ME4245 Quiz 2 **1 hr – Open Book and Notes** **24 Oct 2015**

1. (30 marks) Figure 1 below shows an N-joint robot carrying a tool. Frame E is attached to the end-effector of the robot and Frame R is attached to the base of the robot. Frame G is attached to the tip of the tool. The relative position and orientation of the tool tip and end-effector is given by ${}^{E}T_{G}$ (homogeneous transformation matrix). Frame U is attached to the ground with ${}^{U}T_{R}$ known. At a certain instant of time, the following are known:

${}^{R}J_{E}$ – full manipulator Jacobian relating the joint velocities of the robot's end-effector (Frame E) with respect to base (Frame R).

${}^{R}T_{E}$, - the homogeneous transformation matrix relating the position and orientation of Frame E (end-effector) with respect to Frame R (robot base).

Determine the expression for the full manipulator Jacobian, ${}^{U}J_{G}$, that relates the joint velocities of the tool (Frame G) with respect to universal frame (Frame U). Express this expression in terms of the known quantities. You need not simplify your answer.

2. (30 marks) The robot in Figure 1 carries a wrench. Frame G is attached to the wrench. ${}^{E}T_{G}$ is known. The robot is used to turn a valve as shown in Figure 2, where the tool (Frame G) is holding the valve. The robot exerts a force of 10 N against the valve (in the opposite direction of the Z axis of Frame G) and a torque of 5 N-m in the counter-clockwise direction (around the Z axis of Frame G). The robot is at a configuration shown in Figure 1, while exerting the force and torque on the valve. At this configuration, the Jacobian ${}^{R}J_{E}$ is known.

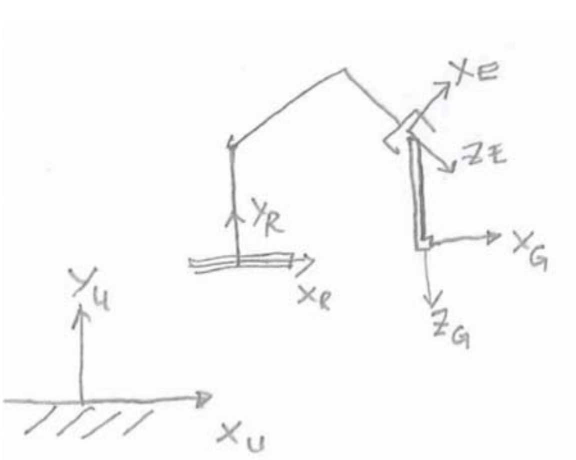


Fig 1

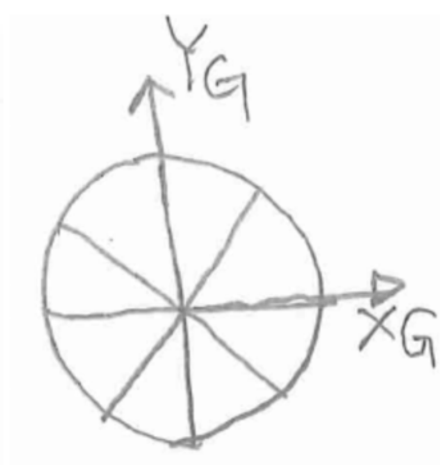


Fig 2

Determine the expression for the joint actuator/forces and torques (τ, N x 1) needed for the robot to exert the required force and torque on the valve. Express this in terms of the known quantities. You need not simplify your expression.

3. (40 marks) Figure 3 shows a robot with 3 joints whose joint coordinates of $q_1 = 0$ (rotational) , $q_2 = 2$ (translational) , and $q_3 = 30°$ (rotational). The arrows indicate the positive directions of motion. At these joint coordinates, the links of the robot are in a vertical plane.

(a) Determine the values of the full manipulator Jacobian, ${}^{U}J_{E}$, that relates the end-effector position and angular velocities with the 3 joint velocities.

(b) The 1st joint is locked in position at the configuration shown in Figure 3. Only the 2nd and 3rd joints are used to position the end-effector along the XY plane of Frame U. Determine the singularities of the robot.

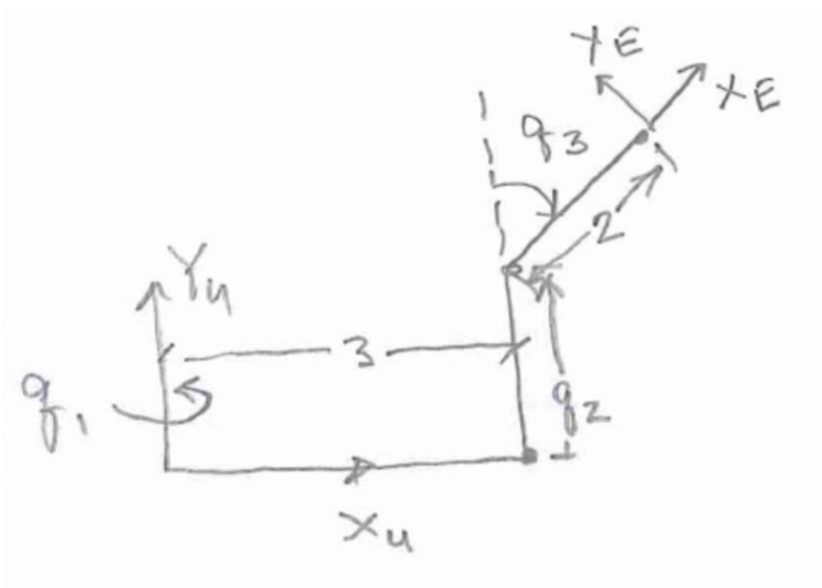
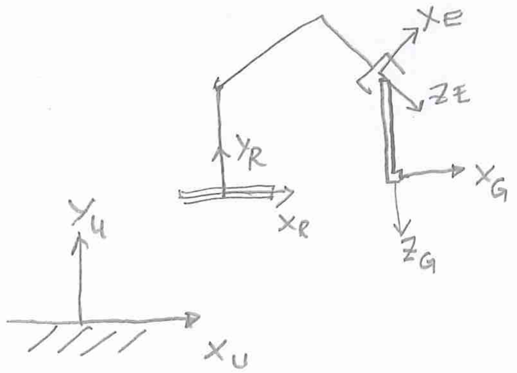


Fig. 3

1.



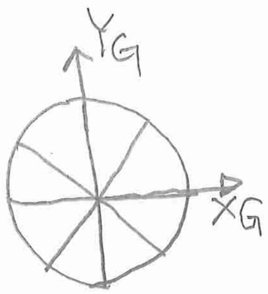
3x3 part of ${}^uT_R$

$$ {}^uJ_E = \begin{pmatrix} {}^uR_R & 0 \\ 0 & {}^uR_R \end{pmatrix} {}^RJ_E $$

$$ {}^uJ_G = \left(\begin{array}{c|c} I & -\widehat{{}^uR_E\,{}^EP_G} \\ \hline 0 & 1 \end{array}\right) {}^uJ_E $$

where ${}^uT_E = \begin{pmatrix} {}^uR_E & {}^uP_E \\ 0 & 1 \end{pmatrix}$ ${}^ET_G = \begin{pmatrix} {}^ER_G & {}^EP_G \\ 0 & 1 \end{pmatrix}$

2.



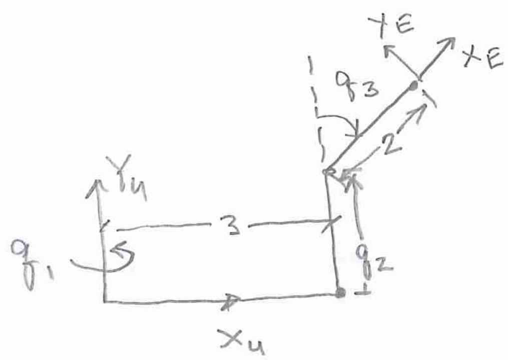
${}^ET_G$ = Known

$$ {}^GF_G = \begin{pmatrix} 0 \\ 0 \\ -10 \\ 0 \\ 0 \\ 5 \end{pmatrix} \qquad \tau = {}^uJ_G^T\,{}^uF_G $$

$$ {}^uF_G = \left(\begin{array}{c|c} {}^uR_G & 0 \\ \hline 0 & {}^uR_G \end{array}\right) {}^GF_G $$

3.

(a)



$$ {}^uJ_E = J_1 \quad J_2 \quad J_3 $$

$$ J_1 = \begin{pmatrix} Y_u \times (P_E - P_0) \\ Y_u \end{pmatrix} \qquad Y_u = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} $$

$$ P_E = \begin{pmatrix} 3 + 2\sin 30^\circ \\ 2 + 2\cos 30^\circ \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 3.73 \\ 0 \end{pmatrix} $$

$$Y_4 \times (P_E - P_0) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 4 \\ 3.73 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix}$$

$$J_1 = \begin{pmatrix} 0 \\ 0 \\ -4 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad J_2 = \begin{pmatrix} Y_4 \\ \\ 0 \end{pmatrix}$$ rotational Joint

$$= \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

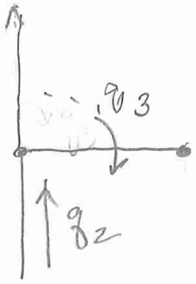
$$J_3 = \begin{pmatrix} Z_2 \times (P_E - P_2) \\ Z_2 \end{pmatrix} \qquad {}^uZ_2 = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \qquad {}^uP_E - {}^uP_2 = \begin{pmatrix} 2\sin 30^\circ \\ 2\cos 30^\circ \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1.73 \\ 0 \end{pmatrix}$$

$$Z_2 \times (P_E - P_2) = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1.73 \\ 0 \end{pmatrix} = \begin{pmatrix} 1.73 \\ -1 \\ 0 \end{pmatrix}$$

$$J_3 = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

$$\therefore J = \begin{pmatrix} 0 & 0 & 1.73 \\ 0 & 1 & -1 \\ -4 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

(b) Singularities (q_2, q_3) only since q_1 is locked



when $q_3 = 90^\circ$

and

$q_3 = -90^\circ$