

QUIZ 1 Solutions (Robotics)

22 Sept 2003

1. ${}^A X_B = \begin{pmatrix} 2-5 \\ 6-6 \\ 5-2 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix} \times \frac{1}{\sqrt{(-3)^2 + (3)^2}}$

8 Marks

${}^A Y_B = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$ (along negative y axis of Frame A) after normalizing

${}^A Z_B = {}^A X_B \times {}^B Y_B$
 ${}^A P_B = \begin{pmatrix} 5 \\ 6 \\ 2 \end{pmatrix} \rightsquigarrow {}^A T_B = \begin{pmatrix} {}^A X_B & {}^A Y_B & {}^A Z_B & {}^A P_B \\ 0 & 0 & 0 & 1 \end{pmatrix} =$

2. ${}^A T_{C_0} = {}^A T_C =$ initially origin of C in A

20 marks ${}^A T_{C_1} = \text{Rot}(x, 30^\circ) {}^A T_{C_0}$ where $\text{Rot}(x, 30^\circ) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ & 0 \\ 0 & \sin 30^\circ & \cos 30^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

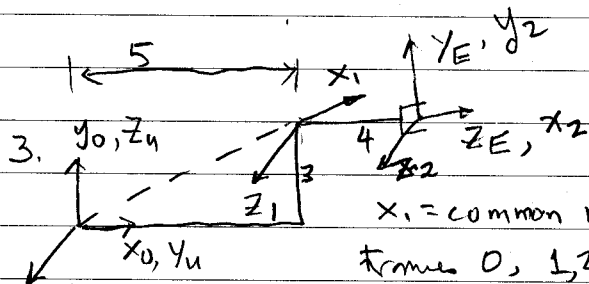
${}^A T_{C_2} = {}^A T_{C_1} \text{Trans}(1, 2, 3)$ where $\text{Trans}(1, 2, 3) = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

${}^B T_{C_2} = ({}^A T_B)^{-1} {}^A T_{C_2}$

${}^B T_{C_3} = \text{Rot}(y, 60^\circ) {}^B T_{C_2}$ where $\text{Rot}(y, 60^\circ) = \begin{bmatrix} \cos 60^\circ & 0 & \sin 60^\circ & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 60^\circ & 0 & \cos 60^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

${}^A T_{C_3} = {}^A T_B {}^B T_{C_3}$

${}^A T_{C_3} = {}^A T_B \text{Rot}(y, 60^\circ) {}^A T_B^{-1} \text{Rot}(x, 30^\circ) {}^A T_{C_0} \text{Trans}(1, 2, 3)$



3. y_0, z_0

25 marks

${}^0 T_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; {}^2 T_E = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

${}^A T_{C_2}$ table:

	θ	r	d	α
1	q_1	0	$\sqrt{3^2+5^2}$	0°
2	q_2	0	4	0°

 ${}^4 T_E = \begin{pmatrix} n_x & o_x & d_x & p_x \\ n_y & o_y & d_y & p_y \\ n_z & o_z & d_z & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$

4. ${}^0 f_E = -\begin{pmatrix} 0 \\ 0 \\ -10 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix}$

14 marks

${}^0 n_E = -{}^0 R_E {}^E P_{cg} \times \begin{pmatrix} 0 \\ 0 \\ -10 \end{pmatrix}$ where ${}^E P_{cg} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix}$

$o_x = a_x = n_y = n_z = 0$
 $n_x = -1, p_x = 0$

$T = {}^0 J_E^T \begin{pmatrix} {}^0 f_E \\ {}^0 n_E \end{pmatrix}$

$T =$ joint actuator forces.

${}^0 R_E = 3 \times 3$ (upper left) of ${}^0 T_E$