\ \\ \\ \end{matrix} \quad (20 \text{ marks})$$

- (b) The rectangle undergoes the following sequence of motions:

- 1st Rotation about X_A by 30 degrees
- 2nd Rotation about Y_E by 45 degrees
- 3rd Translation along Frame E by (10,20,30)

Determine the new position and orientation of Frame E in Frame A. You need not numerically evaluate the position of E, you can just express your answer in terms of known quantities.

Ans:
$$\begin{bmatrix} i & -0.40024 & -0.4 & 0.824504 & 13.1077 \\ & -0.850386 & -0.173205 & -0.496833 & -26.3317 \\ k & 0.341542 & -0.9 & -0.270831 & -21.647 \\ & 0. & 0. & 0. & 1. \end{bmatrix} \begin{matrix} y \\ \\ \\ \end{matrix} \quad (20 \text{ marks})$$

2. A robot has three joints: the 1st joint is rotational, the 2nd is translational, and the 3rd joint is rotational (Fig. 2). The robot operates in a plane parallel to the yz plane of Frame A. The axes of motion of the first and third joints are parallel to the x axis of the base frame. The axis of motion of the second link is on the yz plane of Frame A. The first and last link have lengths of 1 m each, and the middle link can extend from a minimum length of 0 m to a maximum of 1 m. At the configuration shown in Fig. 1, $q_1 = 0^\circ$, $q_2 = 0.75$ m, and $q_3 = 45^\circ$ (with respect to the positive y_A axis). The positive directions of motion are indicated in Fig. 2 (arrows).

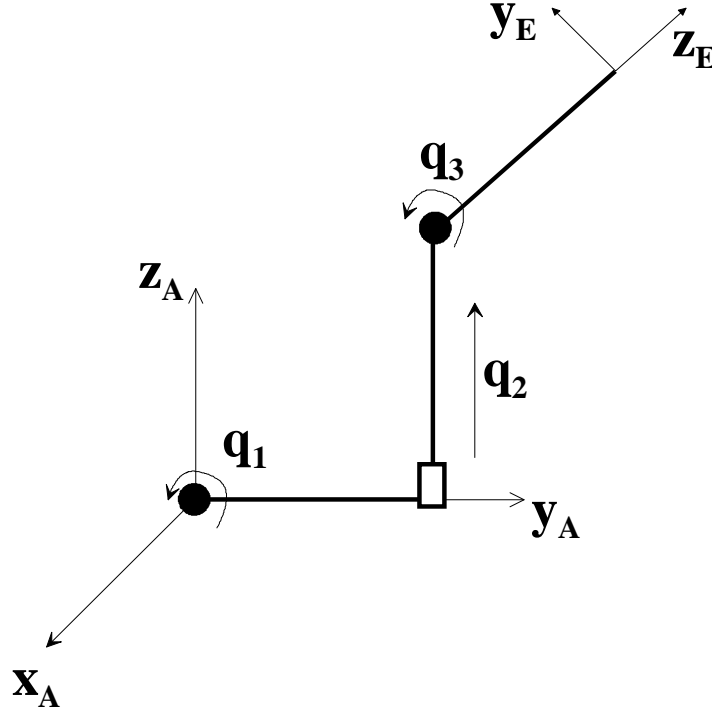


Fig. 2

- Assign frames according to the Denavit-Hartenberg (DH) convention and fill in the table of DH parameters. Indicate which of the parameters are variable. (10 marks)
- Derive expressions for the position and orientation of the end-effector (Frame E) in Frame A as functions of q_1 , q_2 , and q_3 (in Fig. 1). Note that q_1 , q_2 , and q_3 may not necessarily coincide with the corresponding joint coordinate of the DH parameters, i.e., they may have different zero positions; and that the base (0) frame need not coincide with Frame A. (15 marks)
- Derive the expression for the end-effector translational and angular velocity in Frame A as functions of the time derivatives of q_1 , q_2 , and q_3 . (10 marks)
- Given the position of the end-effector, derive expressions for the three joint coordinates q_1 , q_2 , and q_3 . (15 marks)
- Describe the positional and orientational workspace of this robot. Provide expressions when possible. (10 marks)